CONTINUITY OF HOMOMORPHISMS AND DERIVATIONS ON BANACH ALGEBRAS

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The purpose of this thesis is to study the automatic continuity problems for homomorphisms and derivations on Banach algebras.

Let A and B be Banach algebras. The basic automatic continuity problems for homomorphisms are to give algebraic conditions on A or B which ensure that every homomorphism from A into B is continuous. If X is a Banach A-bimodule, then a derivation from A into X is a linear mapping $D : A \longrightarrow X$ such that $D(ab) = aDb + (Da)b$, $a, b \in A$. Of particular importance are the derivations $D : A \longrightarrow A$ on A. The main results on this thesis are the following:

THEOREM. *Let h be an epimorphism from a Banach algebra A onto a Banach algebra B. If the radical R of B satisfies $\bigcap_{n \geqslant 1} R^n = \{0\}$ and is an integral domain, then h is continuous.*

THEOREM. *Let A be a commutative Banach algebra with radical R. Let D be a derivation in A with the separating space $\mathcal{G}$. If there exists an ideal I of A such that $I \subseteq R$, $D(I) \subseteq I$ and $\mathcal{G} \cap R \subseteq I$, then $D(A) \subseteq R$.*

THEOREM. *Let A be a commutative Banach algebra satisfying the following conditions:*

(ⅰ) *There are no closed prime ideals of infinite codimension*
(ⅱ) *For every maximal ideal M, M^2 is finitely generated and of finite codimension in M.*
Then any module derivation from A into a Banach A-bimodule is continuous.

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