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Some Effects of Sliding Velocity on The Elastohydrodynamic Squeeze Films

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탄성 경계상 압착막에 대한 미끄럼 속도의 영향

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요 약

얇은 유막위의 타이어 변형해석이나, 압착막댐퍼 설계등의 해석에 응용되는 변동 하중하의 탄성 경계상 유체 압착막의 해석을 Newton-Raphson 반복법을 사용하여 하였다. 유막두께의 계산은 등계수 요소를 사용하여 정확한 계산을 하였고 슬라이더의 궤적은 강쇄진동의 응답곡선과 유사함을 알 수 있었다.

특히 본 연구에서는 미끄럼 속도의 영향을 고려하였으며 유체가 베어링내에 생긴 포켓에 정체하고 있어서 미끄럼의 영향은 속도가 클때를 제외하고는 큰 영향을 미치지 못함을 알 수 있었다.

1. INTRODUCTION

The analysis of squeeze films between deformable solids through an intervening viscous fluid is one of important elastohydrodynamic (EHD) lubrication problems which arises in the study of natural and artificial joints as well as mechanical joints, compliant slider, elastomeric seals, EHD squeeze film dampers, and the viscous hydroplaning of tires.

The theoretical works reported in [1] and [2] all dealt with cases of deformable solids composed of materials having high elastic moduli and further were restricted to somewhat idealized loadings. Applications and extensions of the work reported in [3] which studied for the case of low modulus material, were made to the viscous hydroplaning problem; [4-5]. Recently the effect of viscoelasticity and fluctuating load was studied in [6] and [7].

Many of the above papers use the solution method described by Rohde and Oh [8] which transforms the original nonlinear equations to the linear ones through a rather complex procedure by using the Newton's method. In recent works [9] and [10], the Newton-Raphson method was adopted to solve the problem.

In this study, the effect of sliding on the squeeze film between a rigid-flat slider and smooth elastic half-space under oscillatory loading is investigated. The half-space is assumed to be a linear elastic solid and the lubricant to be an incompressible, Newtonian fluid. The solution scheme used here is based on that of Yoo [10]. And an attempt is made to calculate the elasticity

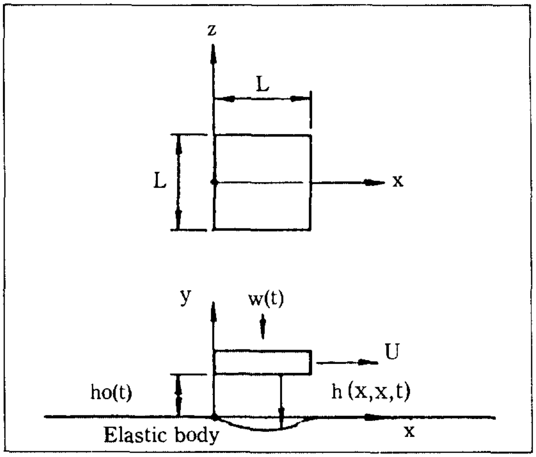

Fig. 1. *Problem geometry*

equation more accurately by taking advantages of characteristic of isoparametric element.

2. ANALYSIS

Fig. 1 shows a schematic configuration studied. A square slider under oscillatory load is sliding on the smooth elastic half-space. Thus a thin viscous fluid film is being squeezed between the slider and the half-space. The analysis of this type of EHD lubrication centers on the calculation of lubricant pressure and film thickness. These two quantities are related by the Reynolds equation on the one hand, and the elasticity equation on the other. The result is a nonlinear integro-differential equation, which may include time as a parameter.

The Reynolds equation for this problem is

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)= -6U\frac{\partial h}{\partial x}+12\frac{\partial h}{\partial t} \tag{1}$$

and boundary and initial conditions are

$$\begin{aligned} &p=0 && \text{at } x=0,\ L\ ;z=L/2 \\ &\frac{\partial p}{\partial z}=0 && \text{at } z=0 \\ &p(x,z,t)\geq 0 && 0\leq x\leq L,\ -L/2\leq z\leq L/2,\ t\geq 0 \\ &p=0 && \text{at } t=0 \end{aligned} \tag{2}$$

The film thickness, h(x, z, t), between the slider and smooth elastic half-space is the sum of $h_o(t)$, the distance between the slider surface and the plane that would be occupied by the surface of half-space when unloaded, and $h_1(x, z, t)$, the vertical displacement of the surface of the elastic half-space when loaded:

$$h(x,z,t)=h_o(t)+h_1(x,z,t) \tag{3}$$

Boussinesq's formula can be used to express $h_1(x, z, t)$ in terms of the surface pressure distribution and the material constant ν (poisson's ratio) and E (Young's modulus) of the elastic half-space; [11].

$$h_1(x,z,t)=\frac{1-\nu^2}{\pi E}\int_{-L/2}^{L/2}\int_0^L\frac{p(x,z,t)}{[(x_0-x)^2+(z_0-z)^2]^{1/2}}dxdz \tag{4}$$

In all cases, we will require that at each instant of time the resultant of the pressure on the surface of the slider equals a prescribed load, w(t), i.e.,

$$\int_{-L/2}^{L/2}\int_0^L p(x,z,t)dxdz=w(t) \tag{5}$$

The preceding formulation was used to analyze the problem of the EHD squeeze film with water being the intervening fluid subject to constraint (5). The low pressure level that is present during such events in addition to the small value for $d\mu/dp$ for water in this pressure range permits the simplifying assumption of a constant fluid viscosity. By using following dimensionless parameters, we can get following non-dimensional equations (7—9) and constraint condition (10).

$$X=\frac{x}{L},\ Z=\frac{z}{L},\ P=\frac{p}{P_{ref}},\ H=\frac{h}{h_{oo}},\ T=\frac{P_{ref}h_{oo}^2}{12\mu L^2}t,\ \text{where } h_{oo}=h_o(0) \tag{6}$$

$$\frac{\partial}{\partial X}\left(H^3\frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial Z}\left(H^3\frac{\partial P}{\partial Z}\right) = -U_s\frac{\partial H}{\partial X}+\frac{\partial H}{\partial T} \tag{7}$$

where $U_s=\dfrac{6U\mu L}{h_{oo}^2P_{ref}}$

$$P=0 \quad \text{at } X=0,\ 1;\ Z=1/2$$
$$\frac{\partial P}{\partial Z}=0 \quad \text{at } Z=0$$
$$P(X,Z,T)\geqq 0 \quad 0\leqq X\leqq 1,\ -1/2\leqq Z\leqq 1/2,\ T\geqq 0 \tag{8}$$
$$P=0 \quad \text{at } T=0$$

$$H=H_o(T)+\text{SCALE}\int\frac{P}{R'}d\Omega \tag{9}$$

where $\text{SCALE}=\frac{(1-\nu^2)}{\pi}\frac{P_{ref}}{E}\frac{L}{h_{oo}}$

R' = distance between P and H points

$$\int P\ d\Omega=W(T) \tag{10}$$

where $W=\frac{w}{P_{ref}L^2}$

We will make use of a fully implicit scheme for the second term of the right side of Eq. (7).

$$\frac{\partial H}{\partial T}=\frac{H(T)-H(T-\Delta T)}{\Delta T} \tag{11}$$

To obtain P(X, Z, T) the finite element method is used. The approximate pressure distribution at time T is thus

$$P(X,Z,T)=\sum_{i=1}^{n}P_i(T)N_i(X,Z) \tag{12}$$

where $P_i(T)$; nodal pressure at node i

n; number of nodes

Applying the Galerkin's approximation to Eq. (7) and (8), we get the following system of equation; [12].

$$K_{ij}P_j+f_i=0 \tag{13}$$

where

$$K_{ij}=\int_\Omega\left(\frac{\partial N_i}{\partial X}H^3\frac{\partial N_j}{\partial X}+\frac{\partial N_i}{\partial Z}H^3\frac{\partial N_j}{\partial Z}\right)d\Omega$$

$$f_i=\int_\Omega\left(-U_s\frac{\partial H}{\partial X}+\frac{\partial H}{\partial T}\right)N_i d\Omega$$

The calculation of the second term of Eq. (9) is done by making use of isoparametric element and Gaussian quadrature.

$$P=\Sigma N_kP_k,\quad X=\Sigma N_kX_k,\quad Z=\Sigma N_kZ_k$$

$$R'=\sqrt{(X_o-X)^2+(Z_o-Z)^2}=\sqrt{(X_o-\Sigma N_kX_k)^2+(Z_o-\Sigma N_kZ_k)^2} \tag{14}$$

where k = 1, 2, … (node)

Thus

$$\text{SCALE}\int\frac{P}{R'}d\Omega=\text{SCALE}\sum_l\int\frac{\Sigma N_kP_k}{R'}\det J\ d\xi d\eta=\text{SCALE}\sum_l\sum_i\sum_j a_ia_j\frac{\Sigma N_kP_k}{R'}\det J \tag{15}$$

where l = 1, 2, … (element)

i, j = 1, 2, … (Gauss order)

a_i, a_j = weighting factor

Since Eq. (9) is highly nonlinear, the convergence of direct iteration is not guaranted. For a fast convergence the Newton-Raphson iteration scheme which will converge in a finite number of steps is introduced here. Eq. (9) can be expressed as

$$\varphi(P)=K(P)_{ij}P_j+f(P)_i=0 \tag{16}$$

The differential form of this equation is

$$\frac{\varphi}{\partial P_i}dP_i=K\ dP_i+dKP_i+df=K\ dP_i+AdP_i+BdP_i \tag{17}$$

where $K_{ij}=\int_\Omega\nabla N_i^TH^3\nabla N_jd\Omega$

$$A_{ij}=\int_\Omega N_i^T(\nabla N_kP_k)N_j3H^2\frac{\partial H}{\partial P}d\Omega$$

$$B_{ij}=\int_\Omega N_i^T\frac{\partial}{\partial P}\left(-U_s\frac{\partial H}{\partial X}+\frac{\partial H}{\partial T}\right)N_jd\Omega$$

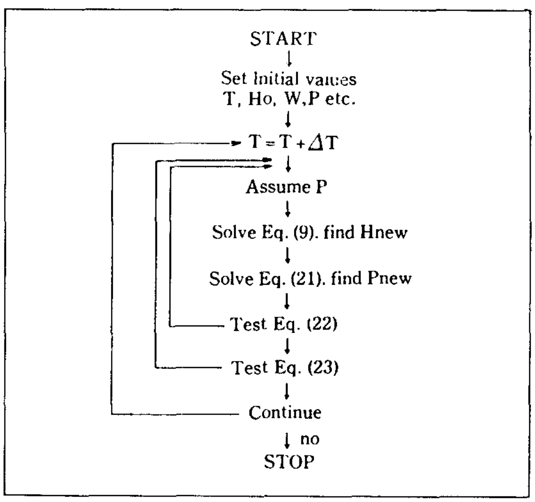

Fig. 2. *Solution procedure*

Usually sliding velocity is independent of pressure P, so B_{ij} is

$$\int_\Omega N_i^T\left(-U_s\frac{\partial}{\partial P}\left(\frac{\partial H}{\partial X}\right)+\frac{\partial}{\partial P}\left(\frac{H(T)-H(T-\triangle T)}{\triangle T}\right)\right)N_j d\Omega$$

$$=\int_\Omega N_i^T\left(-U_s\frac{\partial}{\partial X}\left(\frac{\partial H}{\partial P}\right)+\frac{1}{\triangle T}\frac{\partial H}{\partial P}\right)N_j d\Omega \quad (18)$$

and

$$\frac{\partial H}{\partial P}=\frac{\partial}{\partial P}\left(H_o(T)+\mathrm{SCALE}\int_{-1/2}^{1/2}\int_0^1\frac{P}{((X_o-X)^2+(Z_o-Z)^2)^{1/2}}dXdZ\right)$$

$$=\mathrm{SCALE}\int_{-1/2}^{1/2}\int_0^1\frac{1}{((X_o-X)^2+(Z_o-Z)^2)^{1/2}}dXdZ \quad (19)$$

The above expressin $\partial H/\partial P$ is valid since the integral boundary is independent of P. The tangential matrix $\partial\varphi/\partial P_i$, is

$$\frac{\partial\varphi}{\partial P_i}=K+A+B \quad (20)$$

Using the tangential matrix the iteration procedure is, [12]

$$\left(\frac{\partial\varphi}{\partial P_i}\right)^n\triangle P_i{}^n+\varphi^n=0$$
$$P_i^{n+1}=P_i^n+\triangle P_i^n \quad (21)$$

where $\triangle P^n$; pressure increment

n; number of iteration

The iteration process is terminated when

$$\Sigma\left\{\left(\frac{\triangle P_i^n}{P_i^n}\right)^2\right\}^{1/2}\leq\epsilon_1 \quad (22)$$

and

$$\left|\frac{\int_{-1/2}^{1/2}\int_0^1 P^n dXdZ-W(T)}{W(T)}\right|\leq\epsilon_2 \quad (23)$$

where ϵ_1 and ϵ_2 are suitably small numbers. Fig. 2 shows a schematic solution procedure.

3. NUMERICAL RESULTS AND DISCUSSION

Fig. 3 shows the prescribed non-negative loadings of sinusoidal form, namely

$$W(T)=WO(1+e\cdot\sin(\theta-\frac{\pi}{2})) \quad (24)$$

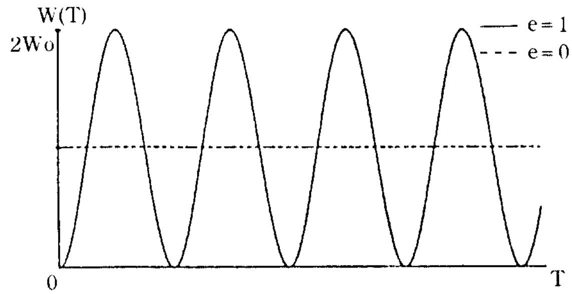

Fig. 3. *Sinusoidal load W versus time T*

where $\theta=\frac{2\pi}{\tau}T$

The bearing volume is defined as following to see the changes of the volume of fluid entrapped in the bearing.

$$B_v=\int H d\Omega \quad (25)$$

Parameters for this problem are the nominal

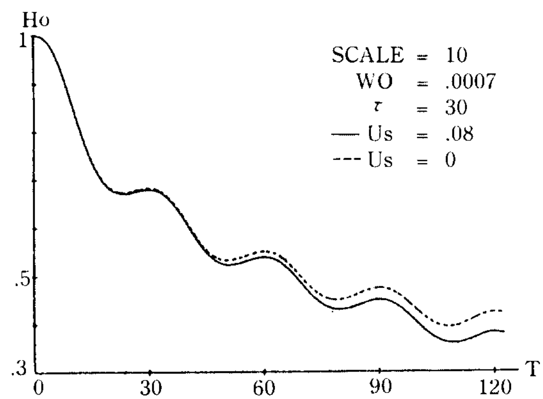

Fig. 4. *Film thickness Ho versus time T*

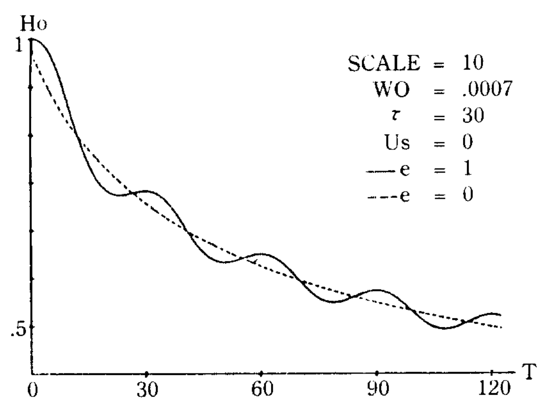

Fig. 5a. *Film thickness Ho versus time T*

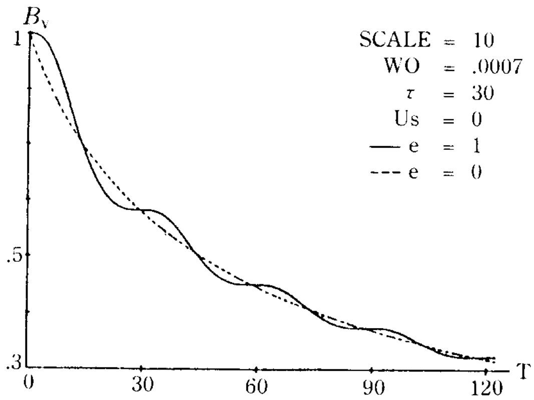

Fig. 5b. *Bearing volume Bv versus time T*

load WO, the period τ, the compliance factor SCALE, and the sliding velocity Us. Fig. 4 to Fig. 6 show some effect of sliding on the trajectories of H_o and Bv. In fact the sliding velocity of 0.08 has no significant effect on the shape of H_o (T) versus

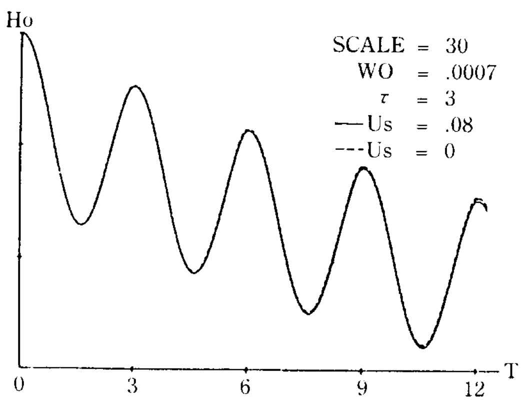

Fig. 6a. *Film thickness Ho versus time T*

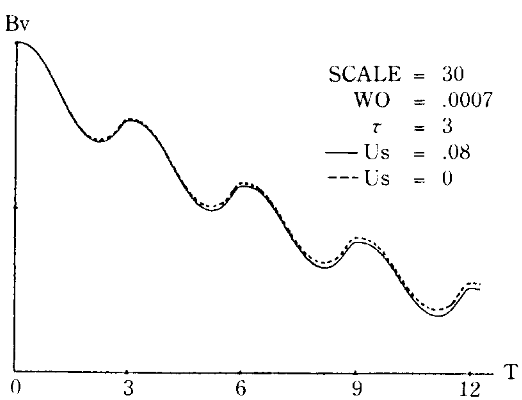

Fig. 6a. *Bearing volume Bv versus thim T*

T curve. Here we can see that the trajectory of H_o with or without sliding velocity resembles response of a damped oscillation. The effect of load conditions and the fluid entrapment phenomenon can also be seen in Fig. 5. The case of small period and large compliance factor is depicted in Fig. 6 where we can see almost periodic response of H_o trajectory.

Fully 3-dimensional film thickness H distributions as function of time T is presented in Fig. 7. As we can see, the film thickness distributions when the time rate of change of load is negative

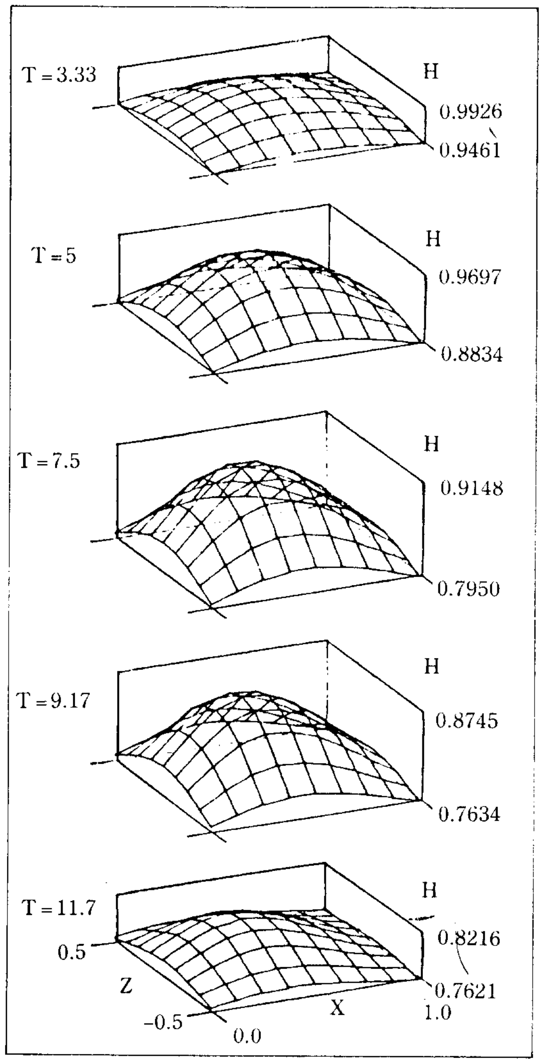

Fig. 7. *Film thickness distribution H as function of time T (SCALE = 30, Wo = 0.0007, τ = 15, Us = 0.08*

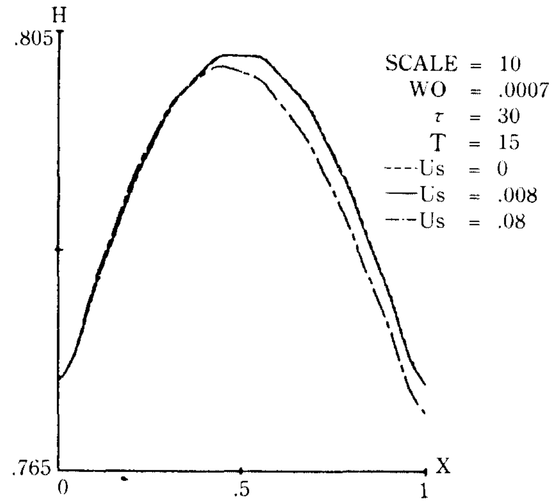

Fig. 8a. *Center line film thickness distribution H as function of sliding velocity Us*

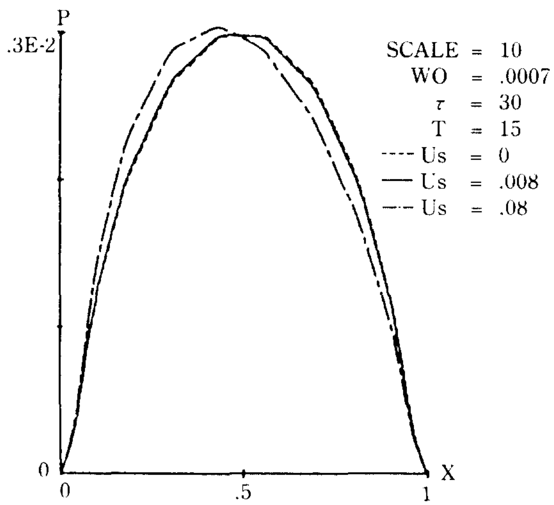

Fig. 8b. *Center line pressure distribution P as function of sliding velocity Us*

are somewhat different at the corner in comparison with the distributions when the time rate of change of load is positive. One Fig. 8 we see the sliding velocities have little effect on the film thickness distribution except large since fluid is entrapped in the pocket which develops in the deformable body.

4. CONCLUSION

A typical cyclic-film-thickness distribution is obtained. As a pocket develops in the deformable body, fluid is entrapped. Thus the pressure distribution in the interior region is almost flat. The slider trajectory with time when the sliding is present is slightly lower than that of no-sliding. The influence of sliding is thus little but large values of sliding versus The almost periodic film thickness varsus time responses are found in the case of small period and large compliance factor.

The numerical scheme used here is general and can be applicable to other problems, such as the dynamically loaded EHD connecting rod problems and viscous hydroplaning of tires.

Nomenclature

SCALE	:	elastic modulus
h, H	:	film thickness; dimensional, non-dimensional
h_o, H_o	:	displacement when unloaded; dimensional, non-dimensional
h_1	:	elastic displacement
L	:	length of the slider
N_i	:	shape function
p, P	:	pressure; dimensional, non-dimensional
P_{ref}	:	reference pressure
SCALE	:	compliance parameter
t, T	:	time, dimensional, non-dimensional
U	:	velocity of the slider
U_s	:	sliding velocity parameter
w, W	:	load; dimensional, non-dimensional
WO	:	non-dimensional nominal load
(x,z), (X,Z)	:	rectangular coordinates; dimensional, non-dimensional
ΔT	:	non-dimensional time increment
ν	:	poisson's ratio
ξ, η	:	natural coordinates
ϵ_1, ϵ_2	:	error limits
τ	:	non-dimensional period
μ	:	viscosity
Ω	:	integral boundary

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