論 文

# 환상대칭 2차원FIR 디지탈필터의 설계

準會員 李 慶 哲* 正會員 姜 哲 豪**

# Design of Circular Symmetric Two-Dimensional FIR Digital Filter

Kyoung Cheol LEE* Chul Ho KANG***Regular Members*

**要 約** 변환을 이용한 2차원 **zero-phase FIR** 디지탈 필터의 설계는 인기가 있고 잘 발전된 기술이나, 현재 설계된 4상한 대칭 필터의 형태는 원점에서 원거리의 주파수에서 정확하게 환상 대칭이 되지 못하는 단점을 가지고 있다. 적절한 급수의 확장 방법에 의한 변환을 이용하여 어떻게 다차원 필터에 대해 임의의 정확성을 갖는 대칭의 필요성이 만족되게 할 수 있는가를 보인다. 이러한 변환 방법으로써 일반적으로 **McClellan** 변환이 알려져 왔다. 본 논문에서는 원점으로부터 원거리의 주파수에서 훌륭한 환상 대칭이 되는 간단한 방법을 제시하였다. 이 방법은 1차원 필터로부터 2차원 환상대칭FIR 디지탈 필터의 설계를 위해서 변형된 **McClellan** 변환을 이용하는 것으로써 **band-pass** 필터와 같은 다수의 **cut-off** 영역을 갖는 2차원 FIR 필터의 설계에 매우 유용하다.

***ABSTRACT*** The design of two-dimensional zero-phase FIR digital filters by transformations is a popular and well-developed technique, but it suffers from the disadvantage that in its current form the filters with only four quadrant symmetry can not be designed exactly circular symmetry at frequencies far from the origin. It is shown how symmetry requirement for the filters with any number of dimensions can be fulfilled with arbitrary precision by adapting transformation with the appropriate series expansions. This transformation has been known generally as the McClellan transformation. In this paper, a simple method for ensuring excellent circular symmetry at frequencies far from the origin is presented. This method which employs the Modified McClellan tansformation for the design of 2-D FIR filter from 1-D FIR filter is quite useful for the design of 2-D FIR filters with multiple cutoff boundaries such as band-pass filters.

## I. INTRODUCTION

An FIR filter is one whose impulse response posesses only a finite number of nonzero samples. For such a filter the impulse response is always absolutly summable and thus FIR filters are always stable.

The design of two-dimensional digital filters have been of great interest lately because of many applications in the area of two-dimensional digital signal processing.

In the case of computer image processing, for example, they are used for the enhacement of low quality images such as X-ray photographs and for the compensation of linear optimal degra-

*, ** 光云大學電子通信工學科
Dept. of Electronic Telecommunication Engineering, Kwangwoon University, Seoul. 132 Korea.
論文番號 : 86-36 (接受 1986. 9. 5)

dations. They are also used for preprocessing data in 2-D pattern recognition system[(1),(12)]. They have found use in nonlinear filtering strategies for dynamic range reduction, contrast enhancement and prefilters in various schemes for picture encoding. 2-D digital filters are also becoming more useful in the processing of sequential data derived from multidimensional arrays as encountered in geophysical exploration, earthquake/nuclear test detection, sonar, radar and radio astronomy[(7)]. For these applications, the filters with linear or zero-phase are often required[(2)]. Such phase characteristics are easily achieved with finite impulse response (FIR) digital Filters.

Several design techniques for two-dimensional linear-phase FIR digital filters are well established. Perhaps the simplest technique with the shortest design time is the windowing methods[(3)].

However, this technique has a few disadvantages. In most cases the resulting filter is not optimal in any sense. Also two-dimensional windows are not well-behaved or well-understood as much as one-dimensional windows. Other methods are frequency sampling[(4)], linear programming and multiple exchange techniques[(5)].

If linear-phase is desired, these three techniques will generally produce a better approximation to an ideal response for a filter of given order but design time required by these algorithms may be tedious. On the other hand, the McClellan transformations not only enjoy a short design time, but also have efficient implementations[(6),(7)].

Transfer functions of two-dimensional filters are often required to exhibit circular symmetry in terms of the spatial frequency variables. It is necessary to ensure that a circularly symmetrical input signal always lead to a circularly symmetrical output signal, more generally, the rotating an arbitrary input signal does not produce any other effect than rotating the corresponding output signal.

In the case of 2-dimensional digital filters, the symmetry requirement still holds inside the basic frequency square which defines the 2-dimensional periodicity of the frequency transformation. A simple and successful method of designing 2-D FIR filters have been to apply a 1-D to 2-D transformation on the 1-D axis.

In this paper, a modification of the transformation which is more circular symmetric than the McClellan transformation is reported. An example of this method is described as a Modified transformation to 2-D linear-phase FIR filters with circular symmetric filter specifications. And it is shown that the filters designed by this new method can be implemented efficiently by computer simulation.

## II. FREQUENCY TRANSFORMATION

### A. Multidimensional Transformation

Let $W$, $W_1$, and $W_2$ be the frequency variables of the one-dimensional and the two-dimensional digital filter, respectively. For optimal circular summetry, the McClellan transformation[(7),(8),(9)] is given by

$$\cos W = (\cos W_1 + \cos W_2 + \cos W_1 \cos W_2 + \cos W_1 \cos W_2 - 1)/2 \qquad (1)$$

Instead of this expression, we shall use the following one;

$$\cos(W/2) = \cos(W_1/2)\cos(W_2/2) \qquad (2)$$

By squaring this last expression and replacing $\cos^2(W/2)$ by $(1+\cos W)/2$ (1) can be seen to follow as (2). Clearly, (2) has a simpler form than (1)and the essential properties of (1) are easily derived from (2).

For our purpose the most interesting aspect about (2) is the fact that it can immediately be generalized to any number of dimensions. Thus, if the number of dimension is n, we replace (2) by

$$\cos(W/2)=\prod_{i=1}^{n}\cos(W_i/2) \tag{3}$$

Squaring this expression again and applying the same type of substitution as before, we now obtain

$$\cos W=-1+2^{1-n}\prod_{i=1}^{n}(1+\cos W_i) \tag{4}$$

which generalized(1).
We still want to show that (3) indeed possesses all the required properties. For this, we observe first that the basic domain

$$-\pi\leqq W_i\leqq\pi \qquad i=1,2,3,\cdots,n \tag{5}$$

is transformed into the basic range

$$-\pi\leqq W\leqq\pi \tag{6}$$

and that, for W satisfying (6), we have $W=0$ if $W_i=0$ for all $i=1$ to n, and $W=\pm\pi$ as soon as a single one of the $W_i$ is equal to $\pm\pi$. Furthermore, within the basic range (5) and (6) the mapping is convex, i.e. such that $|W|$ increase monotonically from 0 to $\pi$ if all $|W_i|$ increase monotonically from 0 to $\pi$. Finally, if we replace all cosine terms in (3) by their power series expansions and neglect all terms of order 4 and higher, (3) reduce to

$$W^2=W_1^2+W_2^2+\cdots+W_n^2 \tag{7}$$

This shows that the desired circular symmetry is obtained at least in neighberhood of the origin.

### B. McClellan Transformation

The McClellan transformation is a transformation of 1-D zero-phase FIR filter into the 2-D zero-phase FIR filter by the substitution of variables. It can be applied to 1-D filter of odd length and in special cases also to filters of even length.

For a 1-D filter of length 2N + 1 to be zero-phase, its impulse response h(n) must have Hermitian symmetric coefficients. Thus if h(n) is real, it must also be even. Denoting the frequency response by $H(e^{jW})$, we can thus write

$$H(e^{jW})=h(0)+\sum_{n=1}^{N}h(n)[e^{-jWn}+e^{jWn}]$$
$$=h(0)+\sum_{n=1}^{N}2h(n)\cos W_n \tag{8}$$

2-D digital filters which have frequency response of the form

$$H(e^{jW1},e^{jW2})=\sum_{m=0}^{M_1}\sum_{n=0}^{M_2}a(m,n)\cdot\cos mW_1\cos nW_2 \tag{9}$$

are examples of 2-D zero-phase filters. The impulse response of this system h(m, n) is a real frequency which is an even function of each of its arguments, namely.

$$a(0,0)=h(n,n) \tag{10a}$$
$$a(0,n)=2h(n,n-n) \qquad n=1,2,\cdots,n \tag{10b}$$
$$a(m,0)=2h(n-m,n) \qquad m=1,2,\cdots,n \tag{10c}$$
$$a(m,n)=4h(n-m,n-n) \qquad m=1,2,\cdots,n,\ n=1,2,\cdots,n \tag{10d}$$

The general McClellan transformation converts 1-D filters of the form (8) into 2-D filters of the form (9) by means of the substitution.

$$\cos W = \sum_{p=0}^{P} \sum_{q=0}^{Q} t(p,q) \cos(pW_1) \cos(qW_2) \quad (11)$$

where $P = Q = 1$

The relation between h(n) and h(m, n) can be seen by rewriting (8) in the form

$$H(e^{jW}) = \sum_{n=0}^{N} b(n) [\cos W]^n \quad (12)$$

and performing the substitution indicated in (11)

$$H(e^{jW_1}, e^{jW_2}) = \sum_{n=0}^{N} b(n) \cdot \left[ \sum_{p=0}^{P} \sum_{q=0}^{Q} t(p,q) \cos(pW_1) \cos(qW_2) \right]^n \quad (13)$$

The coefficients of (11) is chosen for circular contours in the original first order McClellan transformation. This has P = Q = 1 and

$$-t(0,0) = t(1,0) = t(0,1) = t(1,1) = 1/2 \quad (14)$$

This choice of cofficients yields nearly circular contours for low values of W and increasing square contours for large value of W. Thus, the McClellan transformation is quite useful for the design of low or high pass filters with small cutoff radius. However, if either a large cutoff radius or a broad band-pass filter is desired, the McClellan transformation is not quite suitable since it is not capable of providing circular contours in the transition region. A solution to this problem suggested by Mersereau et al,(7) is to determine the transformation coefficients using constrained approximation technique. One can use a higher order transformation. Again, the coefficients of the transformation are obtained using a constrained approximation or an optimization technique. The use of a higher order transformation will naturally result in a higher order filter to take a long design time.

## III. MODIFIED McCLELLAN TRANSFORMATION

The McClellan transformation has been given in terms of cosine function(7), (8).

For applications involving circular symmetry about the origin, it will be convenient if we rewrite the transformation in terms of sine function. Since $\sin\theta \simeq \theta$ for small value of $\theta$, and $\sin\theta$ increases monotonically as $\theta$ increase from zero to $\pi/2$, it will be easier to visualize the mapping. Substituting $\cos W = 1 - 2\sin^2(W/2)$ in (12) and simplifying the resulting equation, we get

$$H(e^{jW}) = \sum_{n=0}^{N} b(n) \left(1 - 2\sin^2(W/2)\right)^n \quad (15)$$

It can be shown as sine polynominal

$$H(e^{jW}) = \sum_{n=0}^{N} b'(n) \left(\sin^2(W/2)\right)^n \quad (16)$$

The Modified transformation which converts 1-D filter into 2-D filter with quadrantal symmetry can be written as

$$\sin^2(W/2) = \sum_{p=0}^{P} \sum_{q=0}^{Q} t(p,q) \sin^2(W_1/2) \cdot \sin^2(W_2/2) \quad (17)$$

and in a general case

$$\sin^2(W/2) = t(0,0) + t(1,0)\sin^2(W_1/2) + t(0,1)\sin^2(W_2/2) + t(1,1)\sin^2(W_1/2)\sin^2(W_2/2) \quad (18)$$

All the coefficients of the transformation t(p, q) can be determined by optimization; however, this will be time consuming, and so certain known restrictions may be imposed on the transformation such that the number of unknowns may be reduced. Some of the restrictions are; 1) the origin (0,0) of the ($W_1$, $W_2$) plane is mapped on to the origin W = 0 of the W axis, 2) the response on the $W_1$ axis is identical to the response on the W axis, 3) the response of the $W_2$ axis is identical to the response on the $W_1$ axis, 4) the response at ($\pi$, $\pi$) is equal to the response at W = $\pi$

We next study the effect of the above constraints that the restrictions 1) - 4) listed above results in the conditions (19a) - (19d) as follows:

1) t(0,0) = 0 (19a)
2) t(1,0) = 1, t(0,0) = 0 (19b)
3) t(0,1) = t (1,0) (19c)
4) t(0,0) + t(1,0) + t(0,1) + t(1,1) = 1 (19d)

If the transformation (18) satisfies all four constraints mentioned above, we get t(0,0) = 0, t(1,0) = t(0,1) = - t(1,1) = 1, i.e. the Modified transformation becomes the original McClellan transformation for circular symmetry from (18):

$$\sin^2(W/2)=\sin^2(W_1/2)+\sin^2(W_2/2)-\sin^2(W_1/2)\sin^2(W_2/2) \quad (20)$$

To show a more circular symmetry than that of the McClellan transformations, we introduce the 2-D frequency plane equal to the response of the 1-D function at $\pi$. Then (20) becomes

$$\sin^2(W/2)=\sin^2(W_1/2)+\sin^2(W_2/2)+t(1.1)\sin^2(W_1/2)\sin^2(W_2/2) \quad (21)$$

Thus, the response of 2-D FIR filter is the following equation.

$$H(e^{jW_1}, e^{jW_2})=\sum_{n=0}^{N} b(n)\cdot\left[1-2\sum_{p=0}^{P}\sum_{q=0}^{Q} t(p,q)\sin^{2p}(W_1/2)\sin^{2q}(W_2/2)\right]^n \quad (22)$$

In (21), when $W_1$ = 0 ($W_2$ axis) $\sin^2(W/2) = \sin^2(W_2/2)$, so the response for $W_2$ is the same as for W. Similarly when $W_2$ = 0 ($W_1$-axis) $\sin^2(W/2) = \sin^2(W_1/2)$, so the response for $W_1$ is the same as for W. In other words the responses at $W_1$- and $W_2$-axis are identical to the response at W-axis. When ($W_1$, $W_2$) is located such that the point is not on $W_1$-axis or on $W_2$-axis, then $\sin^2(W/2)$ depends on t(1,1). Hence for the same point ($W_1$,$W_2$), the W value depends on t(1,1) following the frequency response at ($W_1$,$W_2$), which will be the response of the 1-D prototype at W depends on t(1,1). Now we say different values of t(1,1) gives different responses outside the axis on the 2-D plane.

In the proposed design technique, the optimum value of t(1,1) is determined to mimimize the error of the frequency response in the entire frequency domain, and hence the overall response at frequency plane will approximate of circular symmetry for 2-D FIR filter.

## IV. RESULT OF COMPUTER SIMULATION

### A. Contour Problem

The change of variables (11), (18) defines a mapping from the interval [0,0.5] of the 1-D frequency axis to the square [0,0,5]x[0,0.5] in the 2-D frequency plane. In order to be able to use the change of variables to produce a desired frequency response (i.e., pick t(0,0), t(1,0), t(0.1), and t(1,1)), it is necessary to describe the mechanism of the transformation (11),(17). Solving (11),(17) for $W_2$ as a function of $W_1$:

$$W2 = F(W, W_1, t(0,0), t(1,0), t(0,1))$$

$$= \text{ARCCOS}\left[\frac{\cos W - t(0,0) - t(1,0)\cos W_1}{t(0,1) + t(1,1)\cos W_1}\right]$$

$$W2 = F(W, W_1, t(1,1)) \tag{23}$$

$$= \text{ARCSIN}\sqrt{\frac{\sin^2(W/2) - t(0,0) - t(1,0)\sin^2(W_1/2)}{t(0,1) + t(1,1)\sin^2(W_1/2)}} \tag{24}$$

In (23) Mersereau et al,(7) have obtained contours by the other three variables with the exception of t(1,1) in (14). However, in (24), mapping from 1-D to 2-D depends on only the variable t(1,1) which was discussed in the preceding chapter.

It is easy to see that for a fixed W we get a curve in the $(W_1, W_2)$ plane, and along this curve the transformed 2-D frequency response is constant equal to the value of the 1-D frequency response at W. As W varies we get a family of contours which completely describes the transformed 2-D frequency response.

The contrours of equation(23), (24) are shown in Fig.1 and 2.

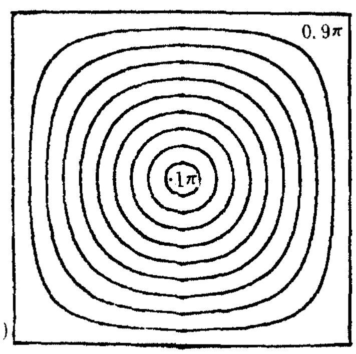


Fig. 1 Contours of constant value for the McClellan transformation of the first order with $-t(0,0) = t(1,0) = t(0,1) = t(1,1) = 1/2$.
The contours are shown for values of W increment of $0.1\pi$.

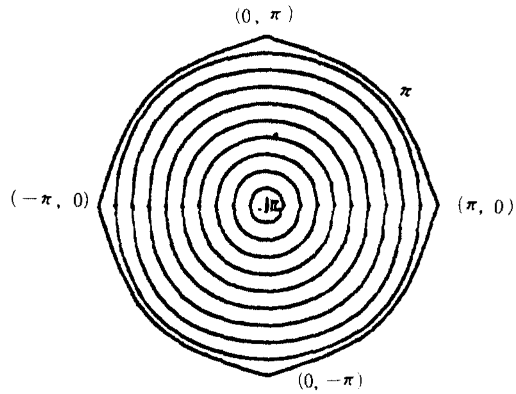


Fig. 2 Contours of constant value for the Modified transformation of the first order with $t(0,0)=0$, $t(1,0)=t(0,1)=1$, $-t(1,1)=0.9$.
The contours are shown for values of W in increment of $0.1\pi$.

## B. Simulation

The VAX 11/750 computer and IBM-PC have been used to implement the proposed design technique and compare it with the design techniques of[(7),(8),(9)].

In order to compare the frequency response of the original McClellan transformation with that of the Modified transformation which are both far from the origin, the prototype of 9 points 1-D band-pass filter is designed similarly to multi-band-pass filter by use of window technique[(10)].

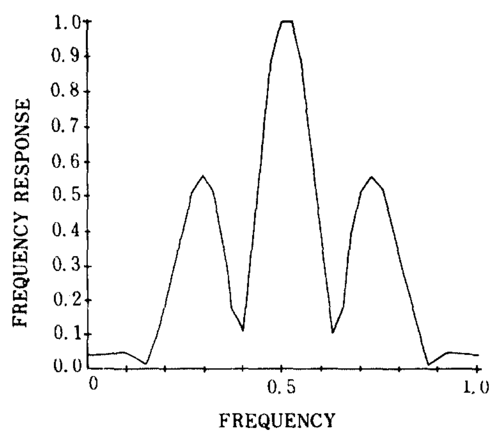


Fig. 3 Frequency response of 9 points 1-D filter.

This 9 points frequency response of 1-D filter from (15) is shown as Fig.3.

Because the contour curve can be defined by t(1,1) value in the Modified transformation, the contour error is minimum at t(1,1) = 0.9.

The error function can be described as the following equation in each frequency(11).

$$E = (W' - W) / W$$

$$\text{where } W' = (W_1^2 + W_2^2)^{\frac{1}{2}} \tag{25}$$

We regard the circular symmetry in center of the origin as an ideal case.

In the original McClellan transformation and the Modified transformation, the errors for each frequency are shown in Fig.4.

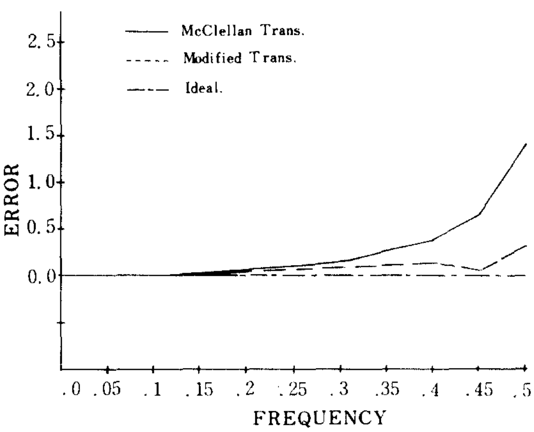


Fig. 4 The errors of the first order McClellan transformation, and the first order Modified transformation.

As seen in Fig.4, we recognize that the error of the Modified transformation at each frequency is smaller than that of the McClellan transformation. Furthermore, we are able to examine the all contour errors. We reduced the error considerably to about a quarter with that of the McClellan transformation being 1.637 and the Modified one being 0.393. This reduction has improved the shapes of the contours into nearly circular ones throughout the 2-D plane as shown in Fig. 1 and 2.

Fig.5 shows the error at each variable t(1,1) value. The error at t(1,1) = 1 is corresponding to the error of the McClellan transformation which is similar to the error about t(1,1) = 0.3, as shown in Fig.5. Besides, the error is minimum at t(1,1) = 0.9.

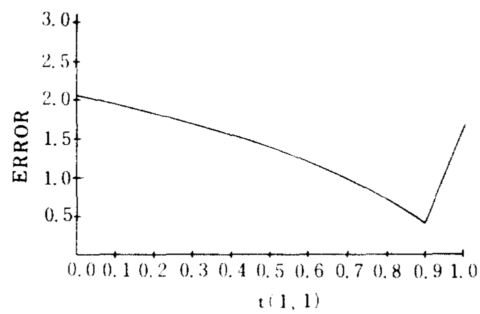


Fig. 5 The errors for each variable t(1, 1) value.

The three dimensional graphics seen from above are shown in Fig.6 and 7 whose magnitude is reduced to one third such that we may see the controur form. The McClellan transformation is

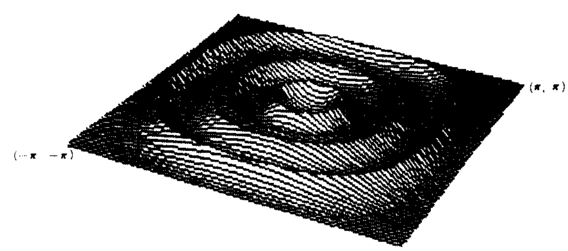


Fig. 6 Three dimensional graphic of the McClellan transformation.

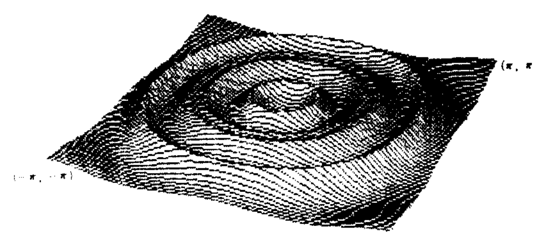


Fig. 7 Three dimensional graphic of the Modified transformation.

shown in Fig.6 and the Modified transformation in Fig.7.

Fig.8 shows an example of 11x11 points circular symmetric FIR band-pass filter designed by adapting the Modified transformation with following specifications:

$0.4\pi \leq r \leq 0.6\pi$ : pass band

$0.0\pi \leq r \leq 0.2\pi$ : stop band

$0.8\pi \leq r \leq 1.0\pi$ : stop band

where $r=(W_1^2+W_2^2)^{\frac{1}{2}}$

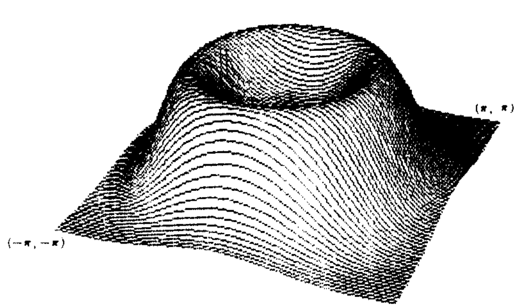


Fig. 8 Three dimensional graphic for the frequency response of 11×11 points band-pass filter by adapting the Modified transformation.

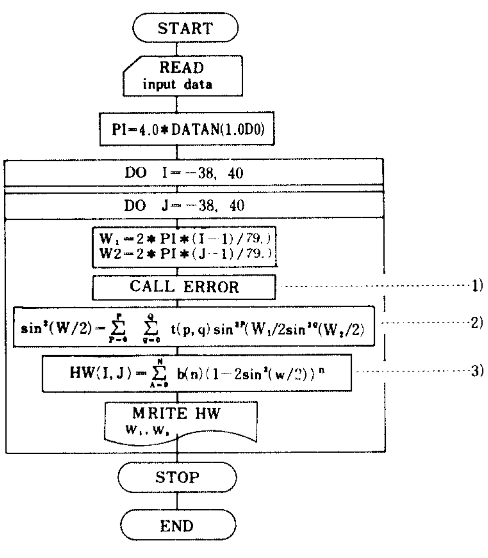


Fig. 9 Flow chart of the Modified transformation.

## C. Flow Chart of the Modified Transformation

The variable t(1,1) value can be minimized by the error which was found by 1).

The contour form can be defined by 2).

The impulse response of 2-D circular symmetric zero-phase FIR digital filter is obtained from 3).

## V. CONCLUSION

The application of the Modified transformation technique for the design of the 2-D FIR digital filters has been proposed. This technique results in almost circular symmetric response in overall 2-D frequency plane compared with the McClellan transformation method. And 1-D FIR prototype can be extended to 2-D FIR digital filters adapting the Modified transformation. Besides, the odd sides (2N+1x2N+1) 2-D FIR digital filter can be designed by simple phase adjustment.

This circular symmetric 2-D zero-phase FIR digital filter will apply to the two-dimensional digital signal processing, for example, computer image processing, sonar, radar, etc..

## 參 考 文 獻


(1) A. V. Oppenheim, Ed., Applications of Digital Signal Processing., Englewood Cliff, Prentice-Hall, NJ: 1974.

(2) T. S. Huang, J. W. Burnett, and A. Gdeczky, "The importance of phase in image processing filters," IEEE Trans. Acoust., Speech, Signal, Processing, vol. ASSP−23, pp. 529−542. Dec. 1975.

(3) T. S. Huang, "Two-dimensional windows," IEEE Trans. Audio and Electroacoust., vol. AU−20, pp. 88−90, Mar. 1972.

(4) J. V. Hu and L. R. Rabiner, "Design techniques for two-dimensional digital filters," IEEE Trans. Audio and Electroacoust., AU−20, pp. 249−257, Oct. 1975.

(5) Y. Kamp and J. P. Thirn, "Chebyshev approximation for two-dimensional nonrecursive digital filters," IEEE Trans. Circuits and Systems, vol. CAS−22. no. 3, pp. 208−218, Mar. 1975.

(6) W. F. G. Mecklenbrauker and R. M. Merseau, "McClellan transformations for two-dimensional digital filtering: Part Ⅱ. Implementation," IEEE Trans, Circuits and Systems., vol. CAS－23, pp. 414－422, July 1976.

(7) R. M. Mersereau and W. F. G. Mecklenbrauker, and T. F. Quatieri. Jr., "McClellan transformations for two-dimensional digital filtering: Ⅰ－Design," IEEE Trans. Circuits and Systems., vol. CAS－23, no. 7, pp. 405－414, July 1976.

(8) J. H. McClellan, "The design of two-dimensional digital filters by transformations, in Proc. 7th Annu. Princeton Conf. Inform. Sci. Syst., pp. 247－251, 1973.

(9) L. R. Rabiner and B. Gold, "Therory and implementation of FIR digital filters," Ch. 3 in Theory and Application of Digital Processing, Prentice-Hall Englewood Cliffs, NJ: 1975.

(10) Digital Signal Processing Committee, "Program for Digital Signal Processing," IEEE, Inc. NY: 1979.

(11) A. Fettwis, "Symmetry Requirements for multidimensional digital filters," Circuit Theory and Applications, vol. 5, pp. 343－353, 1977.

(12) Dane E. Dudgeon and R. M. Mersereau, "Design and Implementatation of two-diensional FIR filters," Ch. 3 in Multidimensional Digital Signal Processing, Prentice-Hall, Englewood Cliffs, NJ. 1984.



**李 慶 哲**(Kyoung Cheol LEE) 準會員

1961年 9月18日生

1985年 2月：光云大學電子通信工學科卒業

1985年 3月－現在：光云大學 大學院 在學中



**姜 哲 豪**(Chul Ho Kang) 正會員

1952年 2月 5日生

1975年 2月：漢陽大學校電子工學科卒業

1979年 2月：서울大學校大學院電子工學科 卒業

1977年～1982年 2月：國方科學硏究所(硏究員)

1982年 3月～現在：서울大學校大學院電子工學科(博士 課程 修了)

1983年 3月～現在：光云大學電子通信科(助教授)