論 文
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행렬부호 함수에 의한 선형 이산치 대단위 계통의 블럭-분해

Block-decomposition of a Linear Discrete Large-scale Systems Via the Matrix Sign Function

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요 약

본 논문에서는 Z-평면상에서 행렬 부호함수를 기초로 한 선형 시불변 이산치 대단위 계통의 블럭—분해 알고리즘을 제시하였다. 이러한 블럭—분해는 새로히 기준원과 사선환(circular stripe) 그리고 투영 연산자를 정의함으로써 수행하였다. 시뮬레이션을 통해 제시된 알고리즘이 다변수 제어계통의 분석과 설계에 아주 유용함을 보였다.

Abstract

An algorithm for block-decomposition of a linear, time-invariant, discrete large-scale systems is presented, based upon the matrix sign function on Z-plane. The block-decomposition is performed by defining a reference circle, a circular stripe and projection operators.

Simulation study shows that the presented algorithm is very useful for multivariable control system's analysis and design.

1. Introduction

For a realistic analysis and design of linear discrete-time large-scale systems which involve interacting dynamic phenomena of widely different modes, the large-scale system is often block-decomposed into two basic structures, the parallel structure and the cascade structure. The two basic structures can be obtained by performing the block-diangonalization and block-triangularization of the state matrix of a dynamic equation.

In the case of discrete-time systems, results concerning the use of block-diagonalization were reported by Phillips[1] and Mahmoud et al.[2]

Though block-diagonalization techniques the major advantage of not having to solve the eigenvalue-eigenvector problem -which is necessary, for example, in modal decomposition technique[3] -they nevertheless re-

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quire the solutions of certain algebraic Riccati and Lyapunov type equation that satisfy a necessary and sufficient condition.[4] Since these equations are generally nonlinear, recursive algorithms for solving them are highly desirable.[5]

The aim of this paper is to develop an alternative block-decomposition algorithm of a discrete-time large-scale system which uses the definition of a reference circle, a circular stripe and a projection operators via the matrix sign function.[6-11]

This paper is organized as follows. In chapter 2, we review the matrix sign function and associated matrix sign algorithm. In chapter 3, we proposed two block-decomposition and multi block-decomposition algorithm based on the nature of the matrix sign function and generalized matrix sign function in Z-plane. An illustrative example is presented in chaper 4 and conclusions are summarized in chapter 5.

2. The matrix sign function in Z-plane

To develop methods for the block-decomposition of a linear discrete large-scale system, we present some definitions of the matrix sign function in Z-plane.

Definition 2.1

The sign function of a complex variable λ with $|\lambda| \neq 1$ in Z-plane is defined by

$$\operatorname{sign}\left(\frac{\lambda-1}{\lambda+1}\right)=\begin{cases}+1 & \text{if } |\lambda|>1\\ -1 & \text{if } |\lambda|<1\end{cases} \tag{2.1}$$

From Definition 2.1 we observe that the sign function is a nonlinear function which maps the outside and inside unit circle of a Z-plane into +1 and -1, respectively. The extension of the sign function to the matrix sign function can be stated as follows.

Let M be the modal matrix of a square matrix A(nxn) and

$$J = WAM = \text{block diag}[J_+,\ J_-] \tag{2.2a}$$

where M and its inverse W are defined as

$$M \triangleq [M_1 \,\vdots\, M_2] \triangleq \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22}\end{bmatrix},\quad M_1 \triangleq \begin{bmatrix} M_{11} \\ M_{21}\end{bmatrix},$$

$$M_2 \triangleq \begin{bmatrix} M_{12} \\ M_{22}\end{bmatrix} \tag{2.2b}$$

$$M^{-1} \triangleq W \triangleq \begin{bmatrix} W_1 \\ W_2\end{bmatrix} \triangleq \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22}\end{bmatrix},\quad W_1 \triangleq [W_{11} \,\vdots\, W_{12}],\quad W_2 \triangleq [W_{21} \,\vdots\, W_{22}] \tag{2.2c}$$

and $J_+ \in R^{n_1\times n_1}$ and $J_- \in R^{n_2\times n_2}$ with $n_1+n_2=n$, are the collections of Jordan blocks associated with the specturm $\sigma(A) \subset Z^+$ and $\sigma(A) \subset Z^-$, respectively, where Z^+ and Z^- are the outside and inside unit circle of the Z-plane, respectively. Then

$$\begin{aligned}\operatorname{sign}(A) &= M[\operatorname{sign}(J_+) \oplus \operatorname{sign}(J_-)]W \\ &= M[I_{n_1} \oplus (-I_{n_2})]W\end{aligned} \tag{2.3}$$

The formal definition of the matrix sign function in Z-plane can be defined by using the Riesz projector[8] as follows:

Definition 2.2

The matrix sign function of a square matrix $A \in R^{n\times n}$ with the eigenvalue spectrum $\sigma(A) \subset Z^+ \cup Z^-$, is defined by

$$\operatorname{sign}(A) \triangleq 2\operatorname{sign}^+(A) - I_n = I_n - 2\operatorname{sign}^-(A) \tag{2.4}$$

or

$$\operatorname{sign}^+(A) \triangleq \frac{1}{2}[I_n + \operatorname{sign}(A)] \tag{2.5a}$$

$$\begin{aligned}\operatorname{sign}^-(A) &\triangleq \frac{1}{2}[I_n - \operatorname{sign}(A)] = \operatorname{sign}^+(A) - \operatorname{sign}(A) \\ &= I_n - \operatorname{sign}^+(A)\end{aligned} \tag{2.5b}$$

To compute sign(A), Roberts proposed an efficient recursive algorithm. This algorithm is as follows:[9]

$$\text{sign}(A) \triangleq S(k+1) = \alpha_k S(k) + \beta_k S^{-1}(k),$$
$$S(0) = A \qquad (2.6)$$

where

$$\alpha_k = \|S^{-1}(k)\| / \|S(k)\| + \|S^{-1}(k)\|$$
$$\beta_k = \|S(k)\| / \|S(k)\| + \|S^{-1}(k)\|$$

For this recursive algorithm, there are some stopping conditions,[10] some of which are

$$S(k+1) - S(k) \leq \mu$$
$$S^2(k+1) - I_n \leq \mu$$
$$\text{Trace}[S^2(k+1)] - n \leq \mu$$

where μ is a small, preselected error bound.

3. Block-decomposition of linear discrete systems

In general, a large-scale system contains widely different modes, and is often required to be block-decomposed into several subsystems with their own characteristics for simulations and designs.

Consider a linear discrete-time system as follows:

$$X(k+1) = AX(k) + Bu(k) \qquad (3.1a)$$
$$y(k) = CX(k) + Du(k) \qquad (3.1b)$$

where $X(k) \in R^{n\times 1}$, $u(k) \in R^{m\times 1}$ and $y(k) \in R^{p\times 1}$ are the state, input and output vectors, respectively. The state vector X(k) is decomposed into the n_1 and n_2 vectors $x_1(k)$ and $x_2(k)$, and the matrices A, B, C, and D have a appropriate dimensions.

In this discrete system, the class of eigenvalues located near the unit circle in the Z-plane are assigned to the slow mode and those located near the origin are assigned to the fast mode. Thus we need the reference circle with radius r for block-decomposition. For system (3·1) the eigenspecturm $\sigma(A)$ is arranged in decreasing order of absolute values: that is

$$\sigma(A) = \{\lambda_1, \cdots, \lambda_{n1}, \lambda_{n1+1}, \cdots, \lambda_n\} \qquad (3.2a)$$

where

$$|\lambda_1| > \cdots > |\lambda_{n1}| > |\lambda_{n1+1}| > \cdots > |\lambda_n| \qquad (3.2b)$$

From Eq.(3-2b), the positive real value $r \neq 1$ is chosen between $|\lambda_{n_1}|$ and $|\lambda_{n_1+1}|$ arbitrary. Thus, the reference circle with radius r is shown in Fig. 1.

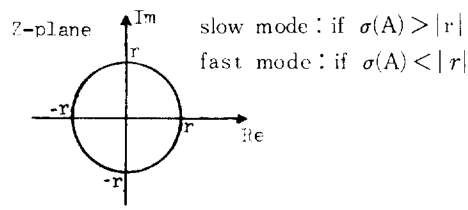

Fig.1. Reference circle on Z-plane.

Now the block-decomposition of a system can be accomplished via the following theorem 3.1.

Theorem 3.1

For discrete system, the eigenvalues of sign $[(A-rI_n)(A+rI_n)^{-1}]$ are ± 1 according to the magnitude of the corresponding eigenvalues of the state matrix.,i.e.

$$\sigma_i\{\text{sign}[(A-rI_n)(A+rI_n)^{-1}]\} = \begin{cases} +1 & \text{if } |\lambda_i| > r \\ -1 & \text{if } |\lambda_i| < r \end{cases} \qquad (3.3)$$

where σ_i is the ith eigenvalue of $\text{sign}[(A-rI_n)(A+rI_n)^{-1}]$ corresponding to the ith eigenvalue λ_i of state matrix A.

Proof) The proof of Theorem3.1 can be found in reference 11).

From Eq.(2-5) and (2-3), we define two projection operators $P_s(A)$, $P_f(A)$ as follows:

$$P_s(A) \triangleq \text{sign}^+[(A-rI_n)(A+rI_n)^{-1}]$$
$$= \frac{1}{2}[I_n + [\text{sign}(A-rI_n)(A+rI_n)^{-1}]]$$

$$=\frac{1}{2}[I_n+M[I_{n1}\oplus(-I_{n2})]W]$$
$$=M[I_{n1}\oplus O_{n2}]W=M_1W_1 \quad (3.4a)$$
$$P_f(A)\triangleq \mathrm{sign}^-[(A-rI_n)(A+rI_n)^{-1}]$$
$$=I_n-\mathrm{sign}^+[(A-rI_n)(A+rI_n)^{-1}]$$
$$=I_n-M[I_{n_1}\oplus O_{n_2}]W$$
$$=M[O_{n_1}\oplus I_{n_2}]W=M_2W_2 \quad (3.4b)$$

It is obvious that Rank $[P_S(A)]=n_1$ and Rank $[P_f(A)]=n_2$.

Let matrices S_1 and S_2 be defined as

$$S_1\triangleq \mathrm{ind}[P_s(A)]=\{s_1, s_2, \cdots, s_{n_1}\}\in R^{n\times n_1}, s_i\in R^{n\times 1} \quad (3.5a)$$
$$S_2\triangleq \mathrm{ind}[P_f(A)]=\{\bar{s}_1, \bar{s}_2, \cdots, \bar{s}_{n_2}\}\in R^{n\times n_2}, \bar{s}_i\in R^{n\times 1} \quad (3.5b)$$

where S_1 (or S_2) is a monic map which contains n_1 (or n_2) independent column vectors of $P_S(A)$ (or $P_f(A)$) in Eq.(3·4). These independent column vectors are selected from the n $\geq n_1$, or $\geq n_2$ column vectors of $P_S(A)$ (or $P_f(A)$) and the number of independent vectors[7], n_1 (or n_2) is equal to the trace of $P_S(A)$ (or $P_f(A)$). Thus, we obtain the results as follows:

Lemma 3.1

There exist nonsingular matrices $H_1\in R^{n_1\times n_1}$ and $H_2\in R^{n_2\times n_2}$ such that

$$S_1=M_1H_1,\ S_2=M_2H_2 \quad (3.6)$$

Where M_1 and M_2 are defined as in Eq.(2.2b, c). Proof) Assume that S_1 contains n_1 columns of $P_S(A)$ with column indices k_i and S_2 contains n_2 columns of $P_f(A)$ with column ind eces $\bar{k}_i$ for i=1, 2, ⋯, n_1 (or n_2). Then, from the definition of S_1 and S_2, we obtain

$$S_1=P_s(A)E_1, \qquad S_2=P_f(A)E_2$$
$$=M_1W_1E_1\triangleq M_1H_1 \qquad =M_2W_2E_2\triangleq M_2H_2 \quad (3.7)$$

where

$$E_1\triangleq\{e_n^{k_1}e_n^{k_2}\cdots e_n^{kn_1}\}\in R^{n\times n_1}$$
$$H_1\triangleq W_1E_1,\ H_2\triangleq W_2E_2$$
$$E_2\triangleq\{e_{n_1}^{\bar{k}}e_n^{\bar{k}_2}\cdots e_n^{\bar{k}n_2}\}\in R^{n\times n_2}$$

According to sylvester's inequality[12], we obtain Rank$(H_1)=n_1$ and Rank$(H_2)=n_2$, or H_1 and H_2 are nonsingular.

(Q.E.D.)

Now define a right block modal matrix M_s as follows

$$M_s\triangleq[S_1 \vdots S_2] \quad (3.8)$$

From lemma 3.1

$$M_s=MH \quad (3.9)$$

where

$$H=H_1\oplus H_2$$

And from Eq·(3·9) and Eq·(2.2a)

$$AM_s=AMH=M\begin{bmatrix} J_+ & O_{n_1\times n_2} \\ O_{n_2\times n_1} & J_- \end{bmatrix}M^{-1}MH$$
$$=M\begin{bmatrix} J_+ & O_{n_1\times n_2} \\ O_{n_2\times n_1} & J_- \end{bmatrix}H$$
$$=MH(H^{-1}\begin{bmatrix} J_+ & O_{n_1\times n_2} \\ O_{n_2\times n_1} & J_- \end{bmatrix}H)=M_sA_R \quad (3.10)$$

where

$$A_R\triangleq H^{-1}\begin{bmatrix} J_+ & O_{n_1\times n_2} \\ O_{n_2\times n_1} & J_- \end{bmatrix}H \quad (3.11)$$

The immediate application of Eq.(3·8 — 3·11) to system theory is the block-decomposition of a system in the following theorem.

Theorem 3.2

Let

$$Z(k)=M_S^{-1}X(k) \quad (3.12)$$

where $Z(k)=[z_1^T(k)\ z_2^T(k)]^T$, $z_1(k)\in R^{n_1\times 1}$ and $z_2(k)\in\omega^{n_2\times 1}$. Then the block-decomposed system of Eq. (3.1) using Eq. (3.12) becomes

$$Z(k+1)=A_RZ(k)+B_Ru(k) \quad (3.13a)$$
$$y(k)=C_RZ(k)+Du(k) \quad (3.13b)$$

where

$$A_R=M_S^{-1}AM_S=\text{block diag}[A_{R_1}, A_{R_2}] \quad (3.13c)$$
$$B_R=M_S^{-1}B=[B_{R_1}^T \vdots B_{R_2}^T]^T \quad (3.13d)$$

$$C_R = CM_S = [C_{R_1} \vdots C_{R_2}] \tag{3.13e}$$

proof) From Eq. (2.2b, c)

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} [M_1 \vdots M_2] = \begin{bmatrix} W_1M_1 & W_1M_2 \\ W_2M_1 & W_2M_2 \end{bmatrix} = \begin{bmatrix} I_{n_1} & O_{n_1 \times n_2} \\ O_{n_2 \times n_1} & I_{n_2} \end{bmatrix} \tag{3.14}$$

Then

$$M_S^{-1}M_S = \begin{bmatrix} S_1^+ \\ S_2^+ \end{bmatrix} [S_1 \vdots S_2] = \begin{bmatrix} S_1^+S_1 & S_1^+S_2 \\ S_2^+S_1 & S_2^+S_2 \end{bmatrix} \tag{3.15a}$$

where '+' denotes pseudo-inverse and

$$S_1^+S_1 = I_{n_1}, \quad S_2^+S_2 = I_{n_2} \tag{3.15b}$$

and from lemma 1

$$S_1^+S_2 = (M_1W_1E_1)^+M_2W_2E_2 = (W_1E_1)^{-1}M_1^+M_2W_2E_2 = (W_1E_1)^{-1}(W_1M_2)W_2E_2 = O_{n_1 \times n_2} \tag{3.15c}$$

$$S_2^+S_1 = (M_2W_2E_2)^+M_1W_1E_1 = (W_2E_2)^{-1}M_2^+M_1W_1E_1 = (W_2E_2)^{-1}(W_2M_1)W_1E_1 = O_{n_2 \times n_1} \tag{3.15d}$$

Substituting M_S and M_S^{-1} into Eq. (3.13c) yields

$$M_S^{-1}AM_S = \begin{bmatrix} S_1^+ \\ S_2^+ \end{bmatrix} A [S_1 \vdots S_2] = \begin{bmatrix} S_1^+AS_1 & S_1^+AS_2 \\ S_2^+AS_1 & S_2^+AS_2 \end{bmatrix} \tag{3.16}$$

Define

$$A_{R_1} = S_1^+AS_1, \quad S_1A_{R_1} = AS_1$$
$$A_{R_2} = S_2^+AS_2, \quad S_2A_{R_2} = AS_2$$

Thus, from Eq. (3.15c, d)

$$S_1^+AS_2 = S_1^+S_2A_{R_2} = O_{n_1 \times n_2}, \quad S_2^+AS_1 = S_2^+S_1A_{R_1} = O_{n_2 \times n_1}$$

(Q. E. D.)

The parallel result of theorem 3.2 can be obtained by theorem 3.3

Theorem 3.3

Let's define a left block modal matrix as

$$M_v = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{3.17a}$$

where

$$V_1 = \{ind[P_S^T(A)]\}^T \in R^{n_1 \times n} \tag{3.17b}$$

$$V_2 = \{ind[P_J^T(A)]\}^T \in R^{n_2 \times n} \tag{3.17c}$$

Let

$$Z(k) = M_vX(k) \tag{3.18}$$

Then, the alternative block-decomposed system using Eq.(3·18) becomes

$$Z(k+1) = A_LZ(k) + B_Lu(k) \tag{3.19a}$$
$$y(k) = C_LZ(k) + Du(k) \tag{3.19b}$$

where

$$A_L = M_vAM_v^{-1} = \text{block diag}[A_{L_1}, A_{L_2}] \tag{3.19c}$$
$$A_{L_1} = V_1AV_1^+ \in R^{n_1 \times n_1}, \quad A_{L_2} = V_2AV_2^+ \in R^{n_2 \times n_2}$$
$$B_L = M_vB = [B_{L_1}^T \vdots B_{L_2}^T]^T \tag{3.19d}$$
$$C_L = CM_v^{-1} = [C_{L_1} \vdots C_{L_2}] \tag{3.19e}$$

Proof) Theorem 3.3 can be proved in a manner similar to theorem 3.2.

If more than two subsystems are needed for the block-decomposition of Eq.(3·1), we can construct new block modal matrices M_S and M_v by using the generalized matrix sign function[8] in Z-plane.

Definition 3.1

Let $A \in R^{n \times n}$ and $\{|\sigma(A)|\} \cup \{r_1, r_2\} = \phi$, where $0 < r_1 < r_2$ and $r_1, r_2 \in R$. The generalized matrix sign function of A with respect to the circular stripe(r_1, r_2) on the Z-plane is defined as

$$\text{sign}_{(r_1, r_2)}(A) = 2\text{sign}^+_{(r_1, r_2)}(A) - I_n = I_n - 2\text{sign}^-_{(r_1, r_2)}(A) \tag{3.20a}$$

where

$$\text{sign}^-_{(r_1, r_2)}(A) \triangleq \frac{1}{2}[\text{sign}_{(r_2)}(A) - \text{sign}_{(r_1)}(A)] \tag{3.20b}$$

$$\text{sign}^-_{(r_1, r_2)}(A) \triangleq I_n - \text{sign}^+_{(r_1, r_2)}(A) \tag{3.20c}$$

and

$$\text{sign}_{(r_i)}(A) = \text{sign}[(A - r_iI_n)(A + r_iI_n)^{-1}], \quad r_i \neq 1 \text{ for } i = 1, 2 \tag{3.20d}$$

The circular stripe(r_1, r_2) on the Z-plane is shown in Figure 2.

Let $A \in R^{n \times n}$ and $\{|\sigma(A)|\} \cap \{r_i, \ i = 0, 1, \cdots, k$

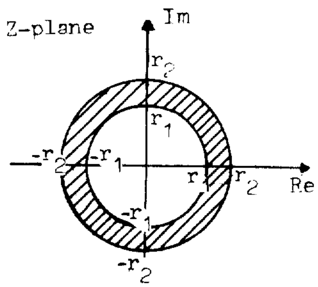

Fig.2 circular stripe (r_1, r_2) on Z-plane

$=\phi$, where $0<r_0<r_1<\cdots<r_k$ and $|r_i \neq 1|$ $\in R$ for $0 \le i \le k$. Define

$$S_i \triangleq \mathrm{ind}[\mathrm{sign}^+_{(r_{i-1}, r_i)}(A)] \triangleq \mathrm{ind}[P_i(A)] \in R^{n\times n_i}, \quad 1 \le i \le k \tag{3.21a}$$

$$V_i \triangleq \{\mathrm{ind}[(\mathrm{sign}^+_{(r_{i-1}, r_i)}(A))^T]\}^T \triangleq \{\mathrm{ind}[P_i^T(A)]\}^T \in R^{n_i\times n}, \quad 1 \le i \le k \tag{3.21b}$$

where

$$P_i(A) \triangleq \mathrm{sign}^+_{(r_{i-1}, r_i)}(A) \tag{3.21c}$$

Theorem 3.4

Assume that $n_i \neq 0$ for $1 \le i \le k$. Then

$$M_s^{-1}AM_s = \text{block diag}[A_{R_1}, A_{R_2}, \cdots, A_{R_k}] \tag{3.22a}$$

$$M_vAM_v^{-1} = \text{block diag}[A_{L_1}, A_{L_2}, \cdots, A_{L_k}] \tag{3.22b}$$

where M_s and M_v are the right and left block modal matrices :

$$M_s = [S_1 \,\vdots\, S_2 \,\vdots\, \cdots \,\vdots\, S_k] \tag{3.22c}$$

$$M_v = [V_1^T \,\vdots\, V_2^T \,\vdots\, \cdots \,\vdots\, V_k^T]^T \tag{3.22d}$$

Let

$$Z(k) = M_s^{-1}X(k) = M_vX(k) \tag{3.23}$$

where $Z(k) = [z_1^T(k)\ z_2^T(k)\ \cdots z_k^T(k)]^T$. Then, the block-decomposed system using Eq. (3.23) becomes

$$Z(k+1) = A_{R_i}\ (\text{or } A_{L_i})\ Z(k) + B_{R_i}\ (\text{or } B_{L_i})\ u(k) \tag{3.24a}$$

$$y(k) = C_{R_i}\ (\text{or } C_{L_i})\ Z(k) + Du(k) \tag{3.24b}$$

where

$$A_{R_i} = S_i^+AS_i \in R^{n_i\times n_i}, \quad A_{L_i} = V_iAV_i^+ \in R^{n_i\times n_i}, \quad 1 \le i \le k$$

$$B_R = M_s^{-1}B = [B_{R_1}^T\ B_{R_2}^T \cdots B_{R_k}^T]^T$$

$$B_L = M_vB = [B_{L_1}^T\ B_{L_2}^T \cdots B_{L_k}^T]^T$$

$$C_R = CM_s = [C_{R_1}\ C_{R_2} \cdots C_{R_k}]$$

$$C_L = CM_v^{-1} = [C_{L_1}\ C_{L_2} \cdots C_{L_k}]$$

S_i^+ and V_i^+ are the left inverse of S_i and the right inverse of V_i for $1 \le i \le k$, respectively.

Proof) Theorem 3.4 can be proven in a manner similar to Theorem 3.2 and 3.3.

To illustrate the theoretical block-decomposition method, we consider a fifth-order of a steam power system in following chapter.

4. Numerical Example

The fifth-order linearized model of a steam power system[3] was discretized with sampling time of 0.7 sec, and it can be represented by:

$$A = \begin{bmatrix} 0.915 & 0.051 & 0.038 & 0.015 & 0.038 \\ -0.03 & 0.889 & -0.0005 & 0.046 & 0.111 \\ -0.006 & 0.468 & 0.247 & 0.014 & 0.048 \\ -0.715 & -0.022 & -0.021 & 0.24 & -0.024 \\ -0.148 & -0.003 & -0.004 & 0.09 & 0.026 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0098 \\ 0.122 \\ 0.036 \\ 0.562 \\ 0.115 \end{bmatrix}$$

The open-loop eigenvalues of A are $\lambda_1 = 0.0295$, $\lambda_{2,3} = 0.2506 \pm j0.1252$ and $\lambda_{4,5} = 0.8928 \pm j0.0937$.

Find the block-decomposition of this system which contains three block-decomposed subsystems. The first block-diagonal matrix contains one small eigenvalue, the second consists of two medium eigenvalues and the third contains two large eigenvalues. Since the eigenvalue of the first subsystem is desired to be lying within the circular stripe[0,0.2], the eigenvalues of the second subsystem lying the circular stripe (0.2, 0.8) and those of the third subsystem lying within the circular stripe(0.8, $\pm\infty$), we can select the real values r_i as $r_0=0$, $r_1=0.2$, $r_2=0.8$ and $r_3=\pm\infty$.

By the recursive matrix sign algorithm Eq.(2·6), $sign_{(0.2)}(A)$ and $sign_{(0.8)}(A)$ yield as follows

$$sign_{(0.2)}(A)=sign[(A-0.2I_5)(A+0.2I_5)^{-1}]=$$

$$\begin{bmatrix} 0.98749\mathrm{D}\ 00 & -0.99845\mathrm{D}{-04} & 0.57292\mathrm{D}{-03} & -0.30817\mathrm{D}{-01} & 0.74027\mathrm{D}{-01} \\ -0.43038\mathrm{D}{-01} & 0.99966\mathrm{D}\ 00 & 0.19703\mathrm{D}{-02} & -0.10598\mathrm{D}\ 00 & 0.25458\mathrm{D}\ 00 \\ 0.19299\mathrm{D}{-01} & 0.15398\mathrm{D}{-03} & 0.99912\mathrm{D}\ 00 & 0.47525\mathrm{D}{-01} & -0.11416\mathrm{D}\ 00 \\ -0.71453\mathrm{D}{-02} & -0.57007\mathrm{D}{-04} & 0.32711\mathrm{D}{-03} & 0.98240\mathrm{D}\ 00 & 0.42266\mathrm{D}{-01} \\ 0.33281\mathrm{D}\ 00 & 0.26553\mathrm{D}{-02} & -0.15236\mathrm{D}{-01} & 0.81955\mathrm{D}\ 00 & -0.96866\mathrm{D}\ 00 \end{bmatrix}$$

$$sign_{(0.8)}(A)=sign[(A-0.8I_5)(A+0.8I_5)^{-1}]=$$

$$\begin{bmatrix} 0.10394\mathrm{D}\ 01 & -0.86774\mathrm{D}{-01} & 0.11581\mathrm{D}\ 00 & 0.28612\mathrm{D}{-01} & 0.59363\mathrm{D}{-01} \\ 0.27302\mathrm{D}\ 00 & 0.95997\mathrm{D}\ 00 & 0.26616\mathrm{D}{-01} & 0.18585\mathrm{D}\ 00 & 0.26616\mathrm{D}\ 00 \\ 0.33788\mathrm{D}\ 00 & 0.13994\mathrm{D}\ 01 & -0.97284\mathrm{D}\ 00 & 0.13527\mathrm{D}\ 00 & 0.19500\mathrm{D}\ 00 \\ -0.21777\mathrm{D}\ 01 & 0.23997\mathrm{D}\ 00 & -0.12283\mathrm{D}\ 00 & -0.10170\mathrm{D}\ 01 & -0.44095\mathrm{D}{-01} \\ -0.56404\mathrm{D}\ 00 & 0.77039\mathrm{D}{-01} & -0.31731\mathrm{D}{-01} & -0.30166\mathrm{D}{-02} & -0.10095\mathrm{D}\ 01 \end{bmatrix}$$

Also, by the definition of the matrix sign function, we obtion

$$sign_{(0)}(A)=-I_5 \quad sign_{(\pm\infty)}(A)=I_5$$

From Eq.(3·20) and Eq.(3·21) we obtain

$$S_1=ind[sign^{+}_{(0,0.2)}(A)]=$$
$$=ind\left\{\frac{1}{2}[sign_{(0)}(A)-sign_{(0.2)}(A)]\right\}$$
$$=[-0.00626\ \ -0.02152\ \ 0.00965\ \ -0.00357\ \ 0.1664]^T$$

$$S_2=ind[sign^{+}_{(0.2,0.8)}(A)]$$
$$=ind\left\{\frac{1}{2}[sign_{(0.2)}(A)-sign_{(0.8)}(A)]\right\}$$
$$=\begin{bmatrix} 0.02596 & 0.15803 & 0.15929 & -1.0853 & -0.44842 \\ -0.04334 & -0.01989 & 0.69962 & 0.12001 & 0.03719 \end{bmatrix}^T$$

$$S_3=ind[sign^{+}_{(0.8,\pm\infty)}(A)]$$
$$=ind\left\{\frac{1}{2}[sign_{(0.8)}(A)-sign_{(\pm\infty)}(A)]\right\}$$
$$=\begin{bmatrix} -1.0197 & -0.13651 & -0.16894 & 1.0888 & 0.28202 \\ 0.04339 & -0.97993 & -0.6997 & -0.11998 & -0.03852 \end{bmatrix}^T$$

and

$$V_1=\{ind[(sign^{+}_{(0,0.2)}(A))^T]\}^T$$
$$=[-0.00626\ \ 0.02596\ \ 0.15803\ \ -1.0197\ \ -0.13651]$$

$$V_2=\{ind[(sign^{+}_{(0.2,0.8)}(A))^T]\}^T$$
$$=\begin{bmatrix} -0.00005 & -0.04334 & -0.01989 & 0.04339 & -0.97993 \\ 0.00029 & 0.05762 & 0.01232 & -0.05791 & -0.01331 \end{bmatrix}$$

$$V_3=\{ind[(sign^{+}_{(0.8,\pm\infty)}(A))^T]\}^T$$
$$=\begin{bmatrix} -0.01541 & 0.02972 & 0.14592 & -0.01431 & -0.09293 \\ 0.03701 & -0.00733 & 0.00579 & -0.02968 & -0.13308 \end{bmatrix}$$

Thus, from Eq.(3·22c) and (3·22d), the right block modal matrix M_s and the left block modal matrix M become as follows:

$$M_s=[S_1 \mid S_2 \mid S_3], \quad M_v=[V_1^T \mid V_2^T \mid V_3^T]^T$$

Now, from Eq.s(3·23) and Eq.s(3·24), the block-decomposed systems and as follows:

$$A_{R_1}=[0.02979]=A_{L_1}$$
$$A_{R_2}=\begin{bmatrix} 0.25903 & -0.01435 \\ 0.05027 & 0.24228 \end{bmatrix}, \quad A_{L_2}=\begin{bmatrix} 0.25705 & -0.01346 \\ 0.05139 & 0.24426 \end{bmatrix}$$
$$A_{R_3}=\begin{bmatrix} 0.89668 & 0.07719 \\ -0.11543 & 0.88921 \end{bmatrix}, \quad A_{L_3}=\begin{bmatrix} 0.89083 & 0.08039 \\ -0.11071 & 0.89506 \end{bmatrix}$$

and

$$B_{R_1}=[-0.71281], \quad B_{L_1}=[-0.00446]$$
$$B_{R_2}=\begin{bmatrix} -0.52229 \\ -0.01523 \end{bmatrix}, \quad B_{L_2}=\begin{bmatrix} 0.01289 \\ 0.08224 \end{bmatrix}$$
$$B_{R_3}=\begin{bmatrix} -0.02593 \\ -0.18915 \end{bmatrix}, \quad B_{L_3}=\begin{bmatrix} -0.01824 \\ -0.1889 \end{bmatrix}$$

The decomposed state matrices eigenvalues using the proposed block-decomposition algorithm are

$$\lambda_1(A_{R_1})=0.02979=\lambda_1(A_{L_1})$$
$$\lambda_{2,3}(A_{R_2})=0.25066\pm j\,0.02554=\lambda_{2,3}(A_{L_2})$$
$$\lambda_{4,5}(A_{R_3})=0.89295\pm j\,0.09434=\lambda_{4,5}(A_{L_3})$$

and are close to original system eigenvalues, the worst error being

$$\frac{\lambda_1(A)-\lambda_1(A_{R_1})}{\lambda_1(A_{R_1})}\times 100=0.97 \text{ percent}$$

5. Conclusions

A linear discrete large-scale system often contains multi-modes for which simulations and designs are extremely difficult. In this paper, an algorithm to block-decomposition of the system is presented by the matrix sign function and the generalized matrix sign function, so that the analysis and the design of the system can be ealily performed.

The main results of this paper are:

(1) we develop a block-decomposition algorithm for the system by defining the reference circle, the circular stripe and the projection operators based upon the matrix sign function in Z-plane.

(2) There is no need to find the eigenvectors of system matrix in this algorithm.

(3) This algorithm do not need a permutation of the system.

(4) There is no need to solve the algebraic Riccati and the Lyapunov-type equations.

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