# Dynamic Shear Modulus and Damping Ratio of Soft Clay

軟弱粘土의 動力學的 剪斷彈性係數 및 減衰比

#### Abstract

Considering the effects of confining pressure, initial shear stress, cyclic stress ratio and number of loading cycles, cyclic triaxial tests are carried out to clarify the soil dynamic properties such as shear modulus and value of material damping of clay under undrained cyclic loading conditions.

The results show that no obvious dependency on initial shear stress and effective confining stress are recognized in the shear modulus and damping ratio plotted versus strain. However, the shear modulus decreases and the damping ratio increases with increasing axial strain. When compared with others, it is also revealed that the shear moduli are distributed within the range curves obtained using empirical equations derived by Marcuson et al. (3) and Kokusho et al. (4), and damping ratios are distributed between the curves obtained by Kokusho et al. (4) and Ishihara et al. (9).

#### 要 旨

有效拘束壓力,初期剪斷應力,應力比 및 反復回數等의 影響을 考慮하면서 非排水條件下에 있는 軟弱粘土試料의 動力學的 剪斷彈性係數 및 減衰比의 變化特性을 把握하기 위하여 一連의 三軸壓縮試驗을 遂行하였다.

그 結果, 初期剪斷應力 및 拘束壓力이 動力學的 土性値에 미치는 影響은 크지 않았지만, 軸方向變形의 增加에 따라 剪斷彈性係數는 減少하고 減衰比는 增加하는 傾向을 나타냈다. 또한, 剪斷彈性係數는 係數는 Marcuson et al.<sup>(3)</sup>과 Kokusho et al.<sup>(4)</sup>에 의해 提案된 經驗式으로 얻어진 범위내에 分布되었고, 減衰比는 Kokusho et al.<sup>(4)</sup>과 Ishihara et al.<sup>(9)</sup>에 의해 얻어진 범위에 分布됨을 보이고 있다.

#### 1. Introduction

A considerable amount of experiments on the cyclic behavior of soft Bangkok clay has been performed by employing one-way cyclic loads,

that is shear stress application only on the triaxial compression side, to undisturbed specimen. However, two-way cyclic loading which generates both compression and extension stress conditions in the triaxial test is more adequate to simulate the stress conditions of a soil element

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subjected to earthquake waves.

The field of this study, therefore, will be narrowed down to the use of two-way cyclic loading under undrained condition on undisturbed Bangkok clay obtained from AIT soil testing site near the test embankment. This study aims to clarify the soil dynamic properties such as shear modulus and value of material damping of clay under undrained cyclic loading conditions considering the effects of confining pressure, initial shear stress, cyclic stress ratio and number of loading cycles.

#### 2. Literature Review

The cyclic behavior of saturated clay has been studied by many researchers on such influencing factors as strain rate and number of load cycles etc. Also many experimental studies have been performed to determine the shear modulus and damping ratio in order to solve dynamic problems of clay grounds.

#### 2.1 Structural Model for Cyclic Loading

Usually the stress-strain relationship in the virgin loading or initial loading may be expressed by mathematical formulae and the curve described is called the skeleton curve or backbone curve. Fig. 1 represents the typical stress-strain relationship during cyclic loading whose skeleton curve is expressed by

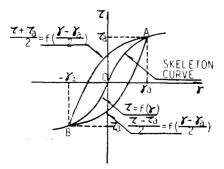


Fig. 1. Structure of Model for Cyclic Loading

where,  $\tau$  and  $\gamma$  are shear stress and shear strain, respectively. In this figure, if loading reversal is considered at point A where  $\gamma = \gamma_a$  and  $\tau = \tau_a$ , then the equation of the stress-strain curve during subsequent unloading from the reversal point is given by

$$\frac{\tau - \tau_a}{m} = f\left(\frac{\gamma - \gamma_a}{m}\right) \dots (2)$$

The above equation is called Masing rule, and usually m=2 is adopted. The hysteresis loop may be obtained by enlarging the skeleton curve by a factor of  $m^{(1)}$ .

#### 2.2 Secant Shear Modulus, G

When the stress-strain relationship during cyclic loading is expressed as a function of strain amplitude, the following formula may be employed:

$$G = \frac{\tau_a}{\gamma_a} = \frac{f(\gamma_a)}{\gamma_a} \quad \dots (3)$$

where, G is the secant modulus at a given strain level,  $\tau_a$  and  $\gamma_a$  denote the amplitude of shear stress and shear strain, respectively. From another point of view, the shear modulus, G is expressed as a function of the shear strain amplitude or the shear stress amplitude.

#### 2.3 Damping Ratio, D

Damping ratio may also be expressed as a function of the shear strain amplitude or the shear stress amplitude. As can be seen in Fig.

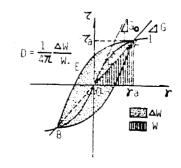


Fig. 2. Definition of the Secant Shear Modulus, G, and Damping Ratio, D.

2, damping ratio is usually defined by

$$D = \left(\frac{1}{4\pi}\right)\left(\frac{\Delta W}{W}\right) \cdots (4)$$

where,  $\Delta W$  is the damping energy, i.e., the area within the hysteresis loop shown in Fig. 2, and W is the equivalent energy defined by:

$$W = \frac{1}{2} f(\gamma_a) \gamma_a \quad \cdots \qquad (5)$$

## 2.4 Initial Shear Modulus at Small Strains

Hardin, et al. (2) suggested an empirical formula for the initial shear modulus as follows:

$$G_0 = 3270 \frac{\{(2.97) - e\}^2}{1 + e} (\sigma_0')^{0.5} \cdots (6)$$

where  $G_0$  and  $\sigma_0'$  are expressed in terms of kN/m<sup>2</sup>. This formula is applicable only for clays of low plasticity or relatively stiff clays having void ratios of approximately  $e=0.6\sim1.5$ . Marcuson et al. (3) proposed the similar formula, for clays with high plasticity index hence with high compressibility, which is expressed as follows:

$$G_0 = 445 \frac{(4.4 - e)^2}{1 - e} (\sigma_0')^{0.5} \cdots (7)$$

This formula is applicable for soft clays with a void ratio range of about  $e=1.5\sim2.5$ . For clay soils from natural deposits such as alluvial and reclaimed deposits, Kokusho, et al. (4) proposed the use of the following formula:

$$G_0 = 90 \frac{(7.32 - e)^2}{1 + e} (\sigma_0')^{0.6} \cdots (8)$$

It is applicable for soft clays with a void ratio range of about  $e=1.5\sim4.0$  and the plasticity index range of about  $PI=40\sim100$ .

#### 3. Experimental Investigation

#### 3.1 Site Conditions

The samples taken from AIT campus (Bangkok, Thailand) is subdivided into very soft clay, soft clay, and medium clay. Fig. 3 shows

the clay layer below the test site.

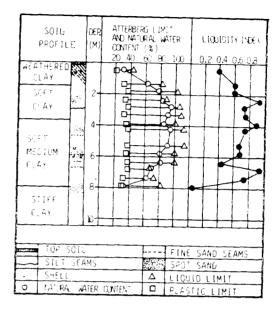


Fig. 3. Subsoil Properties of the Site

As shown in Fig. 3, the weathered crust is extended up to approximately 1.5 m below ground level, and soft, dark grey silty clay and clay layers are located about from 1.5 m to 8.5 m below ground level underlain by a stiff clay layer. In the soft clay layer isolated patches of sand seams and decomposed timber are often contained.

#### 3.2 Sampling

The samples are obtained at depth 5.5 m to 6.0 m below ground level (soft clay zone) using a power auger boring machine equipped with thin walled piston sampler advanced by hydraulic push.

#### 3.3 Testing Methods

Plates 1 and 2 show the triaxial cells and the air and water control unit manufactured by Seiken, Inc., Tokyo, Japen (Model No.: DT 2027).

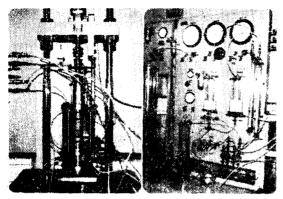


Plate 1. Triaxial Cell Plate 2. Air and Water Control Unit

Besides, a vertical loading unit, a pneumatic sinusoidal loading unit, and a recording unit with digital display and visigraph plotter are used. Using the apparatus, two series of cyclic loading tests are carried out, that is, undrained cyclic loading tests without and with initial shear stress. Table 1 shows the testing types and number of samples employed in the test.

Table 1. Testing Types

Initial Shear Stress (kgf/cm²)	Cyclic Stress Ratio (Z <sub>d</sub> /2σ' <sub>0</sub> )	Number of Samples	
		$\sigma'_0 = 1.0$ (kgf/cm <sup>2</sup> )	$\sigma'_0 = 1.5$ (kgf/cm <sup>2</sup> )
$\tau_i = 0$	0.28	1	1
	0.30	1	1
	0.35	2	2
	0.40	2	2
τ <sub>i</sub> =0.4× (Static Strength)	0.25	1	1
	0.30	1	2
	0.35	2	2
	0.40	2	2
τ <sub>i</sub> =0.6× (Static Strength)	0, 25	1	1
	0.30	1	2
	0.35	2	2
	0.40	- 2	. 2

Application of cyclic stresses is carried out employing a sinusoidal load of about I Hz with a given magnitude. During cyclic loading the cell pressure is kept constant and the back pressure line is closed. After setting the magnitude of cyclic loading and adjusting the measurement equipments, cyclic stress is applied to the specimen.

#### 4. Results

Typical results are demonstrated in Figs. 4 and 5, which represent the results for the tests without and with initial shear stress respectively. As can be seen in Fig. 4, in the case of test without initial shear stress, the axial strain is developed approximately symmetrically with respect to the abscissa. It is also revealed that the axial shear stress decreases when the amplitude of axial shear strain increases to a few percent and the excess pore water pressure at failure state shows irregular sine curves. These phenomena are due to the insufficient compliance of the pneumatic sine loader.

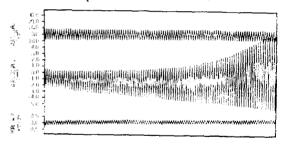


Fig. 4. A Typical Behavior of Clay under Cyclic Loading without Initial Shear Stress.

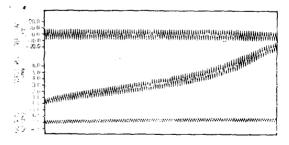


Fig. 5. A Typical Behavior of Clay under Cyclic Loading with Initial Shear Stress,

Fig. 5, which plots the time history of test with initial shear stress, shows some different tendencies. The axial strain curves show an unsymmetrical development with respect to the

abscissa because the compression stress is acted more predominantly than the extension stress during the test with initial shear stress.

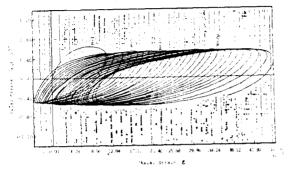


Fig. 6. Shear Stress Versus Shear Strain Behavior of Clay (Test AS4)

Fig. 6 shows a relation between axial stress and axial strain obtained under a given sinusoidal cyclic loading. And the hysteresis loops in Fig. 7 are quoted from the axial stress vs. axial strain curves.

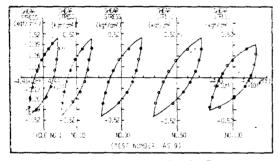


Fig. 7. Typical Stress Strain Curves.

The hysteresis loops are evidently smooth for all strain levels. This indicates that the cyclic triaxial apparatus is free from any mechanical problems of the testing device, and therefore, yields high quality test results<sup>(5)</sup>.

### 4.1 Effect of Number of Loading Cycles on Shear Modulus and Damping Ratio

The shear modulus and the damping ratio in the undrained cyclic loading tests are measured for the number of cycles N=5, 10, 30, 50 and 100, then plotted in Figs. 8 and 9.

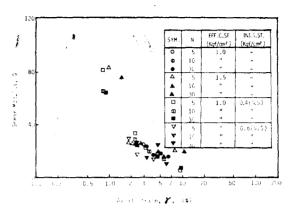


Fig. 8. Relationship Between Shear Modulus and Axial Strain

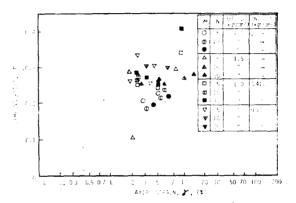


Fig. 9. Relationship between Damping Ratio and Axial Strain

Humphries et al. (6) and Hardin et al. (7) showed the damping ratio decreases approximately linearly with the logarithm of the number of loading cycles in cohesive soils and shear modulus also decreases with increasing number of load cycles.

It is seen that the shear modulus decreases with increasing number of cycles even though the deviations are appeared, and that there exists some sizable decrease of the damping ratio between N=5 and 10 or 30 in Fig. 9. The results show good agreement with the findings of others<sup>(4,6,7)</sup>.

### 4.2 Effect of Confining Stress on Shear Modulus and Damping Ratio

As mentioned previously, some researchers (2,7)

suggested that shear modulus varies with the 0.5 power of effective confining stress for very small strain amplitude, but at large strain amplitude the shear modulus depends on the strength of soil and the variation is larger expressed by the 1.0 power. They also indicated that damping ratio decreases approximately with the 0.5 power of effective confining stress independent of strain amplitude.

Fig. 8 shows the relationships between the shear modulus and the axial shear strains on the semi-logarithmic graph for the different effective confining stress of 1.0 and 1.5 kgf/cm². It is seen in the figure that shear modulus corresponding to each effective confining stress increases slightly as the effective confining stress becomes higher.

However, the results of the tests shown in Fig. 8 indicate an insignificant effect of the confining stress on the strain dependent change of the shear modulus for the cohesive soil.

As a whole, the damping ratio decreases as the confining stress increases from 1.0 to 1.5 kgf/cm², although there are some deviations of the data(Fig. 9). However, even though there were some effects of effective confining stress on damping ratio, a consistent dependency of the damping ratios on the effective confining stress could not be found, implying the lack of a strong connection between the damping ratios and the effective confining stresses.

# 4.3 Comparison of Shear Modulus and Damping Ratio with Other Results

Kokusho et al. (4) compared on the shear modulus versus the void ratio using three experimental equations mentioned in the previous chapter. Fig. 10 shows the range of variation of shear modulus for different effective confining pressures with the results obtained by Kokusho et al.

As can be seen in Fig. 10, the shear modulus

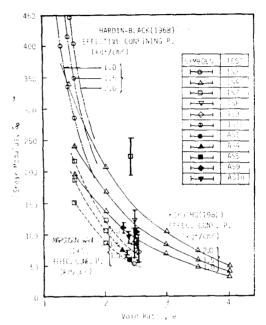


Fig. 10. Comparison of Shear Modulus on Three Experimental Formulae.

obtained in this investigation formed a group between the curves obtained employing the experimental equations derived by Marcuson et al. (3) and Kokusho et al. (4). The variation ranges of the shear modulus for each test were governed by the number of loading cycles, and the values were distributed approximately within 55 kgf/cm² and 140 kgf/cm² as a whole although there exist some scatters. In Fig. 10,

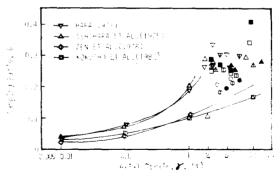


Fig. 11. Comparison of Damping Ratio for Clays,

it is also seen that the shear modulus decreases with increasing void ratio although the values of void ratio are not changed widely, and under the condition with initial shear stress, the shear modulus increased slightly.

On the other hand, the data for damping ratio obtained in this investigation was plotted in Fig. 10 to compare with the other results (5,8,9,10). Because this investigation was carried out with large strain amplitude, data was obtained only for the strain range from 1.5 to 20%. Therefore, the damping ratio was compared with those results for large axial strain range. As shown in Fig. 11, in general, the results of this investigation for damping ratio was a little lower than the curve established by Ishihara et al. (9), and was distributed higher than the curve obtained by Kokusho et al. (4).

#### 5. Conclusion

On the basis of the test results reported herein, it is concluded that the shear modulus decreases with increase in number of loading cycles, on the other hand, the damping ratio decreases slightly with increase in number of loading cycles. When the stress ratio increases, shear modulus increases, while the damping ratio decreases.

No obvious dependency on initial shear stress and effective confining stress are recognized in the shear modulus and damping ratio plotted versus strain. However, the shear modulus decreases and the damping ratio increases with increasing axial strain.

When compared with others, it is also revealed that the shear moduli are distributed within the range curves obtained using empirical equations derived by Marcuson et al. (3) and Kokusho et al. (4), and damping ratios are distributed between the curves obtained by Kokusho et al. (4) and Ishihara et al. (9)).

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