디지틀 전송에서 시스템/랜덤 지터 누적

Systematic and Random Jitter Accumulation in Digital Transmission

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요 약

디지틀 데이타 전송시 수신기에서 또는 중계기에서 송신 데이타를 구하기 위해서는 타이밍 클럭을 복원해야 한다. 대개의 실장된 경우에서는 PLL(phase locked loop)을 이용하는데, 이때 시스템/랜덤 지터가 발생되고 또 중계기가 늘어남에 따라서 생성된 지터가 누적되는데 이것을 해석하였다.

90 Mbps 광통신 시스템에 적용하여 테이블 1, 2, 3, 4와 같은 결과를 얻었으며, 시스템 지터가 랜덤 지터보다 더 급격하 누적됨을 알았고, 또한 댐핑팩터(ξ)가 커짐에 따라서 지터 누적이 약화됨을 알았다.

ABSTRACT

The timing clock is required to be recovered for the detection of original data in a receiver, or a repeater in digital transmission of data signal. In general, phase locked loop (PLL) is usually utilized for timing recovery.

Systematic/random jitters are produced in timing recovery and accumulated with the increase of repeater chains, which have been analyzed in this paper.

The results applicated to 90 Mbps optical communication system are the table 1,2,3,4. The systematic jitter is accumulated more exponentially than the random jitter and the jitter accumulation is weakened according to the increase of damping factor.

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I. INTRODUCTION

From the definition of CCITT, jitter is described as "a short-time variation of the significant instants of a digital signal from their ideal position in time."

In digital system, jitter can be generated in all places but the self-generated jitters produced by the digital lines or multiplexer/ demultiplexer are significant which are composed of systematic jitter, random jitter, and residual jitter (waiting time jitter).

The alignment jitter is the one produced in each repeater (dynamic phase difference between the input signal and the timing clock derived from it). This alignment jitter is made up of systematic and random jitter,increases exponentially if it has the transfer function jitter peaking.

The accumulated jitter is the sum of alignment jitters. Therefore, the accumulated jitter is conclusvely the jitter produced at the end point of digital transmission equipment. [1]-[4]

The systematic jitter is the one produced in the equipment which is correlated with the data pattern whose sources are estimated to be ISI (intersymbol interference), static timing offset, amplitude-to phase conversion (timing noise due to trigger offset).

Since these sources have the energy at zero frequency, the systematic jitters are strongly accumulated in parallel with growth of the number of repeater chain.

The random jitter is the one owing to remaining component (nonsystematic jitter) of self-generated jitter, whose sources are expected to be channel noise, cross talk, random mistuning of timing circuit filter. This random jitter is not strongly accumulated like the systematic jitter.

The residual jitter (waiting time jitter) is produced in multiplexer and demultiplexer because of pulse stuffing mechanism. [6]

In this paper we have studied the systematic/ random jitter accumulation in repeater chain when the 2-nd order PLL is used as the timing-fillter. And we have known the fact that the mean-square systematic jitter is asymtotically proportional to the number of repeater chain when the damping factor (ξ) is large enough. But the accumulation of jitter is exponentially increased in case of small damping factor.

Also, both jitters are decreased in accordance with the increase of damping factor.

II. ANALYSIS OF JITTER ACCUMULATION

In Fig. 1 the model of jitter accumulation is shown.

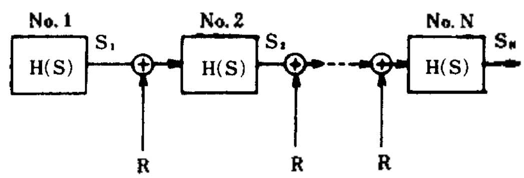

Fig. 1 Equivalent model of repeater chain.

, where H(s) is denoted to be the PLL transfer function for the timing recovery in repeaters.

In the first place, we have considered the systematic jitter accumulation. It is assume that the equivalent jitter applied to each repeater is S(w). The systematic jitter is much accumulated since the correlation is strong between the repeaters.

$S(w)H(jw)\,N-(k-1)$is the jitter spectrum at the N-th repeater output from the jitter source of k-th repeater.

After considering all repeater, the jitter spectrum of the last N-th repeater output is represented as eq. (1).

$$S_N=\sum_{k=1}^{N} S(w)\ H(jw)^{N-k-1}$$
$$=S(w)\,H(jw)\,\frac{1-H(jw)^N}{1-H(jw)} \qquad (1)$$

At this time it is expected that is equal to constant S in the bandwidth of timing filter.

The mean-square system jitter can be described as eq. (3).

$$\overline{S_N^2}=\frac{S}{2\pi}\int_0^\infty |H(jw)|^2\,\left|\frac{1-H(jw)^N}{1-H(jw)}\right|^2 dw \qquad (3)$$

After normalizing by the mean-square jitter in first repeater, the normalized mean-square systematic jitter is obtained in eq. (4).

$$\frac{S_N^2}{S_1^2}=\frac{\frac{S}{2\pi}\int_0^\infty |H(jw)|^2\,\left|\frac{1-H(jw)^N}{1-H(jw)}\right|^2 dw}{\frac{S}{2\pi}\int_0^\infty |H(jw)|^2 dw} \qquad (4)$$

Next we can derived the random jitter accumulation in digital repeater chain.

r(w) is equivalent random jitter and is assumed to be Gaussian.

R(w) is defined to be power spectral density of r(w).

Since the random jitter is uncorrelated with each repeater sources, the power spectral density of the last N-th repeater from the k-th repeater source is expressed as eq. (5).

$$R_{K,N}(w)=R(w)\ |H(jw)|^{2(N-(k-1))} \qquad (5)$$

After transmitting through the N repeaters, the mean-square random jitter is $\overline{R_N^2}$

that is to say.

$$\overline{R_N^2}=\sum_{k=1}^{N}\frac{1}{2\pi}\int_0^\infty R_{K,N}(w)\,dw$$
$$=\frac{R(w)}{2\pi}\int_0^\infty |H(jw)|^2\,\frac{1-|H(iw)|^{2N}}{1-|H(jw)|^2}\,dw \qquad (6)$$

Let's normalizing eq. (6) by the mean-square jitter produced in the first repeater.

Therefore we can come by the eq. (7).

$$\frac{\overline{R_N^2}}{\overline{R_1^2}}=\frac{\int_0^\infty |H(jw)|^2\,\frac{1-|H(jw)|^{2N}}{1-|H(jw)|^2}\,dw}{\int_0^\infty |H(jw)|^2 dw} \qquad (7)$$

III. APPROXIMATION PROCESS

The 2-nd order PLL is generally utilized for timing clock recovery circuit in digital transmission system and the 2-nd order PLL has the 1-st order loop filter.

The transfer function of PLL is represented as eq. (8).

$$H(jw)=\frac{KoKd\ F(jw)}{1+KoKd\ F(jw)} \qquad (8)$$

, where F(jw) is the transfer function of loop filter,

Ko is the VCO gain,

Kd is the gain of phase detector.

Since the F(jw) is the 1-st order transfer function on the whole, the eq. (8) can be converted into eq. (9).

$$H(jw)=\frac{1+j2\xi x}{1-x**2+j2\xi x} \qquad (9)$$

where x equals w/w_n.

w_n is the natural frequency,

ξ is damping factor.

With the eq. (9), we can simply approximate the resulting eqs. (4), (7) without resorting to the generating function.

In the first place, the eq. (7) can be approximated as the following procedure in case that the damping factor is large enough.

$$|H(jw)|^2 = \left|\frac{1+j2\xi x}{1-x**2+j2\xi x}\right|^2$$

$$\simeq \frac{1+(2\xi x)**2}{1+x**4+(2\xi x)**2}$$

$$\simeq \frac{1}{1+(x/2\xi)**2} \quad (10)$$

Therefore the denominator of eq. (7) is integrated as follows.

$$\int_0^\infty |H(jw)|^2 dw = \int_0^\infty \frac{1}{1+(x/2\xi)**2} w_n dx \quad (x=w/w_n)$$

$$\simeq \xi w_n \pi \quad (11)$$

By use of the eq. (10) and variable change of $t=x/2\xi$, the nominator of eq. (7) can be expressed as eq. (12).

$$\frac{1-|H(jw)|^{2N}}{1-|H(jw)|^2} \simeq \frac{\dfrac{\{(1+t**2)**N-1\}}{(1+t**2)**N}}{\dfrac{t**2}{1+t**2}}$$

$$\simeq \frac{N}{1+(N-1)*t**2}$$

$$\simeq \frac{N}{\{1+t**2\}**N} \cdot (1+t**2)$$

$$\simeq \frac{N}{1+N*t**2}(1+t**2) \quad (12)$$

Therefore, the integration of nominator can be accomplished with eqs. (10), (12).

$$\int_0^\infty |H(jw)|^2 \frac{1-|H(jw)|^{2N}}{1-|H(jw)|^2} dw$$

$$= \int_0^\infty \frac{1}{1+t**2} \frac{N}{1+N*t**2} \cdot (1+t**2) dw \quad (13)$$

$$= \xi \pi w_n * SQRT(N) \quad (14)$$

The relationship between the variables is

$$t = x/2\xi = w/2\xi w_n.$$

Therefore, the last conclusion is eq. (14) divided by eq. (11) so that the normalized mean-square random jitter is proportional to SQRT(N) in case of large damping factor, where N is the number of repeater chain.

Likewise, it can be easily shown that the normalized mean-square systematic jitter is proportional to N through the similar procedure in case of a large quantity of damping factor.

but both jitters are exponentially increased according to the number of repeater chain if the damping factor has small value.

Here we have the physical meanings from the above results.

The presence of internal/external noise causes any static phase error from its noise-free level. In turn the presence of a static phase error causes the phase jitter to increase. Then we can study the change of PLL transfer function with the variation of damping factor.

The frequency response of a 2-nd order PLL shows the fact that the loop performs a lowpass filtering operation on its phase inputs, and the bandwidth increases in parallel with the growth of damping factor at a fixed natural frequency. (7)

This fact indicates the following results.

The transient responses of phase errors due to step in phase, or frequency, or ramp in frequency are decreased in a large quantity of ratio according to the increase of damping factor. In addition, if we study the noise performance of PLL, it can be easily deduced that the eq. (11) is equivalent to the noise bandwidth.

At a fixed natural frequency, the noise bandwidth is minimum at damping factor of 0.5 but at this case the jitter becomes rapidly accumulated because the PLL transfer function has the jitter peaking. ($|H(jw)| > 1$)

IV. 90 MBPS OPTICAL COMMUNICATION SYSTEM

The derived above results have been applicated to the implemented 90 Mbps optical communication system. The timing clock recovery PLL circuit is exemplified in Fig. 2.

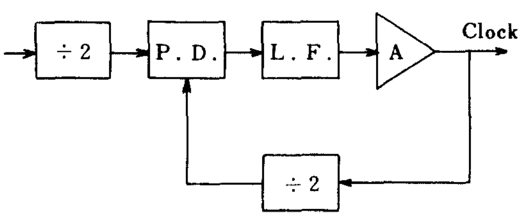

Fig. 2 Block diagram of PLL recovery circuit.

The values of used parameters are as belows.

damping factor = 0.8, 2.0, 4.0, 10.0

Wn(natural frequency) = 3.653 rad/sec

Kd = 0.0593 V/rad

Ko = 31.400 rad/V

BW = 7 kHz

We have obtained the following results in Table 1,2,3,4 through computer simulation in case of variable damping factors.

Table 1. Random/systematic jitters at $\xi = 0.8$.

DAMPING RATIO= .8000 NATURAL FREQ=3653.000 BANDWIDTH=7000.0000

N	RANDOM-JITTER	SYSTIMATIC-JITTER
1	.00000E+00	.00000E+00
11	.18026E+02	.54290E+02
21	.33660E+02	.88283E+02
31	.50149E+02	.12545E+03
41	.66962E+02	.16387E+03
51	.83938E+02	.20281E+03
61	.10101E+03	.24204E+03
71	.11815E+03	.28144E+03
81	.13534E+03	.32098E+03
91	.15257E+03	.36060E+03
101	.16982E+03	.40030E+03
111	.18709E+03	.44006E+03
121	.18159E+03	.47987E+03
131	.18156E+03	.51971E+03
141	.17575E+03	.55959E+03
151	.17955E+03	.59949E+03

Table 2. Random/systematic jitters at $\xi = 2.0$.

DAMPING RATIO= 2.0000 NATURAL FREQ=3653.000 BANDWIDTH=7000.0000

N	RANDOM-JITTER	SYSTIMATIC-JITTER
1	.00000E+00	.00000E+00
11	.11354E+02	.45437E+02
21	.15554E+02	.57044E+02
31	.19045E+02	.65664E+02
41	.22422E+02	.73308E+02
51	.25841E+02	.80695E+02
61	.29339E+02	.88152E+02
71	.32913E+02	.95816E+02
81	.36551E+02	.10372E+03
91	.40239E+02	.11184E+03
101	.43967E+02	.12015E+03
111	.47725E+02	.12859E+03
121	.51508E+02	.13714E+03
131	.55311E+02	.14578E+03
141	.59130E+02	.15448E+03
151	.62963E+02	.16323E+03

V. CONCLUSION

We have derived the normalized mean-square random/systematic jitters from the equivalent model of timing clock recovery PLL circuit. And the approximation can be applied in order to find out the proportionalty in case of a large quantity of damping factor. The both jitters are exponentially increased when the

Table 3. Random/systematic jitters at $\xi = 4.0$.

DAMPING RATIO= 4.0000 NATURAL FREQ=3455.000 BANDWIDTH=7000.0000

N	RANDOM-JITTER	SYSTEMATIC-JITTER
1	.00000E+00	.00000E+00
11	.10626E+02	.46760E+02
21	.13700E+02	.56995E+02
31	.15694E+02	.62500E+02
41	.17247E+02	.66840E+02
51	.18570E+02	.70432E+02
61	.19757E+02	.73549E+02
71	.20861E+02	.76382E+02
81	.21912E+02	.78995E+02
91	.22928E+02	.81457E+02
101	.23925E+02	.83809E+02
111	.24909E+02	.86077E+02
121	.25887E+02	.88288E+02
131	.26865E+02	.90457E+02
141	.27843E+02	.92600E+02
151	.28826E+02	.94727E+02

Table 4. Random/systematic jitters at $\xi = 10.0$.

DAMPING RATIO= 10.0000 NATURAL FREQ=3455.000 BANDWIDTH=7000.0000

N	RANDOM-JITTER	SYSTEMATIC-JITTER
1	.00000E+00	.00000E+00
11	.10434E+02	.47700E+02
21	.13281E+02	.59955E+02
31	.15014E+02	.66678E+02
41	.16270E+02	.70928E+02
51	.17261E+02	.73831E+02
61	.18083E+02	.75991E+02
71	.18788E+02	.77779E+02
81	.19406E+02	.79393E+02
91	.19960E+02	.80898E+02
101	.20461E+02	.82289E+02
111	.20921E+02	.83548E+02
121	.21347E+02	.84680E+02
131	.21744E+02	.85715E+02
141	.22116E+02	.86686E+02
151	.22468E+02	.87616E+02

damping factor has small value.

As the results of Table 1,2,3,4, the normalized mean-square random and systematic jitters are 169.82[dB], 400.3[dB] and 43.967 [dB], 120, 15[dB] and 23.925[dB], 83.809 [dB] and 20.461[dB], 82.289[dB] according to the 0.8, 2.0, 4.0, 10.0 of damping factor when the magnitudes are the output of 101-th repeater using PLL for the timing recovery and are normalized by the output jitter of first repeater. Also, we can recognize the systematic jitter is accumulated more than the random jitter and the magnitude of jitter is decreased according to the growth of damping factor to 10.

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