Transaction of the Korean Society of Mechanical Engineers, Vol. 10, No. 6, pp. 959~965, 1986.

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Kinematic Structures of Manipulators

Won-Gul Hwang*

(Received May 19, 1986)

매니퓰레이터의 運動學的 構造

黃 元 杰

Key Words: Manipulator(매니퓰레이터), Module(모듀울), Maneuverability(기동성), Manipulatability(조작성), Separability(가분성)

抄 錄

모듀울 로봇은 공장 자동화를 달성하기 위한 경제적인 방법으로 기대되고 있다. 모듀울 로봇을 구성하는 모듀울을 개발 설계하기 위해서는 먼저 유용한 로봇을 기본 구조를 연구하여야 한다.

일반적으로 로봇의 처음 3개의 관절은 로봇 손의 위치를 결정하는 데 쓰이고 나머지 관절은 로봇손의 방향을 조정하는데 사용된다. 본 논문에서 매니퓰레이터의 Jacobian 행열은 위치기동행열 (positional maneuverability matrix)과 방향기동행열로 구성됨을 보이고, 가분성(separability)의 조건을 제시하였다. 또한 회전관절이나 미끄름관절로 이루어진 3자유도 로봇 팔의 3차원 공간에서의 기본형태는 12개임을 밝히고, 3자유도 손목이 3개 방향을 완전히 조정할 수 있기 위해서는 3개의 회전관절축이 서로 직교하여야 함을 보였다.

1. Introduction

In robotization of complicated manufacturing systems we must solve problems to cope with complexity, diversity and mobility of manufacturing sites. It is not economically feasible to simply apply conventional general purpose robots to manufacturing systems because most of works in the process do not necessarily require a whole range of capabilities of the general purpose robots.

A robot modularization concept is applied as a new and reliable tool for solving this problem and developing manufacturing-use robots(1). Module robot is a building block system which uses standardized building blocks, called modules, to assemble a complete system. Although the robot modularization concept had been previously introduced in industries, the module robots have not been fully developed yet. A complete module robot system needs particular joints between robot hardware modules and

* Member, Department of Mechanical Engineering, Chonnam National University

hence extra costs. However by systematically utilizing robot hardware modules, the optimum combinations of appropriate robot hardware modules are expected to minimize the types of complete robot systems, and to save the capital investment for robotization of complicated systems.

As a first step to modularization, it is necessary to study the basic structures of robots. There have been research works for this subject[2~4]. Colson[2] surveyed kinematic arrangements of the currently manufactured robotic manipulators. Anisomorphic kinematic chains for major linkages are shown in (3). Milenkovic[4] gives 12 kinematic chains of major linkages, but 4 of them can be obtained from the others by changing the order of connection, resulting in 8 distinct chains. Separability plays an important role in his work, but its condition is not given. On the other hand, the concept of maneuverability is introduced in (5), and it is used to describe some problems of a kinematic configuration of a manipulator.

This paper took analytical approach to examine the structure of usable robots. It shows that the first three rows of the Jacobian matrix are positional maneuverability matrix and the remaining three rows are orientational maneuverability matrix, and it gives a condition for separability of a manipulator in terms of positional maneuverability matrix and orientational maneuverability matrix. It is also shown that there are only 12 distinct types of major robot linkages which are constructed with simple chains, and there is only one combination for 3 degrees of freedom minor linkage which uses simple chains.

2. Position and Orientation of the End Effector

A manipulator consists of a set of $n+1$ rigid links connected in an open-loop chain. Relative motion of the links in the chain results from forces which are applied to the links by an actuator fixed at each joint. To describe the motion of the system of n joints, let $\boldsymbol{q}(t)$ be an n-dimensional vector representing the actual displacement of the n joints.

The coordinate systems are assigned according to (6): Its $\boldsymbol{z}_j$-axis is the axis of rotation of the jth joint which is rotational, or the center line of link $j+1$ and is in the direction of motion if joint $j+1$ is translational. The coordinate system for link j is completed by defining $\boldsymbol{x}_j$ from the cross product $\boldsymbol{z}_{j-1}\times\boldsymbol{z}_j$, which also locates the origin, and $\boldsymbol{y}_j=\boldsymbol{z}_j\times\boldsymbol{x}_j$. The axes $\boldsymbol{x}_j$, $\boldsymbol{y}_j$, $\boldsymbol{z}_j$, etc. are defined as unit vectors. The relative positions of the linkage are determined by n sets of parameters a_j, s_j, α_j, and θ_j. The coordinate system and joint parameters are shown in Fig. 1. For a rotational joint, the parameter θ_j varies and is called the joint variable. For a translational joint, the joint variable is s_j. The joint variable will be denoted as q_j, and this will denote either θ_j or s_j depending on the type of joint.

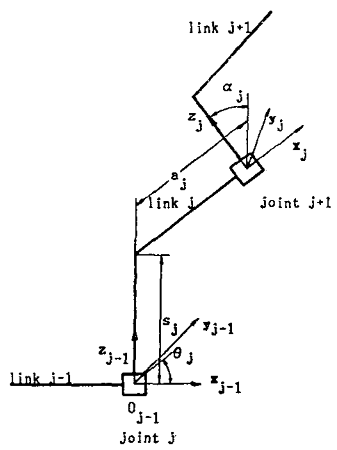

Fig. 1 Coordinate systems and joint parameters

The coordinate transformation from $O_i - x_i y_i z_i$ to $O_{i-1} - x_{i-1} y_{i-1} z_{i-1}$ is defined by the 4×4 matrix ${}^{i-1}A_i$;

$$ {}^{i-1}A_i = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & s_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (1) $$

The position and orientation of the end effector can be described by the vector $\boldsymbol{r} = (x, y, z, \alpha, \beta, \gamma)^T = (\boldsymbol{p}^T, \boldsymbol{\phi}^T)^T$, where vectors $\boldsymbol{p} = (x, y, z)^T$ and $\boldsymbol{\phi} = (\alpha, \beta, \gamma)^T$ are position and orientation of the end effector, respectively. The coordinate transformation from $O_n - x_n y_n z_n$ to $O_0 - x_0 y_0 z_0$ is defined by the matrix ${}^0A_n$;

$$ {}^0A_n = {}^0A_1 {}^1A_2 \cdots {}^{n-1}A_n = \begin{pmatrix} \boldsymbol{n} & \boldsymbol{o} & \boldsymbol{a} & \boldsymbol{p} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (2) $$

where $\boldsymbol{n}$, $\boldsymbol{o}$, and $\boldsymbol{a}$ are the unit normal, unit slide, and unit approach vectors of the hand, respectively.

3. Maneuverability

The kinematic problems of a manipulator with multi-degrees of freedom are considered as the decision of joint types and their arrangement. The position and orientation of the end effector of a manipulator should be sufficiently controlled within work space for industrial handling and other applications. The maneuverability of a manipulator represents the mobility of its configuration while the end effector assumes to be fixed at any position.

If the joint variables $\delta \boldsymbol{q}$ of a manipulator can be determined from $\delta \boldsymbol{p}$, the manipulator is called locally maneuverable with respect to the position. For the change of joint variables, δq_j, $j = 1, 2, \cdots, n$, we have the following relationship:

$$ U_1 \delta q_1 + U_2 \delta q_2 + \cdots + U_n \delta q_n = \delta^0 A_n \qquad (3) $$

where

$$ U_i = \partial^0 A_n / \partial q_i = \begin{pmatrix} \boldsymbol{n}_i & \boldsymbol{o}_i & \boldsymbol{a}_i & \boldsymbol{p}_i \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (4) $$

Now consider the differential translation vector $\delta \boldsymbol{d}$ and differential rotation vector $\delta \boldsymbol{\phi}$ such that

$$ \delta \boldsymbol{d} = \delta d_x \boldsymbol{i} + \delta d_y \boldsymbol{j} + \delta d_z \boldsymbol{k} \qquad (5) $$
$$ \delta \boldsymbol{\phi} = \delta \phi_x \boldsymbol{i} + \delta \phi_y \boldsymbol{j} + \delta \phi_z \boldsymbol{k} $$

where $\delta\phi_x$, $\delta\phi_y$, and $\delta\phi_z$ are differential rotations about $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ axes, respectively. Then the differential translation and rotation transformation $\delta^0 A_n$ is given[7] as

$$ \delta^0 A_n = \begin{pmatrix} 0 & -\delta\phi_z & \delta\phi_y & \delta d_x \\ \delta\phi_z & 0 & -\delta\phi_x & \delta d_y \\ -\delta\phi_y & \delta\phi_x & 0 & \delta d_z \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \delta\boldsymbol{n} & \delta\boldsymbol{o} & \delta\boldsymbol{a} & \delta\boldsymbol{p} \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (6) $$

Since the positions at each joint are given by the right-hand column of U_i, Eqns. (3) and (6) give the following matrix equation:

$$ J_p \delta \boldsymbol{q} = \delta \boldsymbol{p} \qquad (7) $$

where

$$ J_p = [\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_n] \qquad (8) $$

The $3 \times n$ matrix J_p is called the positional maneuverability matrix. It is necessary and sufficient that for Eqn. (7) to be solved, the rank of J_p is 3. Hence we get the following theorem.

Theorem 3-1.[5] If the manipulator has rank $[J_p] = 3$, it is locally maneuverable with respect to the position in 3-dimensional space.

The condition for orientational maneuverability can be obtained similarly. Considering

Eqns. (3), (4), and (6), the rotation with respect to $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$-axis is given by element (3, 2), (1, 3), and (2, 1) of U_i, respectively. Define the orientational maneuverability matrix J_ϕ as

$$J_\phi=[\boldsymbol{\phi}_1\ \boldsymbol{\phi}_2\cdots\boldsymbol{\phi}_n] \tag{9}$$

where $\boldsymbol{\phi}_i=(o_{i3},\ a_{i1},\ n_{i2})^T$. Then we get the following equation similar to Eqn. (7):

$$J_\phi\delta\boldsymbol{q}=\delta\boldsymbol{\phi} \tag{10}$$

From this equation, the following theorem is obtained in the same way as Theorem 3.1.

Theorem 3-2. If rank$[J_\phi]=3$, then the manipulator is locally maneuverable with respect to the orientation in 3-dimensional space.

From the definition of vector $\boldsymbol{r}$ and Eqns. (7) and (10), it follows that

$$\delta\boldsymbol{r}=\begin{pmatrix}\delta\boldsymbol{p}\\ \delta\boldsymbol{\phi}\end{pmatrix}=\begin{pmatrix}J_p\\ J_\phi\end{pmatrix}\delta\boldsymbol{q} \tag{11}$$

But by definition of the Jacobian matrix J,

$$\delta\boldsymbol{r}=J\delta\boldsymbol{q} \tag{12}$$

Comparing Eqns. (11) and (12), it follows that

$$J=\begin{pmatrix}J_p\\ J_\phi\end{pmatrix} \tag{13}$$

which implies that the first three rows of the Jacobian matrix are the positional maneuverability matrix and the remaining rows are the orientational maneuverability matrix. Manipulatability w is defined as[8]

$$w=\sqrt{\det(JJ^T)} \tag{14}$$

It gives a qualitative measure of manipulating ability of robot arms. From Eqn. (13), JJ^T becomes

$$JJ^T=\begin{pmatrix}J_pJ_p^T & J_pJ_\phi^T\\ J_\phi J_p^T & J_\phi J_\phi^T\end{pmatrix} \tag{15}$$

Now suppose that the manipulator is separable,[4] i.e., the position of the end effector is determined from m joints, $q_i, i=n_1, n_2, \cdots, n_m$, and its orientation from the remaining $(n-m)$ joints. For the change of joint variable $\delta\boldsymbol{q}$, we have the following relationships:

$$J_p\delta\boldsymbol{q}=\delta\boldsymbol{p} \tag{16}$$

$$J_\phi\delta\boldsymbol{q}=\delta\boldsymbol{\phi} \tag{17}$$

where

$$J_p=[\boldsymbol{p}_1\ \boldsymbol{p}_2\cdots\boldsymbol{p}_n];\quad \boldsymbol{p}_i=0 \text{ if } i\neq n_1, n_2, \cdots, n_m \tag{18}$$

$$J_\phi=[\boldsymbol{\phi}_1\ \boldsymbol{\phi}_2\cdots\boldsymbol{\phi}_n];\quad \boldsymbol{\phi}_i=0, \text{ if } i=n_1, n_2, \cdots, n_m \tag{19}$$

J_p and J_ϕ are $3\times n$ matrices. From Theorem 3.1 and 3.2, if rank$[J_p]=3$, the manipulator is locally maneuverable with respect to the position, and the condition for the orientational maneuverability reduces to rank$[J_\phi]=3$.

Define positional maneuverability w_p and orientational maneuverability w_ϕ as

$$w_p=\sqrt{\det(J_pJ_p^T)}$$

$$w_p=\sqrt{\det(J_\phi J_\phi^T)} \tag{20}$$

then from Eqns. (15)~(19),

$$JJ^T=\begin{pmatrix}J_pJ_p^T & 0\\ 0 & J_pJ_p^T\end{pmatrix} \tag{21}$$

and hence,

$$w=w_pw_\phi \tag{22}$$

This implies that manipulatability w becomes the product of w_p and w_ϕ if the manipulator is separable. Comparing Eqn. (21) with Eqn. (15) it is evident that the manipulator is separable if $J_pJ_\phi^T=0$.

Now suppose that the position of the end effector is determined from the first joint to mth one and its orientation from the $m+1$th joint to nth joint.

Then

$$J_p=[\boldsymbol{p}_1\ \boldsymbol{p}_2\cdots\boldsymbol{p}_m] \tag{23}$$

becomes $3\times m$ matrix, and

$$J_\phi=[\boldsymbol{\phi}_{m+1}\boldsymbol{\phi}_{m+2}\cdots\boldsymbol{\phi}_n] \tag{24}$$

becomes $3\times(n-m)$ matrix, and the Jacobian matrix J takes the following structure:

$$J=\begin{pmatrix}J_p & 0\\ 0 & J_\phi\end{pmatrix} \tag{25}$$

4. Robot Linkages

The first three joints of a robot from the base are usually designed to perform gross motion of the end effector, and the linkage consisting of the first three joints is called major linkage or arm. The linkage of the remaining joints, used to change its orientation, is called minor linkage or wrist. This feature of a robot manipulator is referred to as separability. The concepts of major and minor linkages apply only to separable robots[(4)].

Suppose that the major linkage of a manipulator consists of the first three joints, and the minor one the remaining three joints. Then J_p and J_ϕ become 3×3 matrices, and hence

$$w_p = |\det(J_p)| \qquad (26)$$

$$w_\phi = |\det(J_\phi)|, \qquad (27)$$

where $|.|$ means the absolute value. The condition that J_p and J_ϕ have full rank are the same as w_p and w_ϕ are not 0.

Those linkages which use only rotary or sliding joints with the joint axes either perpendicular or parallel to each other are defined as simple chains. Major robot linkages tend to use only simple chains. Let us consider the maneuverability of simple chain of three joints. For each joint there are two possibilities, rotational and sliding, denoted by R and S, respectively. Also for relation of two adjacent joint axes there are two choices, i.e., perpendicular and parallel, represented by $\perp$ and $/\!/$, respectively. Hence there are $2^5=32$ possible arrangements for simple chains of three joints. We symbolize ith joint and its relation to the next joint as follows:

$$R_i\perp \quad R_i/\!/ \quad S_i\perp \quad S_i/\!/$$

To examine the positional maneuverability of these 32 arrangements, it is necessary to check the ranks of their J_p matrices. We wrote a PASCAL program to calculate the positional maneuverability for these 32 arrangements, and w_p of 12 arrangements results in 0, which implies that they are not positionally maneuverable in 3-dimensional space. The remaining 20 open kinematic chains of major linkages are useful, and their w_p's are given in Table 1.

Table 1 w_p of the major linkages

$R\perp R\perp R$	$-a_3[a_1(a_2+a_3\cos\theta_3)\sin\theta_3+(a_2\sin\theta_3 -\bar{s}_2\cos\theta_3)\{(a_2+a_3\cos\theta_3)\cos\theta_2 +\bar{s}_3\sin\theta_2\}]$
$R\perp R/\!/R$	$-a_2a_3\sin\theta_3[a_1+a_2\cos\theta_2+a_3\cos(\theta_2+\theta_3)]$
$R/\!/R\perp R$	$a_1a_3\cos\theta_3[(a_2+a_3\cos\theta_3)\sin\theta_2-\bar{s}_3\cos\theta_2]$
$S\perp S\perp S$	$\sin\bar{\theta}_2$
$R\perp R\perp S$	$s_3[a_1+(a_2+a_3\cos\bar{\theta}_3)\cos\theta_2+s_3\sin\theta_2]$
$S\perp R\perp R$	$a_3\cos\theta_3[(a_2+a_3\cos\theta_3)\sin\theta_2-\bar{s}_3\cos\theta_2]$
$R\perp R/\!/S$	$(\bar{s}_2+s_3)[a_2\cos\theta_2+a_3\cos(\theta_2+\bar{\theta}_3)]$
$S/\!/R\perp R$	$a_3(a_3\cos\theta_3+a_2)\sin\theta_3$
$R/\!/R/\!/S$	$a_1[a_2\sin\theta_2+a_3\sin(\theta_2+\bar{\theta}_3)]$
$S/\!/R/\!/R$	$a_2a_3\sin\theta_3$
$R\perp S\perp R$	$a_3(s_2+a_3\sin\theta_3)\sin\bar{\theta}_2\sin\theta_3$
$R/\!/S\perp R$	$-a_3\sin\theta_3(a_1\cos\bar{\theta}_2+a_2+a_3\cos\theta_3)$
$R\perp S/\!/R$	$-a_3(s_2+\bar{s}_3)\cos(\bar{\theta}_2+\theta_3)$
$R/\!/S/\!/R$	$-a_3[a_1\sin(\bar{\theta}_2+\theta_3)+a_2\sin\theta_3]$
$S\perp S\perp R$	$-a_3\cos\bar{\theta}_2\sin\theta_3$
$R\perp S\perp S$	$(s_2+a_3\sin\bar{\theta}_3)\cos\bar{\theta}_2$
$S\perp S/\!/R$	$-a_3\sin(\bar{\theta}_2+\theta_3)$
$R/\!/S\perp S$	$a_1\sin\bar{\theta}_2+s_3$
$S\perp R/\!/S$	$a_2\sin\theta_2+a_3\sin(\theta_2+\bar{\theta}_3)$
$S/\!/R\perp S$	$-s_3$

One of the end links should be connected to the reference frame to function properly. Among the 20 open kinematic chains given above, 8 chains can be obtained from some others by reversing the order, and there remain 12 distinct chains shown below:

$R\perp R\perp R$	$R\perp R/\!/R$	$S\perp S\perp S$
$R\perp R\perp S$	$R\perp R/\!/S$	$R/\!/R/\!/S$
$R\perp S\perp R$	$R/\!/S\perp R$	$R/\!/S/\!/R$
$S\perp S\perp R$	$S\perp S/\!/R$	$S\perp R/\!/S$

'Distinct' means that each linkage is kinematically unique among the twelve categories. Fig. 2 shows one of possible arrangements of joints for them. Note that there can be other configurations for their mechanisms. Milenkovic[4] gives 12 kinematic chains of major linkages, but 4 of them can be obtained from the others by changing the order of connection; SC can be obtained from CS, SN from NS, RN from NR, CR from RC, by reversing the order. Hence he actually gives 8 chains, and $R\perp R/\!/S$, $R/\!/R/\!/S$, $S\perp S\perp R$, and $S\perp R/\!/S$ are not given.

In the same way, the orientational maneuverability of wrist of 3 degrees of freedom is checked. $R\perp R\perp R$ is found the only possible

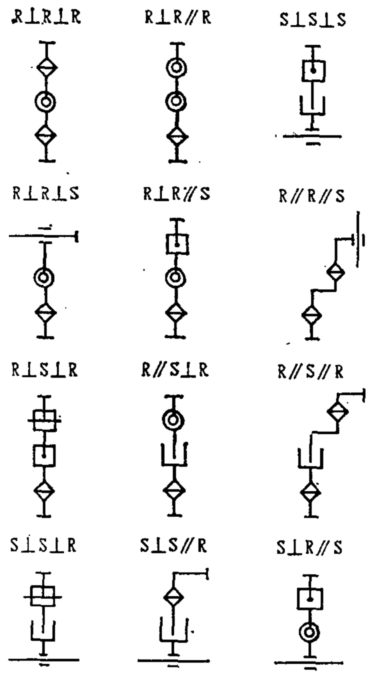

Fig. 2 Kinematic arrangements of major linkages

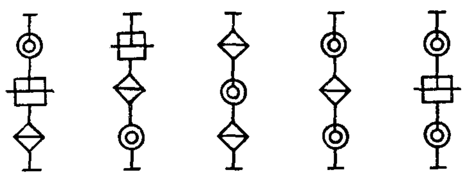

Fig. 3 Wrist $R\perp R\perp R$

arrangement in order to satisfy the condition for orientational maneuverability, and there are 5 arrangements to construct the wrist, as shown in Fig. 3.

5. Conclusions

The variety of possible robot configurations is very large. However robots of practical importance are separable. It is shown that the Jacobian matrix consists of positional maneuverability matrix and orientational maneuverability matrix, and if $J_p J_\phi^T=0$ then the manipulator is separable. It is also shown that the manipulatability is the product of the positional maneuverability and orientational maneuverability if the manipulator is separable.

Simple chains of three joints, which consist of rotary or sliding joints, are examined for positional maneuverability, and it is shown that there exist only 12 distinct categories of major robot linkages which use simple chain mechanism. Connecting one of the end links to the frame results 20 linkages.

Orientational maneuverability is also checked for wrists of 3 degrees of freedom, and it is found that $R\perp R\perp R$ arrangement is the only choice to achieve the orientational maneuverability in 3-dimensional space.

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