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– Research Paper –

An Investigation of the Comparative Rate of Return

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Abstract

The minimun attractive rate of return (MARR) has been used for many years as a decision criterion in engineering economic analysis. Typically, inflation has been either ignored in such studies or considered by adjusting each of the individual cash flows associated with a project for inflation, frequently a lengthy process.

This research investigates a new decision criterion for economic analysis, the comparative rate of return (CRR). The CRR is defined to be the minimum rate of return earned on uninflated cash flows which will result in the MARR being earned on actual deflated cash flows. Given the CRR, analysis of proposed expenditures is simplified, since the analysis can be performed on the uninflated cash flows.

The research presents a derivation of the CRR and investigates its relationships to the MARR, inflation rate, project cash flows and project life.

1. Introduction

The basis for the research is an investigation of problems by using as project decision criteria only the Internal Rate of Return (IRR) and the Minimum Attractive Rate of Return (MARR). The IRR is the interest rate that will cause the present value of cash flows to equal zero (Bussey, 1978, page 212). The MARR should be equal to the highest one of the following: cost of borrowed money, cost of capital, or opportunity cost (Newnan, 1980, page 335). In traditional engineering economy analysis, a project is accepted if the IRR is greater than or equal to the MARR. This may cause an erroneous project decision under inflation. To improve the decision making process, a system must be devised that performs, under the majority of circumstances, better than MARR analysis. This might be called a "Comparative Rate of Return" system and could replace the traditional analysis previously performed.

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2. Application of CRR

The Fundamental Formulas

The general approach of engineering economic analysis involves comparison of cash flow streams which are payments or receipts. Analysis is necessary because a payment or a receipt in a future period is worth less than a payment or a receipt paid in an earlier period. In most practical investment applications it is customary to assume that the current market interest rate, i, will remain constant over time, so that the present worth of cash flow becomes

$$PW = NC_0 + \frac{NC_1}{(1+i)} + \frac{NC_2}{(1+i)^2} + \dots + \frac{NC_N}{(1+i)^N} \tag{1}$$

$$= \sum_{t=0}^{N} [NC_t/(1+i)^t] \tag{2}$$

where

NC_t is the net cash flow for period t.

Equation (2) can be used to find the internal rate of return by setting $PW=0$, thus

$$\sum_{t=0}^{N} [NC_t/(1+i^*)^t] = 0 \tag{3}$$

where

i^*=internal rate of return

Since net cash flow is defined as income less operating cost, equation (3) can be written as follows:

$$\sum_{t=0}^{N} [I_t/(1+i^*)^t] = \sum_{t=0}^{N} [OC_t/(1+i^*)^t] \tag{4}$$

The left-hand side of equation (4) is the present worth of income and the right-hand side is the present worth of costs. The internal rate of return is obtained by finding the interest rate so that the present value of income is equal to the present value of costs.

Present Worth Model with inflation Disregarded

Assuming that each cash flow is discrete and occurs at the end of its interest period, the expression for net present worth before inflation is:

$$PW_A = -P_0 + \sum_{t=1}^{N} \frac{NI_t}{(1+i)^t} \tag{5}$$

Where

PW_A : Present worth before inflation.

P_O : The initial cost of the project.

N : The useful life of the project.

i : The interest rate.

NI_t : The net cash flow (net income) before inflation for any given year t.

The annual net income is given by:

$$NI_t = (GI_t - OC_t) - (GI_t - OC_t - D_t)T_x$$
$$= (GI_t - OC_t)(1 - T_x) + D_tT_x \tag{6}$$

where

GI_t : The project gross income disregarding inflation for any given year t.

OC_t : The project operation cost disregarding inflation for any given year t.

D_t : The cost recovery or depreciation for any given year t.

T_x : The tax rate.

In the above equation, operating costs can be composed of a number of costs. The most important of these are labor, material, and energy costs.

Present Worth Model with inflation Considered

It is assumed that stated inflation rates do not vary from year to year. However, the basic models can be revised to reflect the possibility that not all inputs will be affected equally by inflation.

The basic expression for inflated present worth is:

$$PW_B = -P_O + \sum_{t=1}^{N} \frac{NI_t^*}{(1+i)^t} \tag{7}$$

where

NI_t^*: The inflated net income for any given year t.

The annual inflated net income for any given year t is:

$$NI_t^* = (GI_t^* - OC_t^*) - (GI_t^* - OC_t^* - D_t)T_x$$
$$= (GI_t^* - OC_t^*)(1 - T_x) + D_tT_x$$

where

$$GI_t^* = GI_t(1+F)^t$$
$$OC_t^* = OC_t(1+F)^t$$

So

$$NI_t^* = GI_t(1+F)^t - OC_t(1+F)^t \ (1-T_x) + D_tT_x$$
$$= (GI_t - OC_t)(1+F)^t(1-T_x) + D_tT_x \tag{8}$$

where

F : The uniform rate of inflation.

Since operating costs are usually composed of a number of costs, all of these could be inflated at different rates. As an example, the labor cost may be inflated at the rate of 10 percent, the material cost at 13 percent, and the energy cost at 8 percent. But as an assumption of this paper, the operating cost and gross income are inflated at the same inflation rate, say F.

In order to compare PW_A with PW_B, Oskounejad (1982) deflated NI_t^* of PW_B at the rate of inflation F, giving:

$$PW_B = -P_O + \sum_{t=1}^{N} \frac{NI^*_t}{(1+i)^t(1+F)^t} \tag{9}$$

From this, he demonstrated that the present worth before inflation, PW_A, is greater than the present worth after inflation, PW_B, at any rate of inflation greater than zero.

In traditional engineering economy analysis, a project is customarily considered acceptable if the IRR under the assumption of zero inflation is greater than or equal to the MARR.

If the effects of inflation rate are not included in an engineering economy study, an erroneous project decision can result. Consequently, the objective of maximizing owners' wealth is inadvertently determined. At a time when the most productive use of capital is so important, it would be foolish to ignore the anticipated effects of inflation and take the risk of undesirable project investment because of ignorance in this regard. Omitting inflation from engineering economy studies is the same as assuming that the monetary unit is a constant-valued measure of worth. This is a clearly out of tune with the present and expected future conditions in the business environment.

To avoid this error, a system which performs better than MARR, explicitly considering inflation, should be devised. This will be called the "CRR" method, which is defined to be the minimum rate of return earned on uninflated cash flows which will result in the MARR being earned on actual (constant dollar) cash flows.

If the IRR calculated from uninflated cash flows is greater than or equal to the CRR, then the project is acceptable. If it is less than the CRR, then the project should be rejected. The CRR system could replace the traditional analysis previously performed.

To illustrate, the following example is provided.

Example

Consider a project with an initial cost of \$2500, a gross income of \$5000 per year, and annual operating costs of \$2500. The life of the project is expected to be 5 years with no salvage value. The tax rate is 46 percent, the MARR is 6 percent, and the inflation rate is 8 percent annually. For simplicity, straight line depreciation is assumed without loss of generality.

From equation (6),

$NI_1 = (5000-2500)-(5000-2500-500)(.46)=1580$

$\cdot$

$\cdot$

$\cdot$

$NI_5 = (5000-2500)-(5000-2500-500)(.46)=1580$

The uninflated present worth, PW_A, can be found by using equation (5).

$$PW_A = -P_O + \sum_{t=1}^{N} \frac{NI_t}{(1+i)^t}$$

So

$$PW_A = -2500 + \frac{1580}{(1+C)^1} + \frac{1580}{(1+C)^2} + \cdots + \frac{1580}{(1+C)^5}$$

where

C: Comparative Rate of Return.

Finding the CRR by the approximation method explained at the next section,

CRR $= 7.085\%$

and

$PW_A = \$3964$

To find the present worth after inflation, the gross income and operating cost have been inflated at an annual rate of 8 percent.

From equation (8)

$NI_1^* = (5000-2500)(1+0.08)^1(1-0.46)+500(0.46) = 1688$

$NI_2^* = (5000-2500)(1+0.08)^2(1-0.46)+500(0.46) = 1805$

$NI_3^* = (5000-2500)(1+0.08)^3(1-0.46)+500(0.46) = 1930$

$NI_4^* = (5000-2500)(1+0.08)^4(1-0.46)+500(0.46) = 2067$

$NI_5^* = (5000-2500)(1+0.08)^5(1-0.46)+500(0.46) = 2213$

The present worth after inflation, PW_B, can be found by using equation (7). In order to compare PW_B with PW_A, NI_t^* of PW_B should be deflated at the rate of inflation(8 percent).

From equation (9)

$$PW_B = -2500 + \frac{1688}{(1+0.06)(1+0.08)} + \cdots + \frac{2213}{(1+0.06)^5(1+0.08)^5}$$

$$= \$3967$$

Note that the CRR is greater than the MARR and that the error in the CRR approximation method of \$3 is insignificant.

The example clearly indicates that the use of the MARR(6%) may cause an erroneous project decision under inflation conditions. Economic analysts should use the CRR (7.085%) instead of the MARR (6%) as a project decision criterion.

3. CRR Formulation and Solution

Formulation

The last section was concerned with the development of the idea of CRR and demonstrated that a CRR analysis could replace the more complex traditional analysis methods dealing with inflation.

Two present worth models were developed in the last section:

PW_A = Uninflated present worth before inflation.

PW_B = Inflated present worth after inflation.

Substituting equation (6) into equation (5) yields

$$PW_A = -|Po + \sum_{t=1}^{N} \frac{(GI_t - OC_t)(1-T_x) + D_t T_x}{(1+i)^t} \tag{10}$$

and substituting (8) into (9) gives

$$PW_B = -Po + \sum_{t=1}^{N} \frac{(GI_t - OC_t)(1+F)^t(1-T_x) + D_t T_x}{(1+i)^t(1+F)^t} \tag{11}$$

Substituting CRR for i and MARR for i in equations (10) and (11), respectively, and setting each equation equal to zero will yield the IRR for each case.

PW_A at $CRR = 0$, (12)

PW_B at $MARR = 0$. (13)

Therefore,

$PW_A = PW_B$. (14)

Let C: Comparative rate of return,

M: Minimum attractive rate of return.

Then, equations (10) and (11) become,

$$PW_A = -Po + \sum_{t=1}^{N} \frac{(GI_t - OC_t)(1-T_x) + D_t T_x}{(1+c)^t} \tag{15}$$

$$PW_B = -Po + \sum_{t=1}^{N} \frac{(GI_t - OC_t)(1+F)^t(1-T_x) + D_t T_x}{(1+M)^t(1+F)^t} \tag{16}$$

Setting the right hand sides of equations (15) and (16) equal yields

$$\sum_{t=1}^{N} \left[\frac{(GI_t - OC_t)(1-T_x) + D_t T_x}{(1+C)^t} - \frac{(GI_t - OC_t)(1+F)^t(1-T_x) + D_t T_x}{(1+M)^t(1+F)^t} \right] = 0 \tag{17}$$

For convenience, let

$$\alpha = \frac{1+C}{1+M} = \frac{1+CRR}{1+MARR,} \tag{18}$$

$f=(1+F)$, $X=(1+M)$,

$a_t = GI_t - OC_t$, $b = 1 - T_x$, and $d = T_x$

then (17) becomes

$$\sum_{t=1}^{N} \left[\frac{a_t b(1-\alpha^t) + D_t d(1-(\alpha/f)^t)}{\alpha t \times t} \right] = 0 \tag{19}$$

The task at this stage is to solve equation (19) for the constant α.

For general life $N=n$,

$\alpha^n\ [X^{n-1}(a_1 b + D_1 d/f) + X^{n-2}(a_2 b + D_2 d/f^2) + \ldots + X^1(a_{n-1} b + D_{n-1} d/f^{n-1}) + X^0(a_n b + D_n d/f^n)] - \alpha^{n-1}\ [X^{n-1}(a_1 b + D_1 d)]$

$$-\ldots-\alpha^{1}[X(a_{n-1}b+D_{n-1}d)]-\alpha^{0}[X^{0}(a_{n}b+D_{n}d)]=0 \qquad (20)$$

Note that equation (20) is polynomial in α. Hereafter, the left hand side of equation (20) is denoted $g(\alpha)$. To determine α, the Newton-Raphson method is used since it is one of the most powerful and well-known numerical methods for finding a root of $g(x)=0$.
To illustrate, the following example is provided.

Example

Consider a project with an initial cost of \$5000, annual gross income of \$10000, and operating costs of \$5000. The tax rate is 46 percent, the MARR is 6 percent, and the inflation rate is 8 percent. The life of the project is expected to be 5 years with no salvage value. For simplicity, straight line depreciation is assumed without loss of generality. The problem is to calculate α and the CRR by using the Newton-Raphson method.

From equations (18),

$x = 1.06$

$a = 5000$

$b = 0.54$

$D_t = 1000$

$d = 0.46$

$f = 1.08$

Substituting these values into (20) gives,

$$17309.35\alpha^{5}-3989.43\alpha^{4}-3763.61\alpha^{3}-3550.58\alpha^{2}-3349.6\alpha^{1}-3160=0.$$

With an initial point, α can be determined by using the Newton-Raphson method ($\alpha=1.01004$). From equation (18), CRR is 7.064 percent.

Note that nine iterations were required to find α. This method is complicated and generally impractical without using a computer to find α when $N>5$. Also, the method is even more complex if gross income and operating cost are not constant. Therefore an approximation method for finding α would be quite useful. The following section presents a derivation of such a method.

Approximation Method for Determining CRR

Inspection of equation (18) and experimentation with several example problems reveals that α is typically in the range 1.001 to 1.020. This observation leads to an approximation method approach of performing only one iteration of Newton's Method.
Then,

$$\alpha=1-\frac{g(1)}{g'(1)} \qquad (21)$$

where

$$\alpha=\frac{1+CRR}{1+MARR}$$

In order to simplify equation (20), let

$$A_k=X^{n-k+1}(a_{k-1}b+D_{k-1}d)$$

$$\begin{cases} k \neq 1 \\ k = 2,\ 3, \ldots,\ N+1 \text{ (Integer)} \end{cases}$$

$$A_1 = X^{n-1}(a_1 b + D_1 d/f) + X^{n-2}(a_2 b + D_2 d/f^2) + \ldots + X^1(a_{n-1} b + D_{n-1} d/f^{n-1}) + X^0(a_n b + D_n d/f^n)$$

From equations (20) and (21), for various project lives N,

$N \Rightarrow n$

$$\alpha = 1 - \frac{A_1 - A_2 - \ldots - A_{n+1}}{(n)A_1 - (n-1)A_2 - \ldots - (1) \cdot A_n} \tag{22}$$

and CRR can be found from equations (22) and (18), respectively.

4. Investigation of CRR with Varging Elements

This part presents a more detailed examination of the characteristics of the CRR with varying elements; i.e., the life of the project (N), the ratio of PR, the inflation rate (F), MARR, and depreciation method. The approximation method is used to calculate all CRR values in this section where PR is defined to be the ratio of the annual depreciation, Dep(N), divided by gross income per year, GI, minus annual operating costs, OC:

$$PR = \frac{\text{Dep (N)}}{GI - OC} \tag{23}$$

Effect of Project Life and PR Ratio on the CRR

Project life is varied from one to ten years and the PR ratio ranges from 0.12 to 0.40 in order to determine the effect of these variables on the CRR. The MARR is fixed at 5 percent, the inflation rate (F) at 10 percent, and the tax rate (T_x) at 46 percent. Straight line depreciation is used.

Table 1 shows that the CRR declines as the time period of the project increases. In addition, the CRR increases as the PR ratio increases. Note also that the rate of change (slope) of CRR with increasing life depends only slightly on the PR ratio.

Effect of Inflation Rate and PR Ratio on the CRR

In this section, inflation rate (F) is varied from 6 percent to 16 percent and the PR ratio ranges from 0.12 to 0.40 in order to determine the effect of these variables on the CRR. The MARR is fixed at 5 percent, the project life (N) is fixed at ten years, and the tax rate (T_x) at 46 percent. Straight line depreciation is used.

Table 2 indicates that the CRR values have a nearly straight line increase as the ratio of PR increases for fixed inflation rate (F). The value of the CRR over the various ratios of PR increases as the inflation rate increases.

Effect of MARR on the CRR

MARR is varied from 2 percent to 10 percent in order to determine the effect of this variable on the CRR. The inflation rate (F) is fixed at 10 percent, the tax rate (T_x) at 46 percent, the

PR ratio at 0.2, and the project life (N) at 10 years. Straight line depreciation is used.

Table 3 gives the CRR for various MARR values and shows the relationship of the difference, CRR-MARR, to the MARR. This difference increases exactly linearly as the MARR increases. Note also that the rate of increase in this difference as MARR increases is relatively small.

Effect of Depreciation Method and Project Life on the CRR

Three depreciation/cost recovery methods are used to investigate the effect of these methods on the CRR, with project life varying between three and ten years. These methods are:

(1) Straight Line Depreciation (SL)

(2) Sun-Of-Years Digits Depreciation (SOYD)

(3) Accelerated Cost Recovery System (ACRS)

The ACRS recovery period is three years for a project life of three or four years, and the recovery period is five years for a project life between five and ten years.

The initial cost is $1200, the gross income is $2000 per year, and annual operating cost is $1000. The MARR is fixed at 5 percent, the inflation rate (F) at 10 percent, and the tax rate at 46 percent.

Table 4 shows that the selection of the depreciation method has a small effect on the CRR. This effect, or difference, decreases as the project life increases.

5. Conclusions

There are several elements that influence the value of the CRR:

(1) The life of the project

(2) The annual net income

(3) The annual depreciation/cost recovery

(4) Inflation rate

(5) Minimum Attractive Rate of Return

(6) Tax rate

An exact method for calculating the CRR was derived, using Newton's iterative method. However, this method was found to be difficult to use in calculating the CRR. Therefore, an approximation method was developed to reduce the complexities of the exact method.

This research presented a detailed examination of the characteristics of the CRR with varying elements. The CRR has a nearly linear increase as the PR ratio increases for fixed inflation rate. It was found also that the CRR has a positive correlation with the MARR and inflation rate. However, the CRR decreases as the life of the project increases. The selection of the depreciation method has a small effect on the CRR. This effect, or difference, decreases as the project life increases.

It is recommended that the CRR replace the MARR as a project decision criterion under inflation circumstances. In an economy analysis, a project should be accepted if the internal rate of return (IRR) is greater than or equal to the CRR, and a project should be rejected if the IRR is less than the CRR.

Table 1

CRR Values with Varied Project Life (N) and PR Ratio

F=10%, MARR=5%, T_X=46%

N \ PR	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40
1	5.893	6.157	6.408	6.645	6.871	7.085	7.289	7.483
2	5.873	6.134	6.383	6.620	6.846	7.061	7.267	7.464
3	5.854	6.112	6.359	6.595	6.821	7.038	7.245	7.444
4	5.836	6.091	6.336	6.571	6.797	7.015	7.224	7.425
5	5.818	6.070	6.313	6.548	6.774	6.992	7.202	7.406
6	5.801	6.050	6.291	6.525	6.751	6.969	7.181	7.387
7	5.785	6.031	6.270	6.502	6.728	6.947	7.160	7.368
8	5.769	6.013	6.250	6.481	6.706	6.926	7.140	7.349
9	5.744	5.995	6.230	6.460	6.685	6.905	7.120	7.331
10	5.740	5.978	6.211	6.439	6.664	6.884	7.100	7.313

Table 2

CRR Values with Varied Inflation (F) and PR Ratio

MARR=5%, n=10 years, T_X=46%

PR	C R R (%)					
	F=6%	F=8%	F=10%	F=12%	F=14%	F=16%
0.12	5.493	5.624	5.740	5.845	5.939	6.024
0.16	5.647	5.821	5.978	6.119	6.247	6.363
0.20	5.795	6.013	6.211	6.390	6.553	6.702
0.24	5.940	6.201	6.439	6.657	6.857	7.039
0.28	6.079	6.384	6.664	6.921	7.158	7.376
0.32	6.214	6.563	6.884	7.182	7.457	7.711
0.36	6.345	6.736	7.100	7.439	7.754	8.046
0.40	6.472	6.906	7.313	7.693	8.048	8.380

Table 3

CRR Values with Varied MARR

N=10 years, F=10%, T_X=46%, PR=0.2

MARR	CRR	CRR-MARR
2	3.166	1.166
3	4.181	1.181
4	5.196	1.196
5	6.211	1.211
6	7.226	1.226
7	8.241	1.241
8	9.257	1.257
9	10.272	1.272
10	11.288	1.288

Table 4

CRR Values with Varied Depreciation Method And Project Life (N)

Initial Cost=1200,

Gross Income per year=2000,

Operating Cost per year=1000, MARR=5% F=10% T_X=46%

N	C R R (%)		
	SL	SOYD	ACRS
3	7.445	7.172	7.543
4	6.907	6.645	6.742
5	6.548	6.312	6.592
6	6.292	6.085	6.223
7	6.101	5.920	5.972
8	5.953	5.797	5.794
9	5.836	5.700	5.663
10	5.741	5.623	5.565

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