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Research Paper—

Goal Setting in Multiple Criteria Decision Making

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Abstract

The effects of goal setting in the context of Multiple Criteria Decision Making (MCDM) are classified into two types: internal and external. In the internal models, the impact of the changed goal is limited only to the other goals in MCDM model. By contrast, in the external models, the impact is limited to the factors not included in the MCDM model. In fact, most real world examples of goal setting have the nature of mixed models. To assist in the goal setting process, the framework named Goal Setting Support (GSS) is developed. The GSS helps decision-makers for mixed models to 1) make internal trade-offs in a way that guarantees non-dominancy after the trade-offs, and 2) evaluate achieved goals systematically. The GSS can be used in creating Decision Support Systems that will allow interactive goal setting.

1. Introduction

Setting goals wisely is a very important part of managerial decision making. When there exist some structural relationships among various objectives, goal setting should be undertaken in the context of Multiple Criteria Decision Making (MCDM). Of the several types of MCDM, this research particularly focuses on Multiple Objective Decision Making (MODM) problems as formulated in (1)−(2).

$$\underset{\underline{X}}{\text{maximize}}\ [f_1(\underline{X}),\ f_2(\underline{X}),\ \ldots,\ f_k(\underline{X})] \tag{1}$$

subject to

$$\underline{X} \in C \tag{2}$$

where $\underline{X}$: decision variable vector, $f_i(\underline{X})$: continuous objective function i, and $C = \{\ \underline{X} \mid g_j(\underline{X}) \leqq b_j,\ j=1, \ldots, m\ \}$: a convex set.

We assume that the goal setting process is an adaptive process which requires information about

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1) the impact of a certain goal on the other goals, in the form of marginal rate of substitution, and
2) evaluation of currently achieved goals in comparison with reference values.

Tho objective of this research is therefore to develop a framework namely, the Goal Setting Support (GSS) process that can facilitate the adaptive goal setting process.

The remaining sections are organized as follows. Section 2 investigates the effects of goal setting in MCDM, and section 3 develops a way to generate an Efficient Marginal Rate of Substitution (EMRS) that guarantees a non-dominated solution after trade-offs. Sections 2 and 3 are thus primarily concerned with generating alternative solutions via trade-offs. Section 4 deals with evaluation of goals, and section 5 finally synthesizes the framework of GSS by integrating the notions and techniques developed in the previous sections. Section 6 provides an illustrative example.

2. Effects of Goal Setting in MCDM

The impacts of goal setting can be categorized into two mutually exclusive types : *internal*, in which the impacts of a goal are limited only to the other goals included in the MCDM model ; and *external*, in which the impacts of a goal are limited only to the factors not included in the MCDM model.

2.1. Internal Model

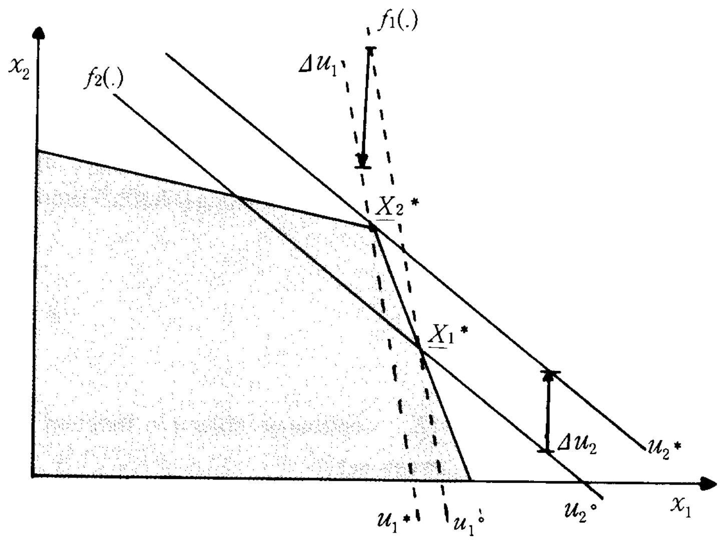

Figure 1. Conflicting Objectives

In the Internal Model, if the goal of an objective $f_p(\underline{X})$, is changed, the change affects only the other goals $f_i(\underline{X})$, $i=1, \ldots, k$, $i \neq p$ in the model (1)–(2). Suppose a simple example in Figure 1 which maximizes two linear objective functions with linear constraints. In the case diagrammed in Figure 1, the initial goal of $f_2(.)$ is set at u_2°. Under these circumstance, the optimal goal of $f_1(.)$ can be found at the non-dominated point $\underline{X}_1^*$. To improve $f_2(.)$ from u_2° to the u_2^* level, $\underline{X}_1^*$ should move toward $\underline{X}_2^*$ along the boundary line, which results in diminishing $f_1(.)$ by Δu_1. In this case, the accomplishment of $f_1(.)$ and $f_2(.)$ are *conflicting* with each other.

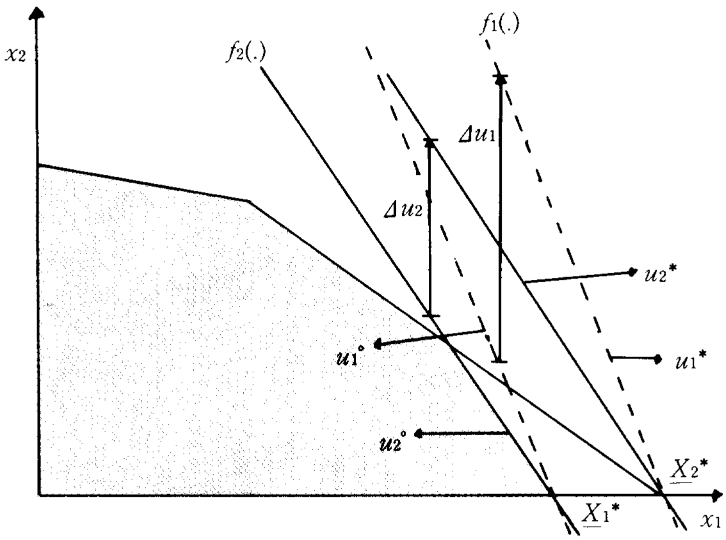

Figure 2. Complementary Objectives

In the case shown in Figure 2, however, the initial goal of $f_2(.)$ is u_2° at point $\underline{X}_1^*$, and the $f_1(.)$ can be improved while the $f_2(.)$ is improved without cost up to the point $\underline{X}_2^*$. In this case, the accomplishment of $f_1(.)$ and $f_2(.)$ are *complementary* with each other. *Complementary* objectives imply that the current solution is a dominated solution. To enhance the goals, the dominated solution should be identified and moved to a non-dominated solution. The issue of finding non-dominated solutions under complementary objectives is handled in the next section.

2.2. External Model

In other cases, the impacts of a goal change are limited to the external factors (such as costs) that are not included in the MCDM model; such cases are examples of the External Model. For example, to expand the goal of plant capacity, we may need actual investment, which may be represented by a step-function as in Figure 3. If we compare the actual cost with the shadow price curve of the goal constraint (which could have been acquired by parametric programming, again as in Figure 3), we can easily identify the upper and lower bounds of beneficial investment, as well as the amount of investment that maximizes the actual benefit, represented as (shadow price – actnal cost).

2. 3. Mixed Model

Cases in which the impacts of a goal change are both internal and external may be classified under the Mixed Model. Most real world decision-making takes place under Mixed Model circumstances.

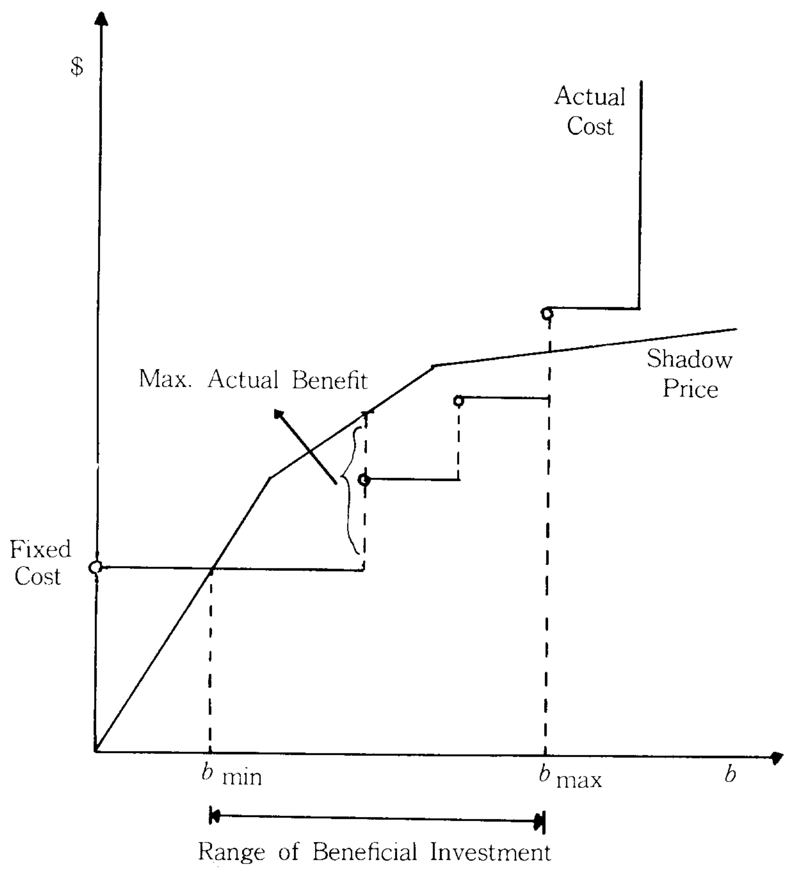

Figure 3. Comparison of Shadow Price Curve and Actual Cost Curve

3. Generation of Efficient Marginal Rate of Substitution

This section describes a process by which the Marginal Rate of Substitutions (MRS) are generated from the model specifications rather than from information provided by the decision-maker (DM), on which the method of Geoffrion [5] and Interactive Goal Programming [3] are dependent. The generated MRS can reduce the burden on the DM by enabling him to concentrate on making a preference judgment based upon the already-generated MRS.

A few methods utilize the generated MRS ; well known methods are the Surrogate Worth Trade-off method (SWT) [7] and the method of Zionts-Wallenius (ZW) [18]. SWT utilizes the generalized Lagrangian multipliers at the non-dominated solution set in the context of nonlinear programming, with the multiplier λ_{ij} representing the MRS between the objectives i and j. Trade-offs in SWT, however, are possible between only two objectives at a time. On the other hand, ZW utilizes W_{ij}, which is the decrease of the objective i that results from introducing a unit of the efficient non-basic variable x_j into the solution. The efficient non-basic variable is one which, when introduced into the basic solution, cannot increase one objective without decreasing at least one other objective.

Suppose the formulation in the model (4)—(6) to generate MRS.

$$\underset{\underline{X}}{\text{maximize}}\ f_p(\underline{X}) \tag{4}$$

subject to

$$f_i(\underline{X}) = u_i\ ,\ i = 1, \ldots, k,\ i \neq p \tag{5}$$

$$\underline{X} \in C \tag{6}$$

Since this study uses the model (4)—(6) to generate the MRS which guarantees non-dominancy after the trade-offs, let us define a term for that concept.

Definition : Efficient Marginal Rate of Substitution(EMRS) is a Marginal Rate of Substitution (MRS) that guarantees the non-dominancy of the new solution achieved through the trade-off that the MRS represents.

The definition of EMRS implies that we first have to find an initial non-dominated solution. The EMRS is particularly useful when the DM feels that some goals are over-satisfied, while some other goals have not been fully met. We will refer to the method of generating EMRS developed in this study as the "Method of Generating EMRS(GEMRS)". The Model (4)—(6) implies that the DM freezes all goals but one, and maximizes the achievement of the unfrozen goal. If there are more than two unfrozen goals, we cannot uniquely define a best solution, unless the preference function between the two goals is explicitly defined. If there is just one unfrozen goal, however, we can easily obtain the most preferred solution by maximizing the achievement of the unfrozen goal. This rationale implicitly forces us to adopt the model (4)-(6) for the development of GEMRS.

The ϵ-constraint method uses a model very similar to (4)-(6) ; the only difference is that the ϵ-constraint method uses inequality goal constraints in (5). It is well known that the ϵ-constraint method provides a non-dominated solution [6 ; p. 54], which makes the model used in that method a reasonable choice for generating a set of non-dominated solutions [8 ; p. 250].

Since the lower bounds in the ϵ-constraint method are not necessarily the same as the achieved goal levels, however, the model (4)-(6) is better to generate MRS in such a way to improve under-satisfied goals at the expense of over-satisfied goals.

The model (4)-(6) used in GEMRS is similar to the goal programming model, as defined by (7)-(10).

$$\underset{\underline{X},\, d}{\text{minimize}} \sum_{i=1}^{k} d_i^- \tag{7}$$

subject to (8)

$$f_i(\underline{X}) + d_i^- = u_i^\circ, \qquad i=1,\dots,k \tag{9}$$

$$d_i^- \geq 0\ , \qquad i=1,\dots,k \tag{10}$$

$$\underline{X} \in C$$

where $\underline{d} = [d_1^-, d_2^-, \dots, d_k^-]$ (11)

However, there is no guarantee that the solution obtained through the goal programming is a non-dominated solution. The other disadvantage of goal programming as a tool for supporting trade-offs is that the goal programming model cannot control the generation of MRS effectively. For these reasons, we adopt the model (4)-(6) in GEMRS.

Let us denote the current goal levels as u_i°, $i=1,\dots,k$. Suppose that the objective function p in (4) is under-satisfied, while some objectives in (5) are over-satisfied. In this case, the values in (5) may be somewhat diminished to improve the achievement of objective p. We assume that *DM* has an idea about how much the objective p should be improved and what sacrifices of goals $i=1,\dots,k$, $i \neq$ p are acceptable in return. We will label the desired increment or decrement of objective i as Δu_i. Note that the Δu_i's are not necessarily the same as the *EMRS*; they represent only marginal rates of substitution desired by the *DM*. To generate the *EMRS*, *GEMRS* utilizes the model (12)-(14) which reflects the desired *MRSs*.

$$\underset{X}{\text{maximize}}\ f_p(\underline{X}) \tag{12}$$

subject to

$$f_i(\underline{X}) = u_i^\circ + \Delta u_i, \quad i=1,\dots,k,\ i \neq p \tag{13}$$

$$\underline{X} \in C \tag{14}$$

The Δu_p is not explicitly shown in the model, but it will be used in the evaluation of the achieved goal p.

Let us denote the optimal feasible solution to (12)-(14) as $\underline{X}_p^*$. If the model (12)-(14) does not have a feasible solution, however, we have to adjust the goals in (13). To assist in this process, the modified formulation of (15)-(18) is very useful.

$$\underset{X,\, d}{\text{maximize}}\ f_p(\underline{X}) - \Big(\sum_{\substack{i=1 \\ i \neq p}}^{k} M_i\, d_i^-\Big) \tag{15}$$

subject to

$$f_i(\underline{X}) + d_i^- = u_i^\circ + \Delta u_i, \qquad i=1,\dots,k,\ i \neq p \tag{16}$$

$$\underline{X} \in C \tag{17}$$

$$d_i^- \geq 0\ ,\ i=1,\dots,k,\quad i \neq p \tag{18}$$

When the desired goals $(u_i^\circ + \Delta u_i)$ in (16) cannot be achieved, the positive d_i^- terms can provide the amount of under-achievement. M_i is a very large positive number that drives the values of the d_i^- toward zero, if possible, with the highest priority. This implies that the achievement of goals in constraints has a higher priority than the improvement of the objective function. If there is a need to set priorities among the objectives $i \neq p$, we may set different levels of values on M_i.

When any d_i^- is positive, we have to adjust the goals in (16) until all d_i^-'s become zeroes. If the diminution of goal by the amount of positive d_i^- is not acceptable to *DM*, such an objective may be set as an objective function in (15). In this way, solve the model (15)-(18) iteratively until all d_i^-'s in the model (15)-(18) are zeroes. That model then becomes for practical purposes the same as (12)-(14). By solving the model (15)-(18) with all $d_i^- = 0$, we can find a tentative MRS :

$$(\Delta u_1,\ \Delta u_2, \ldots, f_p(\underline{X}_p^*) - u_p^\circ, \ldots,\ \Delta u_k) \tag{19}$$

If $f_p(\underline{X}_p^*) - u_p^\circ \geqq \Delta u_p$, the new solution by (15)-(18) provides better goal achievement than the acceptable level of goal p, and the *DM* will therefore prefer the trade-off in (19). If $f_p(\underline{X}_p^*) - u_p^\circ < \Delta u_p$, however, the computed achievement of goal p is worse than the desired goal level, which tends to make the *DM* dislike the current *MRS*. The *DM* may then want to repeat the above process after additional adjustments of the goal levels of $i = 1, \ldots, k$, $i \neq p$. In any case, the current *MRS* still does not have any guarantee of non-dominancy.

To find the *EMRS*, we must check the signs of Lagrangian multipliers in (13). When the quantitative model (12)-(14) is a linear programming model, the shadow prices correspond to the Lagrangian multipliers and appear as by-products of the simplex-method solution process. For the case of linear programming, let us formalize below the condition of being non-dominated.

Theorem 1

A solution is a non-dominated solution to the linear model (15)-(18) if and only if

1) $d_i^- = 0$, $i = 1, \ldots, k$, $i \neq p$
2) All shadow prices in (16) are negative
3) The solution is non-degenerated.

Proof.

1) $d_i^- > 0$ means that the current goal is infeasible. Therefore, all d_i^-'s should be zero to ensure a feasible solution.

2) Assume the solution is non-degenerated. If the shadow price of goal i in terms of objective p, $\lambda_{ip}(i \neq p)$ is positive, then goal i is complementary with goal p, and both goals can be improved simultaneously. Therefore, the current solution is by definition not a non-dominated solution. When the shadow price is zero, goal i $(i \neq p)$ can still be improved without changing goal p. On the other hand, when the shadow price is negative, goal p cannot be improved without diminishing the fulfilment of goal i. Therefore, the solution should have strictly negative shadow price to be a non-dominated solution.

3) If the solution is degenerated, the solution has multiple shadow prices which may include both positive and negative shadow prices as at the point u_0 in Figure 4. To be non-dominated strictly, therefore, the solution should be non-degenerated [1]. [Q. E. D.]

When the model includes the minimization of some objectives, the interpreation of the shadow price and MRS should be the opposite. Nevertheless, the basic principles involved in both models are the same.

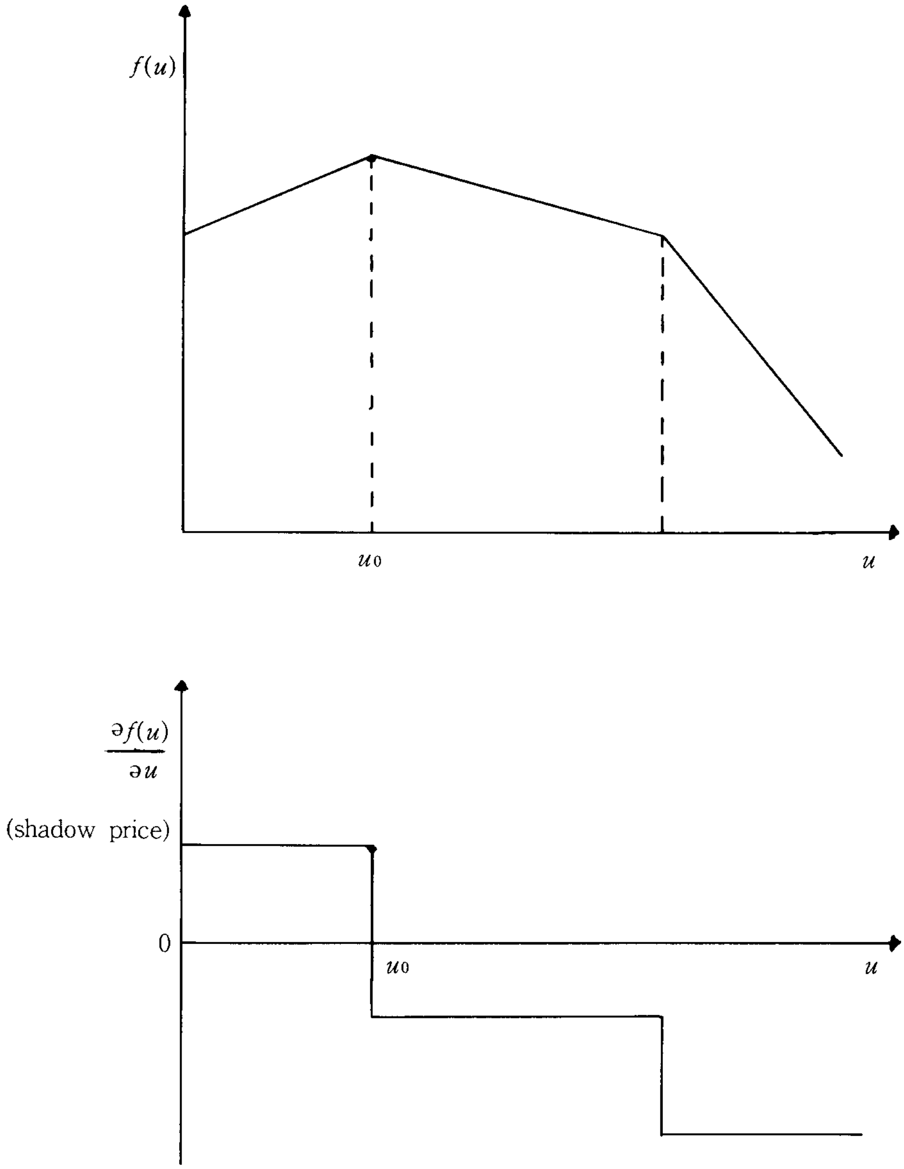

Figure 4. Shadow Prices of Degenerated Solution

4. Evaluation of Goals

In this section, we will focus our attention on the evaluation of goals. An important concept useful for the evaluation of goals is the concept of ideal values, which is used in *STEM* [2], *SEMOPS* [15], *SIGMOP* [16], Method of Displaced Ideal [17], *GPSTEM* [4], and Interactive Sequential Goal Programming [13]. The ideal points $\underline{X}_i^I$, $i=1,\ldots,k$ can be computed by maximizing $f_i(\underline{X})$, $i=1,\ldots,k$ respectively subject to the original constraints in (2). The opposite notion of ideal values is the Feasible Lower Bound (*FLB*).

Definition. The *FLB* of model (1)-(2) is defined as

$$L_i = min\{f_i(\underline{X}_r^I),\ r=1,\ldots,k\},\ i=1,\ldots,k \tag{20}$$

Assume there exists a feasible solution to the original constraints (2).

Theorem 2.
If the *FLB*s are used as lower bounds of goal constraints, there always exists at least one feasible solution.

Proof
The model with goal constraints whose lower bounds are *FLB* is:

$$\underset{\underline{X}}{\text{maximize}} \quad f_p(\underline{X}) \tag{21}$$

subject to

$$\underline{X} \in C \tag{22}$$

$$f_i(\underline{X}) \geqq L_i,\ i=1,\ldots,k,\ i \neq p \tag{23}$$

Since $f_i(\underline{X}_r^I) \geqq L_i$, $r=1,\ldots,k$, for all i by the definition of *FLB*, there always exists an ideal point $\underline{X}_r^I$ that is optimal for (21)-(22) and feasible under (23). [Q. E. D.]

Ideal values and *FLB*s with two linear objectives are graphically illustrated in Figure 5. In this case, $L_1 = min\{f_1(\underline{X}_1^I), f_1(\underline{X}_2^I)\} = f_1(\underline{X}_1^I)$. and $L_2 = min\{f_2(\underline{X}_1^I), f_2(\underline{X}_2^I)\} = f_2(\underline{X}_1^I)$. These values can be used to screen initially desired bounds of goals. *SEMOPS* and *SIGMOPS* use the concept of bounds, and *STEM*, the method of the Displaced Ideal, *GPSTEM*, and others use the concept of ideal points. There has been no consolidation between these two notions, however, despite the close relationship between them. This section therefore deals with association of the bounds of goals with ideal values and *FLB*s for the Internal Model.

When the *DM* initially set the lower bound (LB_i) and upper bound (UB_i) for the goal i, the ideal value $f_i(\underline{X}_i^I)$, L_i, and the bounds must have one of the following six relationships:

1) $L_i \leqq LB_i \leqq UB_i \leqq f_i(\underline{X}_i^I)$
2) $LB_i \leqq L_i \leqq UB_i \leqq f_i(\underline{X}_i^I)$
3) $LB_i \leqq UB_i \leqq L_i \leqq f_i(\underline{X}_i^I)$
4) $L_i \leqq LB_i \leqq f_i(\underline{X}_i^I) \leqq UB_i$
5) $LB_i \leqq L_i \leqq f_i(\underline{X}_i^I) \leqq UB_i$
6) $L_i \leqq f_i(\underline{X}_i^I) \leqq LB_i \leqq UB_i$

1) If $L_i \leqq LB_i \leqq UB_i \leqq f_i(\underline{X}_i^I)$, the whole range of $[LB_i, UB_i]$ is feasible.

2) If $LB_i \leqq L_i \leqq UB_i \leqq f_i(\underline{X}_i^I)$, LB_i does not have to be less than L_i. Hence $[L_i, UB_i]$ is more meaningful and represents tighter bounds.

3) If $LB_i \leqq UB_i \leqq L_i \leqq f_i(\underline{X}_i^I)$, the bounds are too low in comparison with the feasible range. The *DM* should reconsider the bounds and/or the model.

4) If $L_i \leqq LB_i \leqq f_i(\underline{X}_i^I) \leqq UB_i$, UB_i is too high to be achieved by the model. Feasible bounds are $[LB_i, f_i(\underline{X}_i^I)]$.

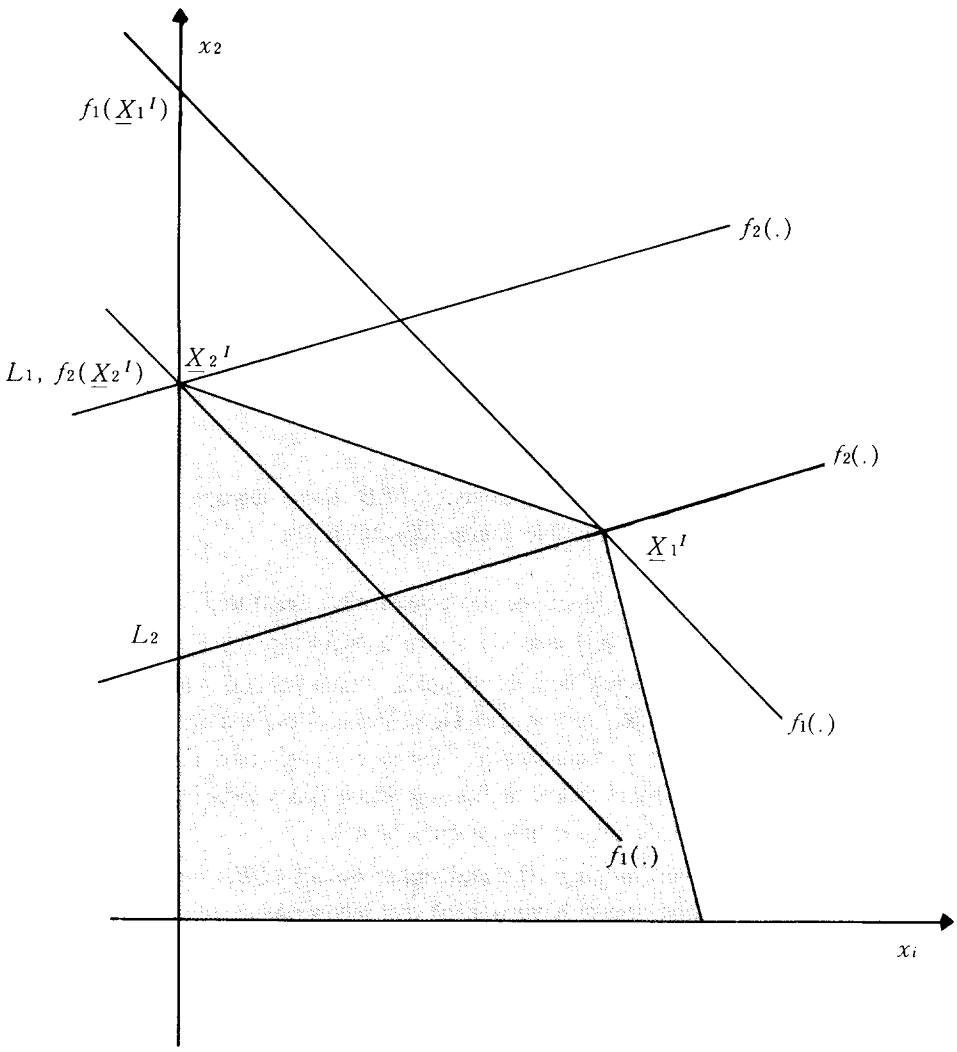

Figure 5. Ideal Points and Feasible Lower Bounds

5) If $LB_i \leqq L_i \leqq f_i(\underline{X}_i{}^I) \leqq UB_i$, the feasible range falls within the bounds. Therefore feasible bounds are $[L_i, f_i(\underline{X}_i{}^I)]$.
6) If $L_i \leqq f_i(\underline{_i}^I) \leqq LB_i \leqq UB_i$, the bounds are too high in comparison with the feasible range. The *DM* should reconsider both the bounds and the model.

Through these adjustments, the initial optimism, pessimism, or modeling errors can be screened out before the main evaluation. Let us denote the screened bounds through the above adjustments $[f_i^L, f_i^U]$. On the other hand, in the External Model, the bounds $[b_{min}, b_{max}]$ from Figure 3 can be used to screen the range of a goal. The bounds obtained from the External Model are not generic, however, because they are depent upon the levels of other goals.

5. Goal Setting Support (GSS) System

We are now ready to synthesize GSS for the mixed model using the techniques described in the previous sections. A skeletal conceptual outline of the GSS procedure runs as follows:

1) Find an initial non-dominated solution.
2) Evaluate the current solution in comparison with the bounds of the goals. If the current solution is satisfactory, stop. Otherwise, proceed to step 3.
3) Suggest a trade-off by the desired MRS.
   Compute the EMRS and evaluate it in the light of the desired MRS. Return to step 2.

The full operational procedure of GSS consists of the following six steps:

1) Formulate the initial model.
2) Set the bounds of the goals.
3) Set the initial targets.
4) Find an initial non-dominated soltion.
5) Evaluate the achieved goals. If all goals are satisfied, them stop. Otherwise, go to step 6.
6) Trade-off using the EMRS by the Internal Model and/or adjust the goal by the External Model. Go to step 5.

Let us describe each step in detail.

Step 1 (Formulate the initial model): The purpose of this step is to identify objectives, decision varibles, constraints, and their functional relationships.

Step 2 (Set the bounds of goals): This step begins by setting the required (lower bound) and aspiration level (upper bound). These bounds will be refined by the ideal values and FLBs as described in section 4, and they are used as reference points for evaluation rather than as constraints.

Step 3 (Set the inital targets): The initial targets T_i's should lie withn the screened bounds. Initial targets that are too high or too low could cause longer iterations to reach the final satisfactory goals.

Step 4 (Find an inital non-dominated solution) : Choose an objective as the objective function of the model (15)-(18). Use the initial targets to find an initial non-dominated solution.

Step 5 (Evaluate the achieved goals) : The optimal objective function value in (15) can be either of the following cases. Let $\underline{X}_p^*$ denote the optimal solution of (15)-(18).

1) $T_i \leqq f_i(\underline{X}_p^*) \leqq f_i^U$
2) $f_i^L \leqq f_i(\underline{X}_p^*) < T_i$
3) $f_i(\underline{X}_p^*) < f_i^L$

Recall that $f_i(\underline{X}_p^*)$ can never exceed f_i^U, since f_i^U has been screened by the ideal value. Let us review some recommendable --althought not required-- steps that the *DM* may take in response to each set of circumstances.

1) When $T_i \leqq f_i(\underline{X}_p^*) \leqq f_i^L$, goal i is over-satisfied. The *DM* may be satisfied with the current goal achievements and stop, or he may proceed to step 6 in an attempt to enhance other goals at the cost of some degradation of goal i.
2) When $f_i^L \leqq f_i(\underline{X}_p^*) < T_i$, goal i is somewhat underatisfied, but not badly so. The *DM* may be satisfied and stop, or he may go on to step 6 to enhance goal i at the cost of degrading other over-satisfied goals.
3) When $f_i(\underline{X}_p^*) < f_i^L$, goal i is absolutely under-satisfied, and the *DM* should proceed to step 6 to enhance goal i.

In addition to the bounds of goals with the original scale, the relative position of the standardized-scale target within the bounds can also be utilized. If the *DM* wants some trade-offs, he should go on to step 6.

Step 6 (Trade-off using the EMRS) : Set the desired MRS first, and choose the least satisfied goal as the objective function. The EMRS will be computed accordingly. To generate the EMRS, utilize the GEMRS as described in section 3. The desired MRS is used to evalute the trade-off by EMRS. The External Model might be used to help goal adjustment process upon the request of DM. At this point, return to step 5 for evaluation of the new solution.

6. An Illustrative Example

This section demonstrates the process of GSS with a numeric example. The functional form of the model is assumed to be linear and continuous. This example has 4 goals, 3 constraints, and 5 decision variables.

Step 1 (Formulate the initial model)

The initial model is (24) – (31) :

$$\text{maximize } f_1(\underline{X}) = 800X_1 + 400X_2 + 600X_3 + 500X_4 + 300X_5 \quad (24)$$

$$\text{maximize } f_2(\underline{X}) = 200X_1 + 300X_2 + 200X_4 \quad (25)$$

$$\text{maximize } f_3(\underline{X}) = 500X_2 + 1000X_3 + 400X_5 \quad (26)$$

$$\text{maximize } f_4(\underline{X}) = 200X_2 + 2000X_4 \quad (27)$$

subject to

$$5X_1 + 2X_2 + 3X_3 + 4X_4 + X_5 \leqq 2000 \quad (28)$$

$$2X_1 + 10X_2 + 5X_4 \leqq 1000 \quad (29)$$

$$4X_2 + 7X_3 + 10X_5 \leqq 1500 \quad (30)$$

$$X_1, \ldots, X_5 \geqq 0 \quad (31)$$

Step 2 (Set the bounds of goals)

The DM would like to set the upper and lower bounds of goals based upon his judgment. Suppose the initial bounds UB_i and LB_i are assigned the values given in Table 1. The DM now wants to compare the bounds with the ideal values and FLB. The ideal values are found by maximizing $f_i(\underline{X})$, $i=1, \ldots, 4$, subject to the constraints (28) – (31); they are summarized in the second column of Table 2. Taking the ideal values into consideration, the DM can screen the intal bounds. In this case, because no ideal values fall below the lower bounds, the DM need to adjust only those upper bounds whose initial level is higher than the ideal value. The screened bounds are given in column 3 and 4 of Table 2.

Step 3 (Set the initial targets)

The DM has set target points that lie within the bounds; both target points and bounds are listed in Table 3. The standardized scale of [0, 1] may also be used, as shown in Table 4.

Table 1. Desired Bounds of Goals

Goal	Lower Bound	Upper Bound
1	300, 000	400, 000
2	60, 000	80, 000
3	150, 000	250, 000
4	200, 000	300, 000

Table 2. Ideal Values and Screened Bounds of Goals

Goal	Ideal Value	Lower Bound	Upper Bound
1	346, 242. 77	300, 000	346, 242
2	84, 782. 61	60, 000	80, 000
3	214, 285. 71	150, 000	214, 285
4	400, 000. 00	200, 000	300, 000

Table 3. Target Points and Bounds of Goals

Goal	Lower Bound	Target Point	Upper Bound
1	300, 000	320, 000	346, 242
2	60, 000	75, 000	80, 000
3	150, 000	180, 000	214, 285
4	200, 000	250, 000	300, 000

Table 4. Standardized Scales of Target Points

Goal	Lower Bound	Target Point	Upper Bound
1	0	.433	1
2	0	.750	1
3	0	.467	1
4	0	.500	1

Step 4 (Find an initial non-dominated solution)

Objective 1 is selected as the objective function. Using the targets in Table 3, we can solve the model (32) − (36) and (28) − (31).

$$\text{maximize } f_1(\underline{X}) - M(d_2^- + d_3^- + d_4^-) \quad (32)$$

subject to

$$f_2(\underline{X}) + d_2^- = 75,000 \quad (33)$$

$$f_3(\underline{X}) + d_3^- = 180,000 \quad (34)$$

$$f_4(\underline{X}) + d_4^- = 250,000 \quad (35)$$

$$d_i^- \geq 0,\ i = 2, 3, 4 \quad (36)$$

and (28) − (31).

M is a very large positive number that forces the DM to fulfill the targets in (33) − (36) if possible, before attempting to improve $f_1(\underline{X})$. The d_i^- terms are used to identify the source of infeasibility if exists. Unfortunately, the model (32) − (36) and (28) − (31) does not have a feasible solution, because $d_2^- = 12,500$ is positive. The levels of goal achievement at this stage are listed in Table 5.

Table 5. Targets and Current Goal Achievements

Goal	Target	Current Level	Difference
1	320,000	322,500	+2,500
2	75,000	62,500	−12,500
3	180,000	180,000	0
4	250,000	250,000	0

To permit a feasible solution, the target of goal 2 should be adjusted to 62,500. After that adjustment is made, an optimal feasible solution is found:

$X_1 = 187.5,$

$X_2 = 0.0,$

$X_3 = 166.7,$

$X_4 = 125.0,$

$X_5 = 33.0.$

Since goal 3 has a positive shadow price (.54), however, this solution is not a non-dominated solution. Values of both goal 1 and goal 3 should be increased until all shadow prices become negative. After these complementary improvements are made, the initial non-dominated solution is found:

$X_1 = 187.5,$
$X_2 = 0.0,$
$X_3 = 179.3,$
$X_4 = 125.0,$
$X_5 = 24.5.$

The goal achievements at this point are summarized in Table 6 [Again, standardized scales like the ones in Table 4 can also be used for this purpose]. This information may also be displayed in graphical form.

Table 6. Status of Goal Achievements

Goal	LB	Target	Achieved	Difference	UB
1	300, 000	320, 000	327, 445	+ 7, 445	346, 242
2	60, 000	75, 000	62, 500	−12, 500	80, 000
3	150, 000	180, 000	189, 130	+ 9, 130	214, 285
4	200, 000	250, 000	250, 000	0	300, 000

Step 5 (Evaluate the achieved goals)

According to the initial non-dominated solution, goal 1 is over-satisfied by 7, 445 ; goal 2 is under-satisfied by 12, 500 ; goal 3 is over-satisfied by 9, 130 ; and goal 4 is exactly satisfied. On the basis of this information, the DM is most concerned about goal 2. Suppose he feels that the original target of goal 2 might have been too high, and now wants to decrease the target of goal 2 from 75, 000 to 70, 000. To achieve the adjusted target, the DM is willing to sacrifice the over-satisfied portions of goals 1 and 3. The desired MRS is then given in the last column of Table 7.

Table 7. Desired MRS

Goal	Current Achievement	New Target	Desired MRS
1	327, 445	320, 000	− 7, 445
2	62, 500	70, 000	+ 7, 500
3	189, 130	180, 000	− 9, 130
4	250, 000	250, 000	0

To focus on the improvement of goal 2, the DM has set the objective 2 as the objective function.

Step 6 (Trade-off using the EMRS)

To generate the EMRS, the model (37) − (41) and (28) − (31) is used :

$$\text{maximize } f_2(\underline{X}) - M(d_1^- + d_3^- + d_4^-) \quad (37)$$

subject to

$$f_1(\underline{X}) + d_1^- = 320,000 \quad (38)$$

$$f_3(\underline{X}) + d_3^- = 180,000 \quad (39)$$

$$f_4(\underline{X}) + d_4^- = 250,000 \quad (40)$$

$$d_1^-,\ d_3^-,\ d_4^- \geqq 0 \quad (41)$$

and (28) – (31).

It turns out that all $d_i^- = 0$ for $i = 1, 3, 4$, and the optimal value of goal 2 is 62,411. However, since the shadow price of goal 1 in (38) is positive (.1787), the current solution is not a non-dominated solution yet. Since goal 1 is the only goal that is complementary with goal 2, the non-dominated solution can be found automatically. While we search for the non-dominated solution, we notice that goals 1 and 3 have zero shadow prices, while goal 4 has a negative shadow price (–.15). Therefore, to improve goal 2, the most effective way is to diminish goal 4 to some extent.

Assume that the DM has decreased goal 4 to 240,000 and that he would like to determine the impact on goal 2. In the same way as before, a new non-dominated solution is found:

$X_1 = 197.3,$

$X_2 = 0.6,$

$X_3 = 166.4,$

$X_4 = 119.9,$

$X_5 = 33.3.$

The EMRS computed using the new non-dominated solution is summarized in Table 8. The EMRS-generated trade-off increases slightly the value of goal 2. If the decreases in goal 3 and 4 are seen as an acceptable cost for improving goal 2, then the DM will prefer the trade-off by this EMRS.

Table 8. The Efficient MRS

Goal	Previous Non-Dominated Solution	Current Non-Dominated Solution	Effcient MRS
1	327,445	327,882	+ 437
2	62,500	63,622	+ 1,122
3	189,130	180,000	− 9,130
4	250,000	240,000	−10,000

At this point, the DM may return to step 5 for evaluation. In this way, steps 5 and 6 can be iterated until the DM is satisfied with all goal achievements.

7. Discussion

Since there can exist multiple paths in finding the negative shadow prices, the solution is path dependent to that extent. Development of a guidance to a preferred path would be a very important future research topic.

Because the goal setting process is an essential part of management, the role of Decision Support Systems in goal setting will become increasingly more important. If the structural relationships between objectives can be quantified, the GSS framework can contribute to the development of DSS. In many cases, however, the goal setting includes many qualitative behavioral impacts, such as the effect of the difficulty of the goal level on performance, the effect of specific goals in comparison to general goals [11, 12], and the effects of subordinate participation in the goal setting process [10, 14]. Therefore, to extend the study on goal setting to include qualitative factors, we need to adopt modeling schemes such as Post-Model Analysis [9] that can incorporate both quantitative and qualitative factors.

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