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Calculation and Estimation of Acoustical Length from Ultrasound Signal for Diffraction Tomography

(초음파 신호로부터의 음장의 계산 및 평가)

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要 約

회절 단층 영상법에서 요구되는 산란데이타의 측정 방법으로써 음장에 관한 정보를 도출함은 매우 중요한 관심사이다.

본 논문에서는 Hilbert 변환을 고려함에 의해 전파시간과 흡수에 관한 정보를 수신된 신호로부터 유도하였다. 또한 이 방법을 확장시켜 초음파 C.T에서 요구되는 투영치의 구성에 관한 방법을 보이고 있다.

마지막으로 본 방법의 유효성을 확인하기위해 행한 계산기 모의실험을 보인다.

Abstract

There are considerabel interests in the use of acoustical lengths for characterization of scattered data required in diffraction tomography. In this paper, we present two new methods, calculation of acoustical lengths by Hilbert transform and estimate on of integrated values on the scan lines from calculated values. These techniques offer insight into the acquisition of projection data in diffraction tomography. The validity of the proposed methods has been confirmed by computer simulation.

I. Introduction

One can determine the spatial distribution of acoustic refractive index within an object from many projections of the object obtained by using ultrasound beams instead of X-rays.

In general, the ultrasonic C.T., based on the assumption that acoustic wave passes straight, employs the X-ray C.T. algorithm, but is inferior to X-ray C.T. due to the error resulting from the scattering of wave.

Because ultrasonic energy does not propagate along straight lines, there is considerable interest in using acoustical lengths to characterize scattered data required in diffraction tomography.

The acoustical lengths are represented by the phase of received ultrasonic signals. Therefore, we investigate the acoustical lengths so as to measure the attenuation coefficient and refractive index.

We introduce two new methods; the first

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is to calculate acoustical lengths by Hilbert transform and the second is to estimate integrated values on the scan lines from calculated values.

These techniques offer insight into the acquisition of projection data in diffraction tomography.

II. Theoretical Details

The basic equation for ultrasound wave propagation [1] is

$$\nabla^2\Psi(\bar{r}) + \frac{\omega^2}{C^2}\Psi(\bar{r}) = 0, \qquad (1)$$

where $\bar{r}$ is a position vector and c is the velocity, $\Psi(\bar{r})$ is the complex valued time independent function.

We define a new function, $\phi(\bar{r})$ such that

$$\Psi(\bar{r}) = \exp\,[\,i K_o \phi(\bar{r})\,]. \qquad (2)$$

Substituting eq. (2) into eq. (1), we obtain

$$iK_o^{-1}\nabla^2\phi - \nabla\phi\cdot\nabla\phi + n^2 = 0, \qquad (3)$$

where n is the refractive index.

Deviding n and ϕ into two parts, we have

$$n = 1 + n_1 \quad , \phi = \phi_0 + \phi_1, \qquad (4)$$

where ϕ_0 is the solution in the unperturbed medium. n_1 and ϕ_1 are randomly varying quantities and their mean values are zero.

Substituting eq. (4) into eq. (3) and neglecting second order terms, we obtain

$$\nabla\phi_0 . \nabla\phi_1 = n_1, \qquad (5)$$

where $\phi_0 = k_0 . \bar{r}$.

Let $\bar{a}$ be an entry point and $\bar{b}$ a received point. Using these variables, eq. (5) can be integrated along a path between a and b.

$$\phi_{1RE}(\bar{b}) - \phi_{1RE}(\bar{a}) = \int_{\bar{a}}^{\bar{b}} n_{1RE}(\bar{r})\,ds$$
$$\phi_{1IM}(\bar{b}) - \phi_{1IM}(\bar{a}) = \int_{\bar{a}}^{\bar{b}} n_{1IM}(\bar{r})\,ds \qquad (6)$$

Where RE and IM indicate the real part and imaginary part.

Eq. (6) can be considered to be two reconstruction formulas [2], one for the speed of sound and one for the attenuation coefficient. Because scattered data required in eq. (6) are acoustical lengths, $\phi_1(\bar{r})$, our attention is given to acquisition of acoustical lengths in next chapter.

III. Calculation of Acoustical Length

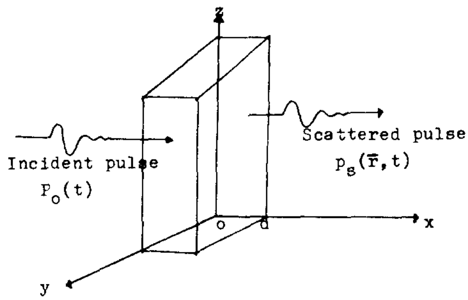


Fig. 1. Bioligical tissue being interrogated by ultrasonic energy.

In order to be explicit, we assume that the random inhomogeneities occur only in the right half-space ($x > 0$) and that the left half-space ($x < 0$) contains no random inhomogeneities. A place wave [3],

$$P_o = A_o e^{-i}(\omega t - K_o x) \qquad (7)$$

advances from the homogeneous to the inhomogeneous medium.

We look for a solution of the wave equation in the right half-space in the form,

$$P_s = A_s(\bar{r})\, e^{-i}\,[\omega t - S(\bar{r})], \qquad (8)$$

where $A_s(\bar{r})$ and $S(\bar{r})$ are unknown functions.

The gist of the method consists in replacing the wave function P_s by another function, $\phi_1(r)$ [4]

$$P_s = Ao\, e^{-i[\omega t - \phi_1(\bar{r})]}, \quad (9)$$

where $\phi_1(\bar{r}) = S(r) - i \log \dfrac{A_s(\bar{r})}{A_o}$

The real and imaginary parts of the function $\phi_1(\bar{r})$ just introduced determine the phase and the logarithm of the ratio of amplitudes, respectively as follows:

$$P_s(\bar{r},t) = A_o e^{-k_o \phi_{1IM}(\bar{r})}\; e^{-i[\omega t - K_o \phi_{IRE}(\bar{r})]}. \quad (10)$$

In practice, the scattered waves are measured as real quantity of pressure. On the other hand, measurements are always real.

$$P_s(r,t) = RE\, [A_o e^{-k_o \phi_{1IM}(\bar{r})}.\; e^{-i[wt - k_o \phi_{IRE(\bar{r})}]} \quad (11)$$

Let $P_s(\bar{r}, t)$ denote an analytic signal [5].

$$P_s(\bar{r},t) = P_s(\bar{r},t)\Big|_{RE} + i\; P_s(\bar{r},t)\Big|_{IM} \quad (12)$$

where, $P_s(\bar{r},t)\Big|_{RE}$ = a real signal of finite energy, and

$P_s(\bar{r},t)\Big|_{IM} = Hi\left[P_s(\bar{r},t)\Big|_{RE}\right]$ = the Hilbert transform of $P_s(\bar{r},t)\Big|_{RE}$.

The imaginary part of analytic signal, $P_s(\bar{r},t)$, can be solved by Hilbert transform of received signal. From eq. (12), we can write the following equation:

$$\phi_{IRE} = -\left[\tan^{-1}\left(\frac{P_s|_{IM}}{P_s|_{RE}}\right) - \omega t\right]$$

$$\phi_{1IM} = -\left[\log(P_s|_{RE}^2 + P_s|_{IM}^2) - \log(A_o^2)\right] / 2K_o. \quad (13)$$

IV. Phase Unwrapping

As defined in the eq. (13), ϕ_{1RE}, as estimated on a digital computer are discontinuous functions [6]. This is due to the fact that arctangent routines in a computer provide only the principal values lying between $-\pi$ and π. Therefore, phase unwrapping is required for the measurement of ϕ_{1RE}.

Let Arg $[\phi_{1RE}(\bar{r})]$ be the principal value and arg $[\phi_{1RE}(\bar{r})]$ be the unwrapping phase.

$$\arg[\phi_{1RE}(\bar{r})] = \text{Arg}[\phi_{IRE}(\bar{r})] + 2\pi \ell_c \quad (14)$$

The phase unwrapping problem amounts to determining the correct integer value ℓ_c. This is carried out by following steps:

We now consider the practical situation, where the principal value and unwrapped phase are available over a discrete set of $\bar{r}$, i.e., $\phi_{1RE}[n]$ are given.

Step : set arg $[\phi_{1RE}[n]]$ = Arg $[\phi_{1RE}[n]]$.

Further, we can write,

$$|\arg[\phi_{1RE}[n]] - \arg[\phi_{1RE}[n-1]] < \pi$$

or equivalently we have

$$|\text{Arg}[\phi_{1RE}[n]] + 2\pi l_2 - \arg[\phi_{1RE}[n-1]]| < \pi. \quad (15)$$

In a like manner, the value of l_c, where c=1.2,3, ..., n, can be obtained. But the above procedure is based on the condition that reference point n satisfying arg $[\phi_{1RE}[n]]$ = Arg $[\phi_{1RE}[n]]$ should be known.

We study the method for the determination of reference point. The time taken to go the distance L is given by the formula.

$$t = \frac{1}{C_o}\int_0^L n(x,y,z)\,dx \quad (16)$$

The mean transit time is

$$t = \frac{1}{C_o}\int_0^L n(x,y,z)\,dx = \frac{1}{C_o}\int_0^L dx, \quad (17)$$

since n=1.

The deviation from the mean is given by

$$\triangle t = t - \bar{t} = \frac{1}{C_O}\int_0^L n(x,y,z)\,dx - \frac{1}{C_O}\int_0^L dx = \frac{1}{C_O}\int_0^L n_1(x,y,z)\,dx.$$

From the equation (18), we can find the distribution of velocity. Therefore, we can determine the reference point in the region that is relatively continuous and consistent with the velocity of biological tissue.

V. Estimation of Projection Data

From the real and imaginary parts of acoustical lengths, we determine the projection data required two reconstruction formulas, one for the speed of sound and another for the attenuation coefficient.

Now we consider an important observation in which the scale variation of the phases and the ratio of amplitudes depend on velocity and attenuation of medium.

The width of each of the scale variation corresponds to the average attenuation coefficient and velocity of propagation on a scan line.

These results based on observation are shown in the following computer simulation.

VI. Computer Simulation

1 Outcome of Simulation Data

(1) Phantom

We assume that the phantom used in simulation is cylinder whose diameter is 20 cm. We have chosen fresh liver tissue as an object. The attenuation coefficient and velocity for the typical soft tissue, that is, fresh liver tissue have the values of approximately 0.1/cm/MHz [7] and 1549m/sec. [8].

(2) Received signal

Received signal can be obtained from Eq. (11) by computing the amplitudes of incident wave and the acoustical lengths. Where the amplitudes A_O's of incident wave are obtained by using transducer whose center frequency is 3.5 MHz.

If refractive index $n_1(\bar{r})$ is

$$n_1(\bar{r}) = n_{1RE}(\bar{r}) - i\,n_{1IM}(\bar{r}), \qquad (19)$$

attenuation coefficient and sound velocity have the relation given in eq. (20).

$$\alpha(\bar{r}) = k_O n_{IM}(\bar{r}),\ n_{1RE}(\bar{r}) = v_O / v(\bar{r}) \qquad (20)$$

Acoustical lengths on received surface are obtained from eq. (6) and (20).

2. Result

The object is insonified by a plane parallel continuous wave. The scattered wave is received along a measured line and its velocity and attenuation are measured. Many $\phi_1(n)$ (n=1, ..., 45) are measured each for a seperate rotation of the object, Θ_n. (Fig. 2) The image reconstruction is done by using the filtered backprojection algorithm.

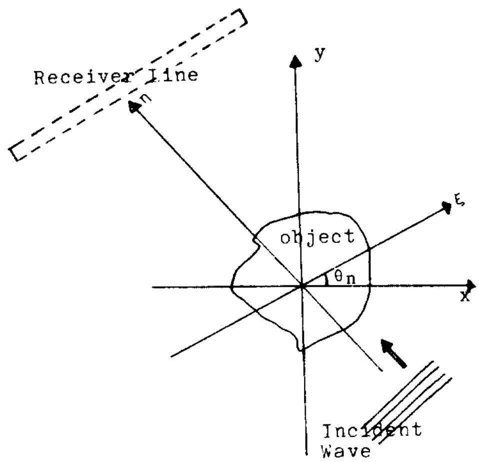


Fig. 2. Coordinate system.

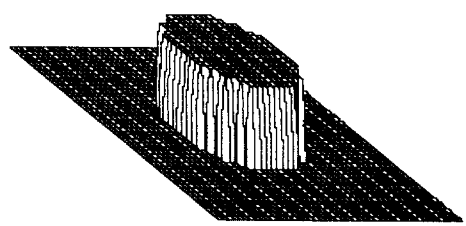


Fig. 3. Phantom (α=0.1/cm/MHz, v=1549m/sec.)

We performed the computer simulation by letting the number of pixels on the reconstructed image be 46x46, with the assumption that the refractive index and the attenuation coefficient per a pixel are 0.955 and 0.35 with the consideration of the cylinder-shaped object.

When we get the values of the right hand side of eq. (13), it is required to get the values of imaginary part. It is obtained by the convolution of the received signal and Kernel, $1/\pi t$, in the discrete frequency domain.

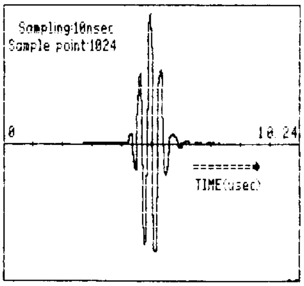


Fig. 4. Transmitted signal.

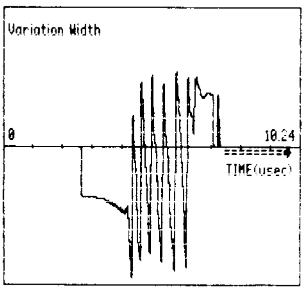


Fig. 5. Variation of real part (ϕ_{1RE} = 0.855).

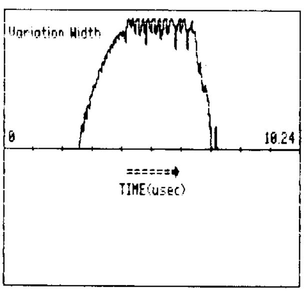


Fig. 6. Variation of imaginary part (ϕ_{1IM}= 6.65).

The waveforms obtained by eq. (13), when the value of the real part and the imaginary part of acoustical length are 0.855 and 6.65, are seen in the Fig. 5 and 6.

In the Fig. 7 and 8, the projection data obtained by this method are compared with the ideal projection data.

In the Fig. 9-b and 11-b, the reconstructed values which is calculated by filtered back-projection algorithm, with the measured projection data, are represented in the three dimension.

In the Fig. 10-b and 12-b, the cross-sectional profiles of reconstructed images, with the measured projection data, are represented.

We have tested the proposed algorithms in computer simulations. On the refractive index, the distribution of the reconstructed values is hindered from blurring, by applying the method presented in this paper, but the values in the object space are a little far from desired values. Meanwhile, on the attenuation coefficient, not only the distribution of the reconstructed values is hindered from blurring, but also good values in the object space can be obtained. But it is susceptible to the noise.

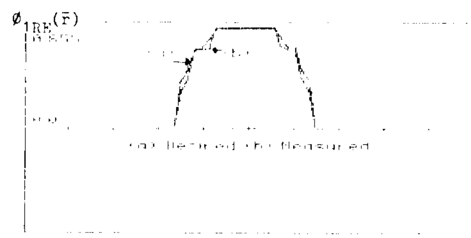


Fig. 7. Projection data of refractive index distribution.

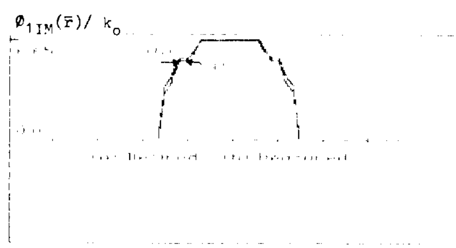
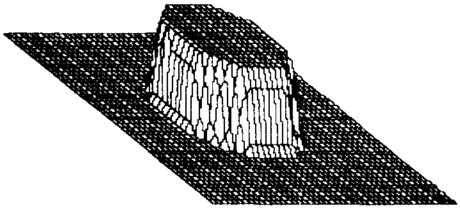
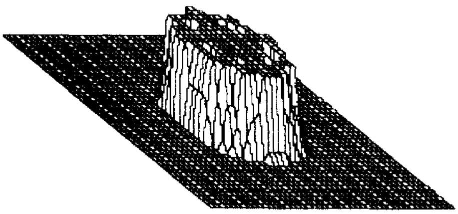


Fig. 8. Projection data of attenuation coefficient distribution.

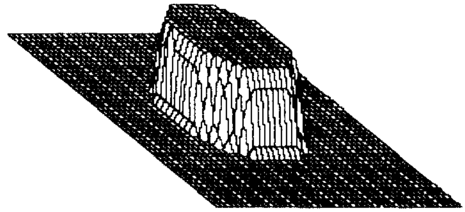


a) Reconstructed image using desired projection data.

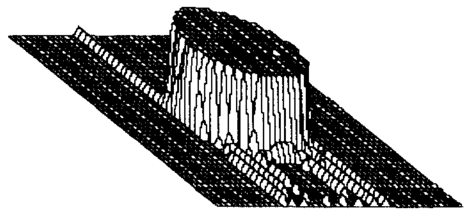


b) Reconstructed image using measured projection data.

Fig. 9. Reconstructed images of refractive index distribution.



a) Reconstructed image using desired projection data.



b) Reconstructed image using measured projection data.

Fig. 11. Reconstructed images of attenuation coefficient distribution.

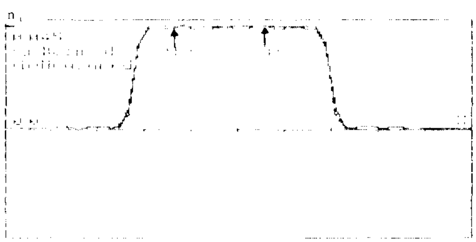


Fig. 10. Center - line profile of reconstructed refractive index distribution.

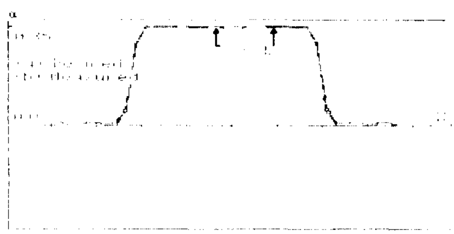


Fig. 12. Center - line profile of reconstructed attenuation coefficient distribution.

The above problems, when the propagation distance is short, result from the small variations of amplitude and refractive index in the time domain.

To solve the problems above, the research on image reconstruction algorithm for diffraction tomography should be added.

The procedure of obtaining solutions of the real and imaginary parts of n was investigated by Greenleaf and Johnson [9], but it possibly becomes an unstable process since a process of deconvolution is involved.

Recently, Greenleaf [2, 10] dealt with the problems about data acquisition, but had some problems in filtering out high frequency terms.

On the contrary, the method presented in this paper was designed to solve the problems arising from deconvolution process and filtering by applying Hilbert transform.

VII. Conclusion

We have studied the problem about the calculation of acoustical length from scattered wave. The results indicate that the problem of profile construction could be solved by values derived from variation widths. It, however, seems that some problems are induced in the case of refractive index if the propagation distance is small.

In the present method, we assumed that the incident wave is a plane wave and each of distribution of refractive index and attenuation coefficients is homogeneous. But, the improved result will be obtained by taking into account multipath effect and the effect on amplitude induced by the refraction.

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