The Effects of Thermal Conductivities of Component Materials on the Heat Flowrates in Composite Longitudinal and Annular Fin Assemblies

직선 및 환상의 Composite Fin Assembly에서 구성 재질들의 열전도율들이 열유동량에 미치는 영향

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초 록

열교환기의 확장 표면이 2가지 또는 3가지의 서로 다른 재질들로써 구성되는 경우, 직선 및 환상의 Composite Fin Assembly에서 구성 재질들의 열전도율들이 각 Assembly에서의 온도 분포와 열유동량에 미치는 영향을 조사하였다.

Composite Fin Assembly에서의 온도 분포를 구하기 위하여 유한 요소법을 사용하여 수치 계산하였다.

Composite Fin Assembly를 구성하는 재료들의 열전도율들이 열유동량에 미치는 효과는 상당히 크며, 따라서 확장 표면의 설계 계산에 있어서 이러한 효과들을 필수적으로 고려하여야 함이 밝혀졌다.

Nomenclature

B_{i1}, B_{i2}, B_i^* = Biot numbers = $h_1 \cdot P/k_W$, $h_2 \cdot P/k_W$ and $h_2 \cdot FL/k_C$, respectively

CT = half-thickness of coated or laminated material

ct, fl, ft, rw, wt = aspect ratios = CT/P, FL/P, FT/P, RW/P and WT/P, respectively

FL = length of composite fin

FT = half-thickness of composite fin

h_1, h_2 = heat transfer coefficients

k1, k2 = thermal conductivity ratios = k_f/k_W and k_C/k_f, respectively

P = half-pitch of composite fin

Q = composite fin assembly heat flowrate

Q* = composite fin assembly heat flowrate for the case k1=1 and k2=1

r, z = cylindrical coordinates

RW = inner radius of cylindrical wall of composite annular fin assembly

T = temperature distribution

T_1, T_2 = fluid temperatures

WT = thickness of composite wall

x, y = Cartesian coordinates

ϵ = constant, $\epsilon = 0$ for longitudinal system and $\epsilon = 1$ for annular system

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k_c, k_f, k_w = thermal conductivities

θ = dimensionless temperature distribution $=(T-T_2)/(T_1-T_2)$

ζ, ξ = dimensionless system coordinates, $\zeta=x/P$ and $\xi=y/P$ for longitudinal system, $\zeta=r/P$ and $\xi=z/P$ for annular system

Subscripts

1 = unfinned side

2 = finned side

c = coated or laminated material (face material)

f = base fin material

w = base wall material

Introduction

The heat flow in a two-dimensional composite fin, which is constructed by coating or laminating an anticorrosive material on the base material under a corrosive environment, has been analyzed on the assumptions that the temperature of the composite fin root is constant and equal to that of the wall surface to which the composite fin is attached, and the composite fin tip is insulated (1-4). In such analyses the composite fins and the supporting wall are treated as completely separate entities as in an extensive range of published literature (5-9) for simple (not coated or laminated) fin heat transfer.

Some studies (10-12) of the temperature depression at the root of a simple fin, however, indicate that the addition of fins induces substantial reduction of the fin root temperature and quite significant two-dimensional effects within the supporting wall. Thus it would be most desirable to simultaneously examine the heat flows within both the fins and the supporting wall by employing a multidimensional analysis in order to prevent errors from the use of the one-dimensional approximation or the employment of the analysis method which the fins and the supporting wall are treated as separate entities.

This study examines the effects of thermal conductivities of the base fins and the face material (coated or laminated material) on the temperature distributions and the heat flowrates for two-dimensional longitudinal and annular composite rectangular fin assemblies with real constraints, as shown in Fig. 1.

The two-dimensional analysis of the conductive-convective heat flow through either longitudinal or annular composite fin assembly requires the solution of a Laplacian mixed boundary-value problem (10-13). To the best knowledge of the authors, any analytic method may not be suitable for the solutions of the complicated conductive-convective heat flow problems in the composite fin assemblies. The problems of this type can easily be treated by various numerical techniques such as the finite difference and finite element methods. Therefore, in this study, the solutions of the problems are computed employing the finite element method.

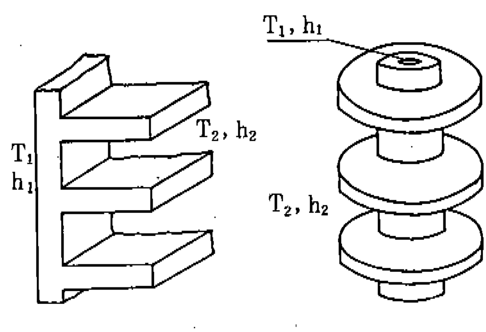

Fig. 1 Configurations of Composite Longitudinal and Annular Fin Assemblies

Analysis

Consider a heat exchanger comprised of equally spaced either longitudinal rectangular fins attached to a plain wall or annular rectangular fins attached to a cylindrical tube wall, of which all exposed surfaces (namely, surfaces of base fins and interfin base wall of finned side) to a corrosive environment are coated with a face material, as depicted schematically in Fig. 1.

All exposed surfaces; both finned and unfinned sides are subjected to convective heat transfer. There is no heat source in either longitudinal nor annular system. The heat flow through the complete composite fin assemblies is steady state two-dimensional, especially for the annular system that is both radial and axial. General consideration is given to the case in which the base wall, the base fins and the face material coated or laminated on the finned side surface exposed to corrosive coolant have different but constant characteristics. The material of each region is isotropic and homogeneous. Furthermore, it is assumed that the heat transfer coefficients of both finned and unfinned sides are uniform and equal over the corresponding surfaces respectively, and that perfect contacts are maintained at all interfaces.

In order to develop a mathematical representation and simplify the analysis on the basis of the above restrictions and assumptions it is necessary to set which section of the composite fin assembly system should be investigated. The geometrical and thermal symmetry of the composite fin assembly configurations as the consequence of the previous assumptions indicates that it will suffice to examine the temperature distribution and the heat flow rate within the region bounded by the contour ABC'D'E'F'A Fig. 2. This is the smallest elementary section to be analyzed which maintains all the essential features of the complete composite fin assembly systems.

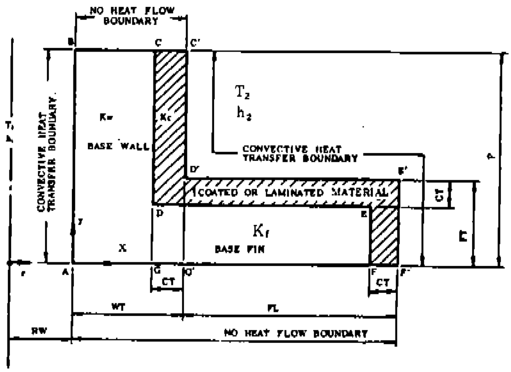

Fig. 2 Nomenclature for Composite Longitudinal and Annular Fin Assemblies

On the basis of the preceeding development, for the steady-state two-dimensional conductive-convective heat flow through a composite fin assembly, the determination of the temperature distribution, $\theta(\zeta, \xi)$, within the assembly requires the simultaneous solution of

$$\frac{\partial^2\theta_c}{\partial\zeta^2} + \frac{\epsilon}{\zeta}\frac{\partial\theta_c}{\partial\zeta} + \frac{\partial^2\theta_c}{\partial\xi^2} = 0$$

within the face material region (CC'D'E'F'FEDC, Fig. 2),

$$\frac{\partial^2\theta_f}{\partial\zeta^2} + \frac{\epsilon}{\zeta}\frac{\partial\theta_f}{\partial\zeta} + \frac{\partial^2\theta_f}{\partial\xi^2} = 0$$

within the base fin material region (DEFGD, Fig. 2)

and
$$\frac{\partial^2\theta_w}{\partial\zeta^2} + \frac{\epsilon}{\zeta}\frac{\partial\theta_w}{\partial\zeta} + \frac{\partial^2\theta_w}{\partial\xi^2} = 0$$

within the base wall material region (ABCDGA, Fig. 2)

subjected to the boundary conditions

on AB $\dfrac{\partial \theta_w}{\partial \zeta} = -B_{i1}(1-\theta_w)$

on BC $\dfrac{\partial \theta_w}{\partial \xi} = 0$

on CC′ $\dfrac{\partial \theta_c}{\partial \xi} = 0$

on AG $\dfrac{\partial \theta_w}{\partial \xi} = 0$

on GF $\dfrac{\partial \theta_f}{\partial \xi} = 0$

on FF′ $\dfrac{\partial \theta_c}{\partial \xi} = 0$

on C′D′ $\dfrac{\partial \theta_c}{\partial \zeta} = -\dfrac{B_{i2}}{k1 \cdot k2}\theta_c$

on D′E′ $\dfrac{\partial \theta_c}{\partial \xi} = -\dfrac{B_{i2}}{k1 \cdot k2}\theta_c$

on E′F′ $\dfrac{\partial \theta_c}{\partial \zeta} = -\dfrac{B_{i2}}{k1 \cdot k2}\theta_c$

and the matching conditions at the interfaces

on CD $\theta_w=\theta_c$ and $\dfrac{\partial\theta_w}{\partial\zeta} = k1 \cdot k2 \dfrac{\partial\theta_c}{\partial\zeta}$

on DG $\theta_w=\theta_f$ and $\dfrac{\partial\theta_w}{\partial\zeta} = k1 \dfrac{\partial\theta_f}{\partial\zeta}$

on DE $\theta_f=\theta_c$ and $\dfrac{\partial\theta_f}{\partial\xi} = k2 \dfrac{\partial\theta_c}{\partial\xi}$

on EF $\theta_f=\theta_c$ and $\dfrac{\partial\theta_f}{\partial\zeta} = k2 \dfrac{\partial\theta_c}{\partial\zeta}$

where ζ=x/P, ϵ=0 and ξ=y/P for the longitudinal case, and ζ=r/P, ϵ=1 and ξ=z/P for the annular configuration.

These equations are solved numerically employing the finite element method. The two-dimensional solution regions to be considered herein are divided into a total of 296 three-node linear triangular elements with 178 nodal points for the longitudinal system and equal numbers of triangular ring element for the annular system in the manner of representing the parting lines of the materials by the boundaries of triangles and making the distribution of the nodal points to accomodate the rapidity of the temperature variations, as shown in Fig. 3. The heat flowrates through the assemblies are evaluated from temperature distributions within the composite fin assemblies generated by the finite element formulations.

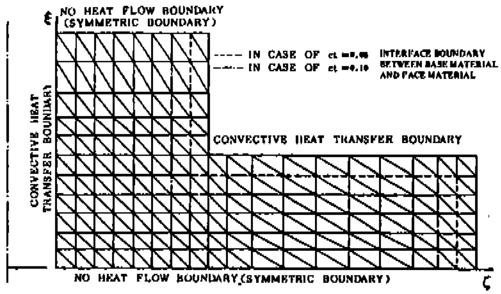

Fig. 3 Finite Element Model of the Composite Fin Assembly

It would be superfluous to present details of the finite element formulations herein because the application of the finite element formulation method to problems of heat transfer analysis is described in several journal and textbook articles. It will suffice to note that the computer program used to calculate the finite element formulations was validated by comparison with the exact analytic solutions for a conductive-convective two-dimensional heat transfer problem reported by Jo (13).

Results, Discussion and Conclusions

The temperature distributions and the heat flow for a composite fin assembly are both functions of B_{i1}, B_{i2}, k1, k2, ct, fl, ft, rw and wt (where the parameter rw is excluded in case of the longitudinal system). The main objective of this study is to investigate the effects of the thermal conductivities of the base fin material and the face material on the heat flowrates through the composite fin assemblies. Therefore, to examine all combinations of above total system parameters is beyond the scope of this work. For both longitudinal and annular systems, the study has been accomplished for the fixed values of the system variables, as given in the following:

$B_{i1} = 0.5$; $B_{i2} = 0.05$; $fl = 2$; $ft = 0.3$; $wt = 0.6$; $rw = 1.4$

where the value of rw is necessary only for the annular case.

Since the thickness of the face material is generally much smaller than that of the base material in the composite fin assembly, the value of ct is chosen to be either 0.05 or 0.1 for all cases under consideration. The two remainder parameter ranges investigated are k1:1-8 and k2:0.025-4. The foregoing fixed values of the system parameters were not taken from any practical application, so no attempt is made to maintain the optimum conditions for the present models.

Fig. 4 presents isothermal lines for typical cases in the composite longitudinal fin assembly, plotted from the dimensionless temperature distributions taken from the numerical solutions. It is shown that increasing the thermal conductivity ratio, k2 increases the temperature throughout the composite fin region, and hence the temperature gradient in the x-direction become larger. This phenomenon is physically plausible. That is, the large value of k2 means that the conductive resistance of the composite fin region is low, so that larger quantity of heat must be brought by conduction to the composite fin region from the composite wall region, and this conductive transport causes remote wall region temperature drop and fin temperature gain. Since the convective heat transfer is proportional to the temperature difference between the convective boundary surface and the convective fluid on the basis of the assumption that the convective heat transfer can be described using Newton's Law of Cooling (14), the temperature drop on the unfinned side (AB, Fig. 2) and the temperature gain on the finned side (C'D'E'F', Fig. 2) simultaneously will result in an increase in the overall heat flowrate through the composite fin assembly.

The transverse temperature variations at cross sections of the composite fin region become significant as the ratio of the thermal

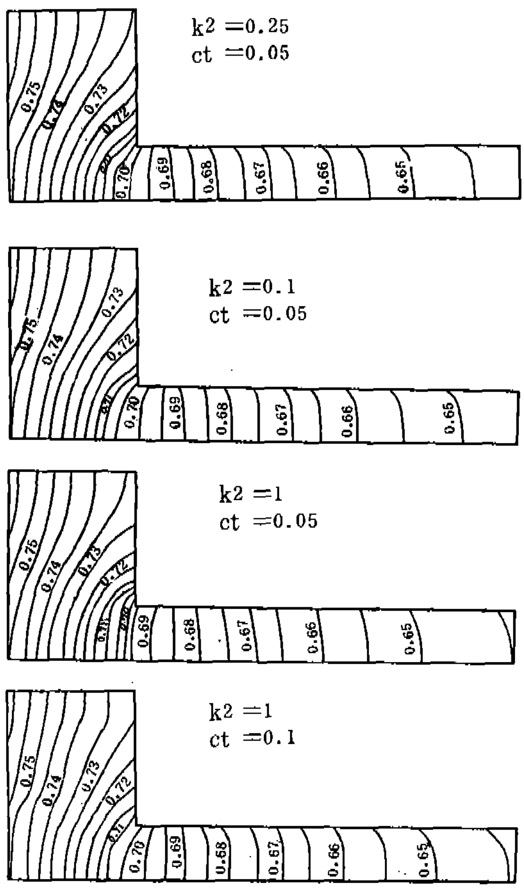

Fig. 4 Isotherms for k1 = 4

conductivities, k2, decreases. This tendency is more significant in the face material region, as one would expect.

Representative dimensionless plots of temperature distribution θ versus ζ along the surface of the composite fin (D'E', Fig. 2) are shown in Figs. 5 and 6, which respectively corresponds to longitudinal and annular system. In each of the graphs presented in these figures, the parameters, k1 and ct, are assigned as prescribed values, and then the dimensionless temperature is plotted for four different values of k2. Figs. 5 and 6 show that the surface temperature at the composite fin tip (point E') is significantly affected by variations in the thermal conductivity of the face material. This effect is quite substantial for lower value of k2.

It is evident from Figs. 5 and 6 that for fixed values of system variables except the thermal conductivity of the face material, a lower value of k2 results in a reduction in the composite fin surface temperature gradient, as investigated precedently from Fig. 4. This seems to be contrary to the observations of Chu et al. (4), which are obtained by analyzing the transient response of a composite longitudinal single fin with end insulated on the basis of uniform fin root temperature. However, unfortunately they could not succeed in the examinations of the effects of variations in the coated material thermal conductivity on the surface temperature distributions and the heat flowrates, because the fixed value of the Biot number, B_i^* as defined in the normenclature, which is one of the prescribed conditions for comparison of the results implies that the value of $h_2 \cdot FL$ should be directly proportional to the value of the coated material thermal conductivity k_C, so that comparison can not be justified.

Further inspection of Figs. 5 and 6 indicates that the temperature gradient of the annular system generally differs from that of the longitudinal system under the same conditions. The surface temperature of the longitudinal system gradually varies with longitudinal position in all cases. On the other hand, for the annular situation, the surface temperature variations in radial direction are very large from the fin root to a zone and become very small from the zone to the composite fin tip (of course, the temperature drop will exist near the tip when k2 has a small value, but this case is excluded in the present examination). This aspect of the results can be explained as follows. Becuase of the geometrical configuration of the

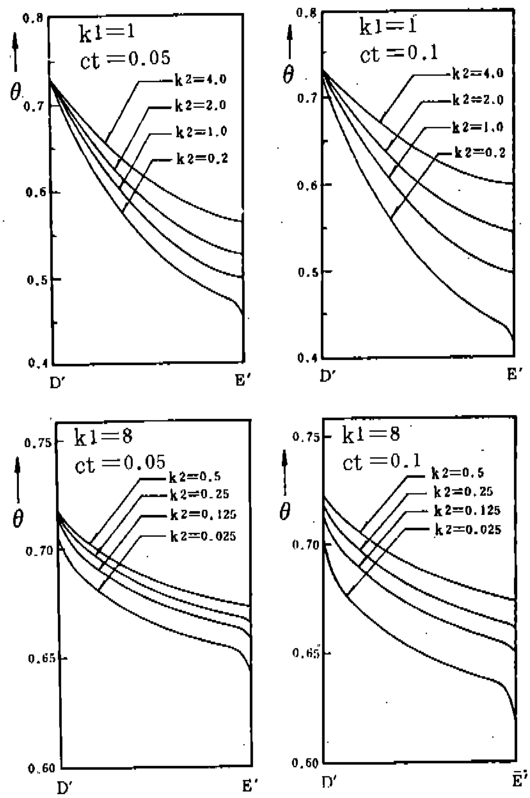

Fig. 5 Distributions of Composite Longitudinal Fin Surface Temperature

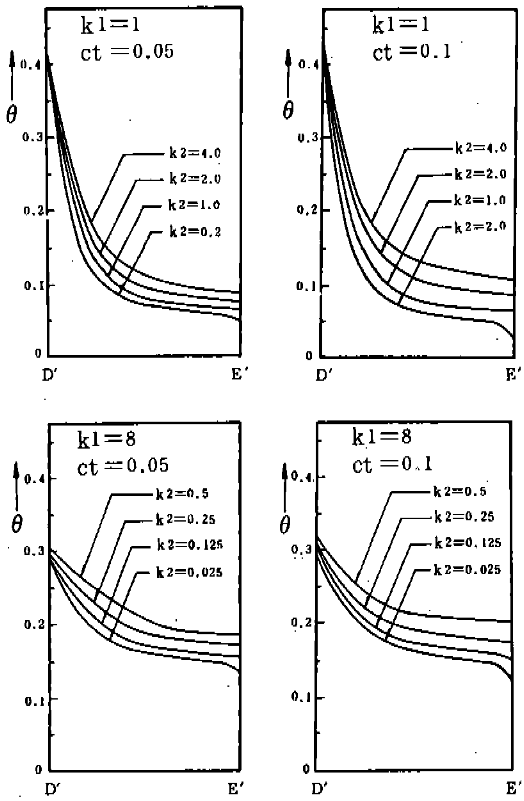

Fig. 6 Distributions of Composite Annular Fin Surface Temperature

annular system, the convective surface area of the composite fin increases in proportion to $r^2-(RW+WT)^2$ as the radial coordinate r varies in a direction toward the composite fine tip, and hence, most of the heat passing into the composite fin at its root will be dissipated through the lateral surface near the root of the composite fin.

The effects of thermal conductivities of base fin and face materials on the heat flowrates for longitudinal and annular systems, respectively, are shown in Figs. 7 and 8. In each of the graphs presented in these figures, the percentage differences in Q and Q* are plotted against k1·k2 for ct=0.05 and 0.1 and for four different values of k1.

For both geometrical configurations, a most significant feature of the results is the manner in which the percentage difference in Q and Q* increases, and inevitably approaches some limiting value as the thermal conductivity ratio of face material-to-base wall material ($=k1\cdot k2 = k_c/k_w$) is increased. Closer inspection of the results presented in Figs. 7 and 8 indicates that the heat flowrate for both composite longitudinal and annular fin assemblies increases rapidly as k1·k2 increases up to a value of about 4, except only for ct of 0.1 and k1 of 2 or less in case of annular situation. Thereafter, using higher thermal conductivity materials as base fins and face materials produces little advantage, both in the increasing of the heat flowrate and in the composite fin assembly efficiency. Figs. 7 and 8 show that the variations in the differences between Q and Q* are substantial for large values of ct and small values of k1 and k2. For greater value of k1, the effect of ct on the heat flowrate may not be significant. These phenomena are much more prominent in the annular situation than in the longitudinal case generally, and should be considered in extended surface design. For the ranges of system parameters indicated previously, the maximum difference between Q and Q* in the longitudinal situation is some 12.5 percent and that in the annular case is some 30 percent, as shown in Figs. 7 and 8.

From the results of this investigation, it may be concluded that the extended surface design can not be properly accomplished without due consideration of effects of variations in the face material thermal conductivity as well as those of the base wall material and the base fin material which play an important role in improving the performance of the extended surface.

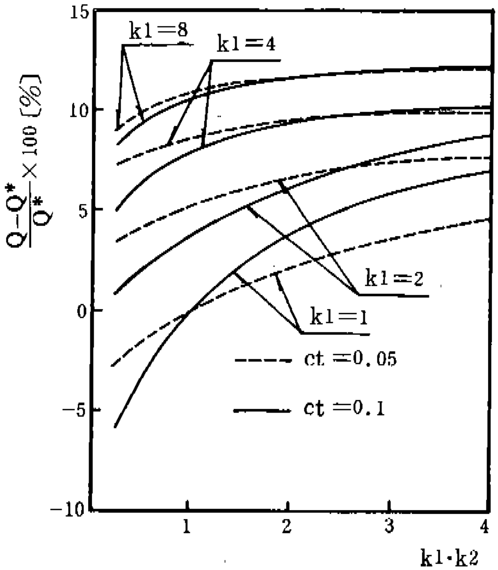

Fig. 7 The Percentage Difference in Q and Q* versus kl · k2 for Longitudinal System

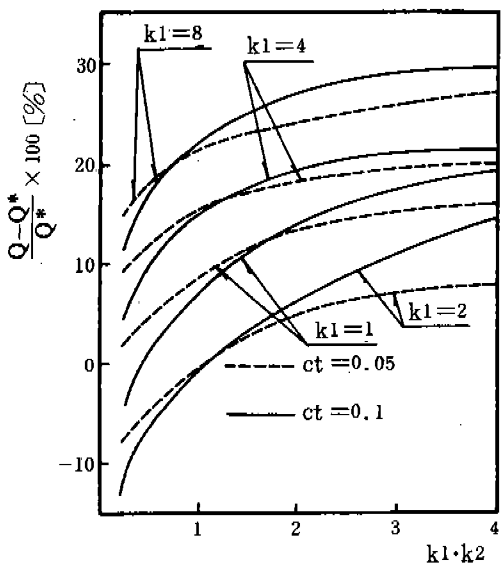

Fig. 8 The Percentage Difference in Q and Q* versus kl · k2 for Annular System

References

1. Barker, J.J., "The Efficiency of Composite Fins," Nuclear Science and Engineering, Vol. 3, 1958, pp. 300-312.
2. Tittle, C.W., "Boundary Value Problems in Composite Media: Quasi-Othogonal Functions," Journal of Applied Physics, Vol. 36, No. 4, 1965, pp. 1486-1488.
3. Feijoo, L., Davis, H.T., and Ramkrishna, D., "Heat Transfer in Composite Solids with Heat Generation," ASME Journal of Heat Transfer, Vol. 101, 1979, pp. 137-143.
4. Chu, H.S., Weng, C.I., and Chen, C.K., "Transient Response of a composite Straight Fin," ASME Journal of Heat Transfer, Vol. 105, 1983, pp. 307-311.
5. Avrami, M., and Little, J.B., "Diffusion of Heat through a Rectangular Bar and of the Cooling and Insulating Effect of Fins," Journal of Applied Physics, Vol. 13, 1942, pp. 255-264.
6. Keller, H.H., and Somers, E.V., "Heat Transfer from an Annular Fin of Constant Thickness," ASME Journal of Heat Transfer, Series C, Vol. 81, No. 2, 1959, pp. 151-156.
7. Lau, W., and Tan, C.W., "Errors in One-Dimensional Heat Transfer Analysis in Straight and Annular Fins," ASME Journal of Heat Transfer, Series C, Vol. 95, 1973, pp. 549-551.
8. Kern, D.Q., and Kraus, A.D., Extended Surface Heat Transfer, McGraw-Hill, New York, 1972.
9. Eckert, E.G.R., and Drake, R.M., Analysis of Heat and Mass Transfer, McGraw-Hill, New York, 1972.
10. Sparrow, E.M., and Lee, L., "Effects of Fin Base-Temperature Depression in a Multifin Array," ASME Journal of Heat Transfer, Vol. 97, 1975, pp. 463-465.

11. Suryanarayana, N.V., "Two-Dimensional Effects on Heat Transfer Rates from an Array of Straight Fins," ASME Journal of Heat Transfer, Vol. 99, 1977, pp. 129-132.

12. Heggs, P.J., and Stons, P.R., "The Effects of Dimensions on the Heat Flowrate through Extended Surfaces," ASME Journal of Heat Transfer, Vol. 12, 1980, pp. 180-182.

13. Jo, J.C., "A Study on the Heat Transfer in a Straight Fin of Rectangular Profile," M.S. Thesis, Hanyang University, 1980.

14. Kreith, F., Principles of Heat Transfer, Harper and Row, New York, 1976.