論 文
35~4~5

스펙트럼 추정을 위한 공분산 기구변수 격자 앨고리즘

Covariance Lattice Instrumental Variable Algorithm for Spectral Estimation

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요 약

본 논문에서는 순환 매개변수 앨고리즘에 관한 코베리언스형을 사용한 격자형 앨고리즘을 제시하였다. 이 앨고리즘은 주로 적응신호처리와 스펙트럼 추정에 적용할 수 있고 또한 시스템 식별에도 사용할 수 있다. 앨고리즘이 복잡하기는 하나 계산량은 차수에 비례하여 계산상에 있어서 효과적이라 할 수 있다. 또한 이 격자형 앨고리즘은 하드웨어 구성에 있어서 큰 장점을 가지고 있고, 몇가지 경우의 스펙트럼 추정에 대한 컴퓨터 시뮬레이션을 통해 좋은 추정결과를 얻었다.

Abstract

The last few years have seen a rapid development of so-called lattice algorithms for the fast solution of finite date algorithms. So far, most of the work on ladder form has been done for the prewindowed case.

In this paper, the covariance lattice algorithm for instrumental variable recusions is presented. This algorithm can be used in various areas of adaptive signal processing, spectral estimation and system identification. The behavior of the proposed algorithm is illustrated by some simulation results for spectral estimation.

1. Introduction

The spectral analysis from given data can be widely applied to system theory, optics, spectroscopy. oil exploration, earth quake analysis and many other areas.

The techniques based on Fast Fourier Transform (FFT) are computationaly efficient and produce reasonable results for a large data length but provide poor resolution for short data records and show Gibb's phenomenon. To alleviate this inherent limitation of FFT, the parametric procedure have been proposed 1), 2).

The signal can be modeled by time series model. Using this model, spectral estimation can be reduced to estimate time series coefficients. The system function of this model can be realized in many different forms. A recent researches show the lattice structure has more attractive features 4), 5); 1) It is computationally efficient because it belongs to fast algorithm. 2) It has an orthogonality property i.e. the (N+1)-th order lattice predictor is the same as the N-th order predictor

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接受日字 : 1986年 2月 5日

except the last section. 3) In normalized version of lattice filter, all the values are automatically scaled, making it possible to use fixed point computations.

Lattice algorithms have different forms by choosing the type of windowing the observed data, i.e. autocorrelation form, prewindowed form and covariance form 5). The covariance form is widely used in speech processing and spectral estimation, and is generally considered superior to the prewindowed and postwindowed forms 9).

To eliminate bias problem of least squares algorithms, instrumental variable (IV) method is known to be one of the most powerful algorithm 11). And the Lattice implementations of Recursive IV (RIV) for prewindowed form were presented. 4), 6), 10).

In this paper, the RIV lattice algorithms for covariance form is derived. This algorithm gives better resolution than any other lattice algorithms. And computer simulation is presented to show efficiency of this algorithm.

2. Instrumental Variable Spectral Estimation

Consider the problem of estimating the AR parameters of an autoregressive moving-average (ARMA) process. Let y(t) be a scalar ARMA process of order (N, M).

$$y(t) = -\sum_{i=1}^{N} A(i)y(t-i) + w(t) \qquad (2.1a)$$

$$w(t) = \sum_{i=0}^{M} C(i)v(t-i) \qquad (2.1b)$$

where y(t) is the output signal and v(t) is white process.

The spectral estimation problem is reduced to estimated ARMA coefficients, and the estimated spectrum is given by

$$S(w) = \frac{C(e^{jw})\ C(e^{-jw})}{A(e^{jw})\ A(e^{-jw})} \qquad (2.2)$$

where

$$A(z) = 1 + A(1)z^{-1} + \cdots + A(N)z^{-1}$$
$$C(z) = C(0) + C(1)z^{-1} + \cdots + C(M)z^{-1}$$

One of the more successful techniques for ARMA spectral estimation is based on a two step procedure in which the AR and MA parts of the model are estimated seperately. But the quality of the spectral estimates obtained by this method will depend strongly on the accuracy of the AR parameter estimates.

Since the LS method gives biased parameter estimates when disturbance process w(t) is nonwhite, the IV method is used to estimate the AR parameters.

The IV estimates of A(z) are described by

$$\theta = (Z'Y)^{-1} Z \cdot y \qquad (2.3)$$

where

$$\theta = [A(1), \cdots, A(N)]'$$
$$y = [y(0), \cdots, y(t)]'$$
$$Y = \begin{bmatrix} -y'(N-1), \cdots, -y'(0) \\ \text{〃} \qquad\qquad \text{〃} \\ \text{〃} \qquad\qquad \text{〃} \\ -y'(t-1), \cdots, -y'(t-N) \end{bmatrix}$$

Z is the instrumental variable matrix, and Z' is transpose of Z.

The different type of instrumental variants are presented in 11).

3. The General Update Formulas

In this section, we consider the ARMA model of the type depicted in (2.1a). Suppose we wish to estimate the coefficients A(i) based on data y(t) observed over an interval $S \leq t \leq T$. A least-squares estimation problem can be formulated by writing down a set of equations for the forward prediction error as follows:

$$e(t) = y(t) + \sum_{i=1}^{N} \hat{A}(i)\ y(t-i). \qquad (3.1)$$

these equations can take the several forms, depending on the range of t in (3.1). The so called prewindowed case corresponds to $O \leq t \leq T$ where y(t) is assumed zero for t<O. The prewindowed case is obtained by setting $O \leq t \leq t+N$, where y(t) is assumed to be zero for t<O and for t>T.

In this paper we will deal with the covariance case (also known as the nonwindowed case) where $S \leq t \leq T$ and no assumptions are made for t<S or for t>T. Then (3.1) is expand to

$$\begin{vmatrix} e'(S+N) \\ e'(S+N+1) \\ \cdot \\ \cdot \\ e'(T) \end{vmatrix} = \begin{vmatrix} y'(S+N) \\ y'(S+N+1) \\ \cdot \\ \cdot \\ y'(t) \end{vmatrix} + \begin{vmatrix} y'(S+N-1) \cdots y'(S) \\ y'(S+N) \cdots y'(S+1) \\ \cdot \qquad\qquad \cdot \\ \cdot \qquad\qquad \cdot \\ y'(T-1) \cdots y'(T-N) \end{vmatrix} \begin{vmatrix} \hat{A}'(1) \\ \hat{A}'(2) \\ \cdot \\ \cdot \\ \hat{A}'(N) \end{vmatrix} \qquad (3.2)$$

which can be written more compactly as

$$E_{N,S,T} = x_{S+N:T} + X_{N,S:T-1}\ A' \tag{3.3}$$

Note that the notation we will be using throughout: $X_{N,}$ $_{s:t}$ will always denote a matrix containig the observed data (y(t),S≦t≦T), arranged in block clumns, where N is the model order, as indicated on the right hand side of (3.2). For the case N = 1 (or when the order subscript is meaningless) we will drop the first subscirpt. Thus, e.g.,$X_{S+N:T} = X_{1,\ S+N:T}$ = [y(S+N), ..., y(T)]'. In the following we will define various quantities such as residuals and refection coefficients, which are obtained by projections on the space spanned by the columns of $X_{n,\ S:T}$. In this case the first subscript will again denote the order of underlying model, and the second third subscripts the range of data points on which the quantity in question depends.

The least-squares solution for A′ in (3.3) is the one which minimize the cost function

$$V_{N,S:T} = tr\{E'_{N,S:T}\ E_{N,S:T}\}. \tag{3.4}$$

As is well known, $A'_{N,\ S:T}$ satisfies normal equation

$$R_{N,S:T-1}\ A'_{N,S:T} = (X'_{N,S:T-1}\ X_{N,S:T-1})\ A'_{N,S:T} = -X'_{N,S:T-1}\ X_{S+N,T} \tag{3.5}$$

wholse solution is

$$A'_{N,S:T} = -(X'_{N,S:T-1}\ X_{N,S:T-1})^{-1} X'_{N,S:T-1}\ X_{S+N:T} \tag{3.6}$$

The parameter estimate $A'_{N,S:T}$ by instrumental variable method is given by

$$A'_{N,S:T} = -(Z'_{N,S:T-1}\ X_{N,S:T-1})^{-1} Z'_{N,S:T-1}\ x_{S+N:T} \tag{3.7}$$

The associated error vector is given by

$$\begin{aligned} E_{N,S:T} &= x_{S+N,T} - X_{N,S:T-1}(Z'_{N,S:T-1} X_{N,S:T-1})^{-1} Z'_{N,S:T-1}\ x_{S+N:T} \\ &= [I - X_{N,S:T-1}(Z'_{N,S:T-1}\ X_{N,S:T-1})^{-1}\ Z'_{N,S:T-1}]x_{S+N,T} \end{aligned} \tag{3.8}$$

The last entry e(T) of the error vector is given by

$$\begin{aligned} e(T) &= x'_{S+N:T}[I - Z_{N,S:T-1}\ (X'_{N,S:T-1}\ Z_{N,S:T-1})^{-1}\ X'_{N,S:T-1}]\sigma = x'_{S+N:T}[I - P_{ZX}]\sigma \\ &= x'_{S+N:T} P^{c}_{ZX}\sigma \end{aligned} \tag{3.9}$$

Where

$\sigma = [0 \cdots 0I]'$

$\pi = [I0 \cdots 0]'$

$P_{ZX} = Z_{N,S:T-1}(X'_{N,S:T-1}\ Z_{N,S:T-1})^{-1} X'_{N,S:T-1}$

$P^{c}_{ZX} = I - P_{ZX}$; nonsymmetric projection operator

The projection operator can be recursively updated as the projectin space X and Z is changed by the derivation of columns of x and z. More specifically, the following update formula can be derived

$$P^{c}_{Z+z,\ X+x} = P^{c}_{ZX} - P^{c}_{ZX}x\ [x'P^{c}_{ZX}z]^{-1}x'P^{c}_{ZX} \tag{3.10}$$

$$U'P^{c}_{Z+z,\ X+x}V = U'P^{c}_{ZX}V - U'P^{c}_{ZX}x\ [x'P^{c}_{ZX}z]^{-1}x'P^{c}_{ZX}V \tag{3.11}$$

where Z + z = [Z z], X + x = [X x]; matrix augmented form For time updates, $x = \sigma$, $Z = \sigma$.

$$U' \begin{vmatrix} P^{c}_{N,S:T-1} & 0 \\ 0 & 0 \end{vmatrix} V = (U'P^{c}_{N,S:T}V) - (U'P_{N,S:T}\sigma) \cdot (\sigma' P^{c}_{N,S:T}\sigma)^{-1}(\sigma' P^{c}_{N,S:T}V) \tag{3.12}$$

This is the unnormalized forward time update formular for the projection operator. By using a similar method we can obtain the following unnomalized backward time update formula:

Table 1. The definitions and the various substitutions for the correlation coefficient updates.

Z	X	z	x	U	V	$U'P^{c}_{zx}V$	Z	X	z	x	U	V	$U'P^{c}_{zx}V$
Z	X	z-	x-	x+	σ	$e^{x}_{N,s:T}$	Z	X	π	π	x+	σ	$e^{x}_{N,s:T}$
Z	X	z-	x-	x+	π	$d^{x}_{N,s:T}$	Z	X	σ	σ	x+	π	$d^{x}_{N,s:T}$
Z	X	z+	x+	x-	σ	$r^{x}_{N,s-1:T-1}$	Z	X	π	π	x-	σ	$r^{x}_{N,s-1:T-1}$
Z	X	z+	x+	x-	π	$q^{x}_{N,s-1:T-1}$	Z	X	σ	σ	x-	π	$q^{x}_{N,s-1:T-1}$
Z	X	z-	x-	π	σ	$b^{x}_{N,s:T-1}$	Z	X	π	π	σ	σ	$1-g_{N,s:T}$
Z	X	z+	x+	π	σ	$b^{z}_{N,s:T-1}$	X	Z	π	π	z+	σ	$e^{z}_{N,s:T}$
Z	X	z+	x+	π	π	$1-f_{N,s:T}$	X	Z	σ	σ	z+	π	$d^{z}_{N,s:T}$
Z	X	z+	x+	σ	σ	$1-g_{N,s:T}$	X	Z	π	π	z-	σ	$r^{z}_{N,s-1:T-1}$
X	Z	x-	z-	z+	σ	$e^{z}_{N,s:T}$	X	Z	σ	σ	z-	π	$q^{z}_{N,s-1:T-1}$
X	Z	x-	z-	z+	π	$d^{z}_{N,s:T}$	Z	X	σ	σ	x+	z-	$K^{zx}_{N,s-1:T}$
X	Z	x+	z+	z-	σ	$r^{z}_{N,s-1:T-1}$	Z	X	σ	σ	x-	z+	$K^{xz}_{N,s-1:T}$
X	Z	x+	z+	z-	π	$q^{z}_{N,s-1:T-1}$	Z	X	π	π	x+	x+	$K^{xz}_{N,s:T}$
X	Z	x-	z-	π	σ	$h^{z}_{N,s:T-1}$	※						
X	Z	x+	z+	π	σ	$h^{z}_{N,s:T-1}$	X = $X_{N,s:T-1}$				,	Z = $Z_{N,s:T-1}$	
Z	X	z-	x-	x+	z+	$K^{xz}_{N,s:T}$	x+ = $x_{s+N:T}$				,	z+ = $z_{N,s:T-1}$	
Z	X	z+	x+	x-	z-	$K^{xz}_{N,s-1:T-1}$	x- = $x_{s-1:T-N-1}$				,	z- = $z_{s-1:T-N-1}$	

$$U'\begin{vmatrix}0 & 0\\ 0 & P^{C}_{N,\,S+1:T}\end{vmatrix}V=(U'P^{C}_{N,\,S:T}V)-(U'P^{C}_{N,\,S:T}\pi)\cdot(\pi'P^{C}_{N,\,S:T}\pi)^{-1}(\pi'P^{C}_{N,\,S:T}V) \quad (3.13)$$

The proof of this is given in 4). Using the update formula (3.11) we can derive several versions of the lattice RIV by proper choice of Z, X, z, x, U, V 4), 10). The definitions of the correlation coefficients are presented in Table 1. The recursions in Table 2 are obtained by making the substitutions depicted in Table 1, in the update formula (3.11). Consider the basic matrix $X_{N,\,S:T-1}$ and observe that it can be augmented as follows. Adding the column $X_{S+N:T}$ to its left corresponds to forward combined order/time update. Adding the column $X_{S-1:T-N-1}$ to its right corresponds to backward combined order/time update. σ and π act as "time anihilators" when added to the column space of $X_{N,\,S:T}$ The details are given in 4), 9).

Table 2. Updates for the correlation coefficients.

Order updates :

$$e^{x}_{N+1,\,s-1:T}=e^{x}_{N,\,s:T}-K^{xz}_{N,\,s-1:T}(K^{zz}_{N,\,s-1:T-1})^{-1}r^{x}_{N,\,s-1:T-1}$$
$$d^{x}_{N+1,\,s-1:T}=d^{x}_{N,\,s:T}-K^{xz}_{N,\,s-1:T}(K^{zz}_{N,\,s-1:T-1})^{-1}q^{x}_{N,\,s-1:T-1}$$
$$r^{x}_{N+1,\,s-1:T}=r^{x}_{N,\,s-1:T-1}-K^{zx}_{N,\,s-1:T}(K^{xx}_{N,\,s:T})^{-1}e^{x}_{N,\,s:T}$$
$$q^{x}_{N+1,\,s-1:T}=q^{x}_{N,\,s-1:T}-K^{zx}_{N,\,s-1:T}(K^{xx}_{N,\,s:T})^{-1}d^{x}_{N,\,s:T}$$
$$h^{x}_{N+1,\,s-1:T-1}=h^{x}_{N,\,s:T-1}-q^{z'}_{N,\,s-1:T-1}(K^{zz}_{N,\,s-1:T-1})^{-1}r^{x}_{N,\,s-1:T-1}$$
$$h^{x}_{N+1,\,s:T}=h^{x}_{N,\,s:T-1}-d^{z'}_{N,\,s:T}(K^{xx}_{N,\,s:T})^{-1}e^{x}_{N,\,s:T}$$
$$f_{N+1,\,s:T+1}=f_{N,\,s:T}+d^{z'}_{N,\,s:T}(K^{xx}_{N,\,s:T})^{-1}d^{x}_{N,\,s:T}$$
$$g_{N+1,\,s:T+1}=g_{N,\,s:T}+e^{z'}_{N,\,s:T}(K^{xx}_{N,\,s:T})^{-1}e^{x}_{N,\,s:T}$$
$$e^{z}_{N+1,\,s-1:T}=e^{z}_{N,\,s:T}-K^{xz'}_{N,\,s-1:T}(K^{zz'}_{N,\,s-1:T-1})^{-1}r^{z}_{N,\,s-1:T-1}$$
$$d^{z}_{N+1,\,s-1:T}=d^{z}_{N,\,s:T}-K^{xz'}_{N,\,s-1:T}(K^{zz'}_{N,\,s-1:T-1})^{-1}q^{z}_{N,\,s-1:T-1}$$
$$r^{z}_{N+1,\,s-1:T}=r^{z}_{N,\,s-1:T-1}-K^{zx'}_{N,\,s-1:T}(K^{xx'}_{N,\,s:T})^{-1}e^{z}_{N,\,s:T}$$
$$q^{z}_{N+1,\,s-1:T}=q^{z}_{N,\,s-1:T}-K^{zx'}_{N,\,s-1:T}(K^{xx'}_{N,\,s:T})^{-1}d^{z}_{N,\,s:T}$$
$$h^{z}_{N+1,\,s-1:T-1}=h^{z}_{N,\,s:T-1}-q^{x'}_{N,\,s-1:T-1}(K^{zz'}_{N,\,s-1:T-1})^{-1}r^{z}_{N,\,s-1:T-1}$$
$$h^{z}_{N+1,\,s:T}=h^{z}_{N,\,s:T-1}-d^{x'}_{N,\,s:T}(K^{xx'}_{N,\,s:T})^{-1}e^{z}_{N,\,s:T}$$
$$K^{xz}_{N+1,\,s-1:T}=K^{xz}_{N,\,s:T}-K^{xz}_{N,\,s-1:T}(K^{zz}_{N,\,s-1:T-1})^{-1}K^{zz}_{N,\,s-1:T}$$
$$K^{zx}_{N+1,\,s-1:T}=K^{zx}_{N,\,s-1:T-1}-K^{zx}_{N,\,s-1:T}(K^{xx}_{N,\,s:T})^{-1}K^{xx}_{N,\,s-1:T}$$

Time updates :

$$e^{x}_{N,\,s+1:T}=e^{x}_{N,\,s:T}-d^{x}_{N,\,s:T}(1-f_{N,\,s:T})^{-1}h^{x}_{N,\,s:T-1}$$
$$d^{x}_{N,\,s:T-1}=d^{x}_{N,\,s:T}-e^{x}_{N,\,s:T}(1-g_{N,\,s:T})^{-1}h^{z'}_{N,\,s:T-1}$$
$$r^{x}_{N,\,s:T-1}=r^{x}_{N,\,s-1:T-1}-q^{x}_{N,\,s-1:T-1}(1-f_{N,\,s:T})^{-1}h^{x}_{N,\,s:T-1}$$
$$q^{x}_{N,\,s-1:T-2}=q^{x}_{N,\,s-1:T-1}-r^{x}_{N,\,s-1:T-1}(1-g_{N,\,s:T})^{-1}h^{z'}_{N,\,s:T-1}$$
$$g^{z}_{N,\,s+1:T}=g^{x}_{N,\,s:T}+h^{z'}_{N,\,s:T-1}(1-f_{N,\,s:T})^{-1}h^{x}_{N,\,s:T-1}$$
$$e^{z}_{N,\,s+1:T}=e^{z}_{N,\,s:T}-d^{z}_{N,\,s:T}(1-f_{N,\,s:T})^{-1}h^{z}_{N,\,s:T-1}$$
$$d^{z}_{N,\,s:T-1}=d^{z}_{N,\,s:T}-e^{z}_{N,\,s:T}(1-g_{N,\,s:T})^{-1}h^{x'}_{N,\,s:T-1}$$
$$r^{z}_{N,\,s:T-1}=r^{z}_{N,\,s-1:T-1}-q^{z}_{N,\,s-1:T-1}(1-f_{N,\,s:T})^{-1}h^{z}_{N,\,s:T-1}$$
$$q^{z}_{N,\,s-1:T-2}=q^{z}_{N,\,s-1:T-1}-r^{z}_{N,\,s-1:T-1}(1-g_{N,\,s:T})^{-1}h^{x'}_{N,\,s:T-1}$$
$$K^{zx}_{N,\,s-1:T-1}=K^{zx}_{N,\,s-1:T}-e^{x}_{N,\,s:T}(1-g_{N,\,s:T})^{-1}r^{z'}_{N,\,s-1:T-1}$$
$$K^{xz}_{N,\,s-1:T-1}=K^{xz}_{N,\,s-1:T}-r^{x}_{N,\,s-1:T-1}(1-g_{N,\,s:T})^{-1}e^{z'}_{N,\,s:T}$$
$$K^{zx}_{N,\,s+1:T}=K^{zx}_{N,\,s:T}-d^{x}_{N,\,s:T}(1-f_{N,\,s:T})^{-1}d^{z'}_{N,\,s:T}$$

Moreover, a very similar approach can be used to derive a complete set of recursions for the various whitening filters ($A_{N,\,S:T}$ and similar quantities). The definitions of the whitening filters are as follows.

$$A_{N,\,S:T}(z^{-1})=I-x'_{S+N:T}\,X^{R}_{N,\,S:T-1}\,[z^{-1}I\cdots z^{-N}I]' \quad (3.14)$$

$$B_{N,\,S-1:T-1}(z^{-1})=z^{-N}I-x'_{S-1,\,T-N-1}\cdot X^{R}_{N,\,S:T-1}\,[I\ z^{-1}I\cdots z^{-N+1}I]' \quad (3.15)$$

$$C_{N,\,S:T-1}(z^{-1})=-\sigma'\,X^{R}_{N,\,S:T-1}\,[I\ z^{-1}\ I\cdots z^{-N+1}I]' \quad (3.16)$$

$$D_{N,\,S:T-1}(Z^{-1})=-\pi'\,X^{R}_{N,\,S:T-1}\,[I\ z^{-1}\ I\cdots z^{-N+1}I]' \quad (3.17)$$

where $X^{R}=Z(X'Z)^{-1}$

The update formulas for the whitening filters are

$$[X\,\vdots\,x]^{R}=[X^{R}\,\vdots\,0]+P^{C}_{ZX}z(x'P^{C}_{ZX}z)^{-1}\,[-x'X^{R}\,\vdots\,I]$$

Premultiply by U' as follows:

$$U'[X\,\vdots\,x]^{R}=[U'X^{R}\,\vdots\,0]+(U'P^{C}_{ZX}z)(x'P^{C}_{ZX}z)^{-1}\,[-x'X^{R}\,\vdots\,I] \quad (3.18)$$

The following dual formula is similarly obtained

Table 3. Updates for the whitening filters:

Order updates :

$$A_{N+1,\,s-1:T}(Z^{-1})=A_{N,\,s:T}(Z^{-1})-K^{xz}_{N,\,s-1:T}(K^{zz}_{N,\,s-1:T-1})^{-1}Z^{-1}B_{N,\,s-1:T-1}(Z^{-1})$$
$$B_{N+1,\,s-1:T}(Z^{-1})=Z^{-1}B_{N,\,s-1:T-1}(Z^{-1})-K^{zx}_{N,\,s-1:T}(K^{xx}_{N,\,s:T})^{-1}A_{N,\,s:T}(Z^{-1})$$
$$C_{N+1,\,s-1:T-1}(Z^{-1})=C_{N,\,s:T-1}(Z^{-1})-r^{z'}_{N,\,s-1:T-1}(K^{zz}_{N,\,s-1:T-1})^{-1}B_{N,\,s-1:T-1}(Z^{-1})$$
$$D_{N+1,\,s-1:T-1}(Z^{-1})=D_{N,\,s:T-1}(Z^{-1})-q^{z'}_{N,\,s-1:T-1}(K^{zz}_{N,\,s-1:T-1})^{-1}B_{N,\,s-1:T-1}(Z^{-1})$$
$$C_{N+1,\,s:T}(Z^{-1})=Z^{-1}C_{N,\,s:T-1}(Z^{-1})-e^{z'}_{N,\,s:T}(K^{xx}_{N,\,s:T})^{-1}A_{N,\,s:T}(Z^{-1})$$
$$D_{N+1,\,s:T}(Z^{-1})=Z^{-1}D_{N,\,s:T-1}(Z^{-1})-d^{z'}_{N,\,s:T}(K^{xx}_{N,\,s:T})^{-1}A_{N,\,s:T}(Z^{-1})$$

Time updates :

$$A_{N,\,s:T-1}(Z^{-1})=A_{N,\,s:T}(Z^{-1})-e^{x}_{N,\,s:T}(1-g_{N,\,s:T})^{-1}Z^{-1}C_{N,\,s:T-1}(Z^{-1})$$
$$A_{N,\,s+1:T}(Z^{-1})=A_{N,\,s:T}(Z^{-1})-d^{x}_{N,\,s:T}(1-f_{N,\,s:T})^{-1}Z^{-1}D_{N,\,s:T-1}(Z^{-1})$$
$$B_{N,\,s:T-1}(Z^{-1})=B_{N,\,s:T}(Z^{-1})\quad r^{x}_{N,\,s:T}(1-g_{N,\,s+1:T+1})^{-1}C_{N,\,s+1:T}(Z^{-1})$$
$$B_{N,\,s+1:T}(Z^{-1})=B_{N,\,s:T}(Z^{-1})-q^{x}_{N,\,s:T}(1-f_{N,\,s+1:T+1})^{-1}D_{N,\,s+1:T}(Z^{-1})$$
$$D_{N,\,s:T-1}(Z^{-1})=D_{N,\,s:T}(Z^{-1})\quad h^{x}_{N,\,s:T}(1-g_{N,\,s:T+1})^{-1}C_{N,\,s:T}(Z^{-1})$$
$$C_{N,\,s+1:T}(Z^{-1})=C_{N,\,s:T}(Z^{-1})-h^{x'}_{N,\,s+1:T}(1-f_{N,\,s:T+1})^{-1}D_{N,\,s:T}(Z^{-1})$$

$$U'[x \vdots X]^R = [0 \vdots U'X^R] + (U'P^C_{ZX}z)(x'P^C_{ZX}z)^{-1} [I \vdots -x'X^R] \quad (3.19)$$

The recusions of the whitening filters are depicted in Table 3. This algorithm can be used as the least square ladder algorithm 8), 9) by setting Z = Y.

Table 4. Unnormalized growing memory covariance RIV algorithm for computing reflection coefficients from data.

Initialization :

$$K^{xz}_{0,-1} = K^{zz}_{0,-1} = \delta I$$

Reflection coefficients and backward prediction errors are all initialized to zero.

For T = 0, 1, ⋯ , LT DO :

$$e^x_{0,T} = r^x_{0,T} = y_T, \qquad e^z_{0,T} = r^z_{0,T} = z_T$$

$$f_{0,T} = g_{0,T} = h^x_{0,T} = h^z_{0,T} = 0$$

$$K^{xz}_{0,T} = K^{zz}_{0,T} = K^{xz}_{0,T-1} + y_T z'_T$$

For N = 0, 1, ⋯ , min(P,T)-1 DO :

$$d^x_{N,T} = d^x_{N,T-1} + e^x_{N,T}(1 - g_{N,T})^{-1} h^{z'}_{N,T-1}$$

$$d^z_{N,T} = d^z_{N,T-1} + e^z_{N,T}(1 - g_{N,T})^{-1} h^{x'}_{N,T-1}$$

$$e1^x_{N,T} = e^x_{N,T} - d^x_{N,T}(1 - f_{N,T})^{-1} h^x_{N,T-1}$$

$$e1^z_{N,T} = e^z_{N,T} - d^z_{N,T}(1 - f_{N,T})^{-1} h^z_{N,T-1}$$

$$g1_{N,T} = g_{N,T} + h^z_{N,T-1}(1 - f_{N,T})^{-1} h^x_{N,T-1}$$

$$K^{xz}_{N,T} = K^{xz}_{N,T-1} + e1^x_{N,T}(1 - g1_{N,T})^{-1} r^z_{N,T-1}$$

$$K^{zz}_{N,T} = K^{zz}_{N,T-1} + r^z_{N,T-1}(1 - g1_{N,T})^{-1} e1^{z'}_{N,T}$$

$$g_{N+1,T+1} = g_{N,T} + e^{z'}_{N,T}(K^{xz}_{N,T})^{-1} e^x_{N,T-1}$$

$$h^x_{N+1,T} = h^x_{N,T-1} - d^{z'}_{N,T}(K^{xz}_{N,T})^{-1} e^x_{N,T}$$

$$h^z_{N+1,T} = h^z_{N,T-1} - d^{x'}_{N,T}(K^{xz}_{N,T})^{-1} e^z_{N,T}$$

$$f_{N+1,T+1} = f_{N,T} + d^{z'}_{N,T}(K^{xz}_{N,T})^{-1} d^x_{N,T}$$

$$K1^{xz}_{N,T} = K^{xz}_{N,T} - d^x_{N,T}(1 - f_{N,T})^{-1} d^{z'}_{N,T}$$

$$e^x_{N+1,T} = e1^x_{N,T} - K^{xz}_{N,T}(K^{zz}_{N,T-1})^{-1} r^x_{N,T-1}$$

$$e^z_{N+1,T} = e1^z_{N,T} - K^{zz'}_{N,T}(K^{zz'}_{N,T-1})^{-1} r^z_{N,T-1}$$

$$r^x_{N+1,T} = r^x_{N,T-1} - K^{zz}_{N,T}(K1^{xz}_{N,T})^{-1} e1^x_{N,T}$$

$$r^z_{N+1,T} = r^z_{N,T-1} - K^{xz'}_{N,T}(K1^{xz'}_{N,T})^{-1} e1^z_{N,T}$$

$$K^{xz}_{N+1,T} = K1^{xz}_{N,T} - K^{xz}_{N,T}(K^{zz}_{N,T-1})^{-1} K^{zz}_{N,T}$$

$$K^{zz}_{N+1,T} = K^{zz}_{N,T-1} - K^{zz}_{N,T}(K1^{xz}_{N,T})^{-1} K^{xz}_{N,T}$$

※ LT : Last of time , P : Estimated order

4. The Growing Memory Covariance Lattice RIV Algorithm

The growing-memory covariance algorithm uses at each time point all the available data-(y(O), ... ,y(t)) (S = 0 case). In this respect, it is similar to the prewindowed algorithm and to the classical autocorrelation method. This is, however, where the similarity ends since the sample-covariance matrices are different, hence the resulting algorithms.

The algorithm is obtained from the general set of recursions given in section III, by selecting a minimal set of equations that makes the algorithm well defined. The general procedure doing this is as follows.

The forward residuals $e^x_{N,T}$ (simple notation for $e^x_{N,O:T}$) and the corresponding whitening filter, $A_{N,T}(Z^{-1})$ (simple notation for $A_{N,O:T}(Z^{-1})$) are considered to be primary quantities. Thus, we start by writing down their corresponding recursions.

All other quantities are considered as secondary. Thus they are used only to the extent needed to update the primary quantities.

Continuing this way, we end up with a minimal set of quantities and a coressponding minimal set of recursions in the correct order of evaluation.

The updated quantities 'e', 'r', 'h' can be viewed as

Table 5. The whitening filter for the growing memory covariance method of RIV.

For I = 0, 1, ⋯ , P DO :

$$A_{0,I} = B_{0,I} = 1, \qquad C_{0,I} = D_{0,I} = 0 \qquad I = 0$$

$$A_{0,I} = B_{0,I} = C_{0,I} = D_{0,I} = 0 \qquad I > 0$$

For N = 0, 1, ⋯ , P-1 DO :

$$C1_{N,I} = C_{N,I} - h^{z'}_{N,T}(1 - f_{N,T+1})^{-1} D_{N,I}$$

$$D1_{N,I} = D_{N,I} - h^x_{N,T}(1 - g_{N,T+1})^{-1} C_{N,I}$$

$$A1_{N,I} = A_{N,I} - d^x_{N,T}(1 - f_{N,T})^{-1} D1_{N,I-1}$$

$$B1_{N,I} = B_{N,I} - r^x_{N,T}(1 - g1_{N,T+1})^{-1} C1_{N,I}$$

$$A_{N+1,I} = A1_{N,I} - K^{xz}_{N,T}(K^{zz}_{N,T-1})^{-1} B1_{N,I}$$

$$B_{N+1,I} = B1_{N,I-1} - K^{zz}_{N,T}(K1^{xz}_{N,T})^{-1} A1_{N,T}$$

$$C_{N+1,I} = C1_{N,I} - r^{z'}_{N,T}(K^{zz}_{N,T-1})^{-1} B_{N,I}$$

$$D_{N+1,I} = D1_{N,I-1} - d^{z'}_{N,T}(K^{xz}_{N,T})^{-1} A_{N,I}$$

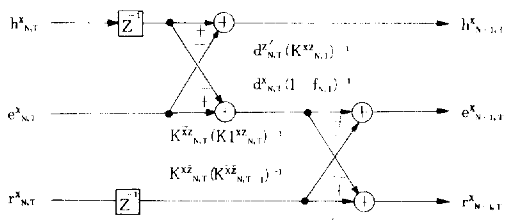

Fig. 1. Ladder form for the growing memory covariance RIV algorithm.

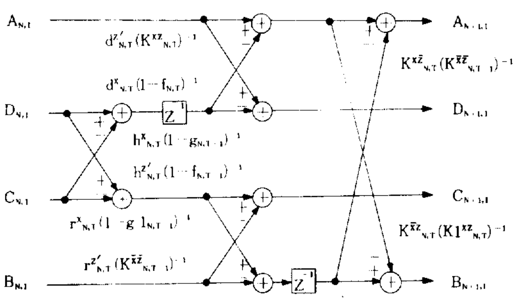

Fig. 2. Ladder form realization of the growing memory whitening filter for RIV.

'signals', while 'd', 'k' are the best viewed as 'gains'. Note that the backward residual 'q' does not appear at all, as it is not needed in growing memory covariance algorithm, but in sliding memory covarance algorithm. The sliding memory covarance algorithm can be easily derived from the updates of the previous section by setting the window fixed 9)

The growing meory covariance RIV lattice Algorithm for computing reflection coefficients from data is presented in Table 4. and that for the whitenng filter from reflection coefficients is summarized in Table 5. In whitening filter algorithm, it is not necessary to update whitening filter, at every time point, but only at will. Setting D=O in this algorithm the whitening filter for the prewindowed case. The structures of these lattice filters are depicted in Fig.1 and Fig. 2.

5. Simulation Results

Computer simulations are presented to illustrate the behavior of the RIV lattice alrorithm.

Example 1:

This example is the one studied by Ulrych and Bishop (see chap. 7 in [7]) which is (4.1) ARMA process:

$$y(t) = -2.7607y(t-1) + 3.8106y(t-2) - 2.6535y(t-3) + 0.9238y(t-4) + w(t)$$
$$w(t) = v(t) + 0.446v(t-1) \qquad (5.1)$$

where v(t) is white Gaussian noise with zero mean and unit variance.

This model was estimated by LS case and IV case, respectively. The estimated parameters in each case are presented in Table 6. Note that the IV parameters are more consistent than the corresponding LS parameters. The comparison of the estimated spectra of the true parameter and IV parameter is presented in Fig. 3. In each case 500 data pointe were used.

Example 2:

A time series was generated by

$$y(t) = 20\sin(0.4t) + 6.3\sin(0.425t) + w(t)$$
$$w(t) = v(t) + 0.446v(t-1)$$

Table 6. The comparison of the estimated parameters by LS and IV algorithm.

True para.		A(1)	A(2)	A(3)	A(4)
Bati. para.		-2.7607	3.8106	-2.6535	0.9238
Data					
50	LS case	-3.6486	6.1168	-5.0170	1.9132
	IV case	-3.2893	5.1324	-3.6935	1.4526
100	LS case	-3.0250	4.4097	-3.2237	1.1314
	IV case	-2.9134	4.0604	-2.8349	1.4526
200	LS case	-2.8755	4.0634	-2.8907	1.0111
	IV case	-2.8057	3.8887	-2.7149	0.9406
300	LS case	-2.8589	4.0018	-2.8215	0.9752
	IV case	-2.7831	3.8228	-2.6467	0.9087
400	LS case	-2.8401	3.9740	-2.8049	0.9708
	IV case	-2.8002	3.8842	-2.7186	0.9441
500	LS case	-2.8246	3.9740	-2.7817	0.9759
	IV case	-2.7849	3.8581	-2.6982	0.9409

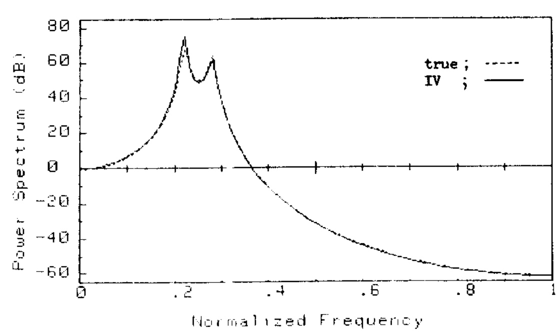

Fig. 3. The comparision of the true spectrum and the IV spectrum estimate, Example 1.

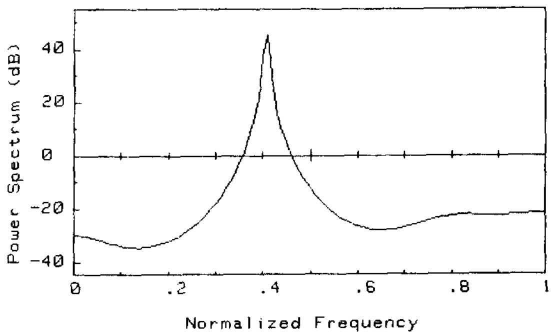

Fig. 4 (a). Ths LS spectral estimate, Example 2.

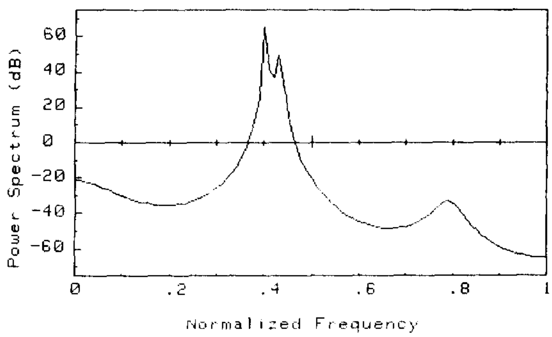

Fig. 4 (b). The IV spectral estimate, Example 2.

where v(t) is a unit variance white noise process. This was estimated, based on 500 data points, order of 8 in each case. The spectra for the LS case and the IV case are presented in Fig. 4 (a) and Fig. 4 (b), respectively.

6. Conculsion

The recursive instrumental variable ladder algorithm for covariance form was presented. This algorithm is the generalized form of lattice algorithms so that it can be used as the least square ladder algorithm by setting $Z = Y$ (Giving a unified description of lattice algorithms has allowed us to point out how these algorithms are related.). This can be used in various areas of signal processing, spectral estimation and system identification. Computer simulation showed that this algorithm gives better spectral resolution than any other lattice algorithm.

This RIV algorithm can be used to estimate the AR part of an ARMA process. The prediction error sequence produced by the RIV can be used as the input to an adaptive MA modeling algorithm. Finally the algorithm described has been given in "unnormalized" form and that "normalized" versions can be derived by using the operator approach.

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