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ALMOST-INVERTIBLE SPACES

PAUL E. LONG, LARRY L. HERRINGTON AND DRAGAN S. JANKOVIĆ

1. Introduction

A topological space (X, τ) is called *invertible* [7] if for each proper open set U in (X, τ) there exists a homeomorphsim $h: (X, \tau) \longrightarrow (X, \tau)$ such that $h(X-U) \subset U$. Doyle and Hocking [7] and Levine [13], as well as others have investigated properties of invertible spaces. Recently, Crosseley and Hildebrand [5] have introduced the concept of semi-invertibility, which is weaker than that of invertibility, by replacing "homemorphism" in the definition of invertibility with "semihomeomorphism", A space (X, τ) is said to be *semi-invertible* if for each proper semi-open set U in (X, τ) there exists a semihomeomorphism $h: (X, \tau) \longrightarrow (X, \tau)$ such that $h(X-U) \subset U$. The purpose of the present article is to introduce the class of almost-invertible spaces containing the class of semi-invertible spaces and to investigate its properties. One of the primary concerns will be to determine when a given local property in an almost-invertible space is also a global property. We point out that many of the results obtained can be applied in the cases of semi-invertible spaces and invertible spaces. For example, it is shown that if an invertible space (X, τ) has a nonempty open subset U which is, as a subspace, H-closed (resp. lightly compact, pseudocompact, S-closed, Urysohn, Urysohn-closed, extremally disconnected), then so is (X, τ).

2. θ-Homeomorphisms.

We recall that a subset A of a space (X, τ) is said to be ***regular-open*** (resp. *regular-closed*) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). The family of all regular-open (resp. regular-closed) subsets of a space (X, τ) is denoted by $RO(X, \tau)$ (resp. $RC(X, \tau)$). The topology τ_s on X which has as its base $RO(X, \tau)$ is called the *semiregularization* of τ. A

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space (X,τ) is said to be *semiregular* if $\tau_s=\tau$. A function $f:(X,\tau)\longrightarrow(Y,\tau)$ is called *θ-continuous* [8] if for each $x\in X$ and for each open set V containing $f(x)$ there exists an open set U containing x such that $f(cl(U))\subset cl(V)$.

DEFINITION 2.1 *A bijection $f:(X,\tau)\longrightarrow(y,\sigma)$ is a θ-homeomorphism if both f and f^{-1} are θ-continuous.*

Evidently, every homeomorphism is a θ-homeomorphism, but the converse is not ture since the identity function from a non-semiregular space (X,τ) onto (X,τ_s) is a θ-homeomorphism which is not a homeomorphism.

The following characterization of θ-homeomorphisms is established in [11].

THEOREM 2.2 *A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is a θ-homeomorphism if and only if $f:(X,\tau_s)\longrightarrow(y,\sigma_s)$ is a homeomorphism.*

A topological property P is said to be *semiregular* if it is shared by a space and its semiregularization. Combining Theorem 2.2 and the fact that the identity function from a space onto its semiregularization is a θ-homemorphism we obtain the following corollary.

COROLLARY 2.3 *A topological property is semiregular if and only if it is preserved under θ-homeomorphisms.*

There some other observations about θ-homeomorphisms as well as regular-open and regular-closed sets that we wish to make before proceeding to our weaker form of invertibility. The first of these properties are found in the next lemma and have rather straightforward proofs which are omitted.

LEMMA 2.4 *Let (X,τ) be a space and let Y be an open subspace of* (X,τ).

(a) $int_Y(cl_Y(A))=Y\cap int(cl(A))$ *for each subset A of y.*

(b) $R0(Y,\tau|Y)=\{Y\cap U:U\in R0(X,\tau)\}$.

(c) $RC(Y,\tau|Y)=\{Y\cap F:F\in RC(X,\tau)\}$.

(d) $(\tau|Y)_s=\tau_s|Y$.

Combining Theorem 2.2 and Lemma 2.4 we have the following result.

THEOREM 2.5 *Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a θ-homeomorphism. Then*

(a) $U \in \tau_s$ *implies that* $f|U : U \longrightarrow f(U)$ *is a* θ*-homeomorphism.*

(b) $F \in RC(X, \tau)$ *implies that* $f|F : F \longrightarrow f(E)$ *is a* θ*-homeomorphism.*

Recall that a subset A of a space (X, τ) is said to be *semi-open* [12] (resp. *α-set* [15]) if $A \subset cl(int(A))$ (resp. $A \subset int(cl(int(A)))$). The family of all semi-open sets (resp. α-sets) in a space (X, τ) is denoted by $SO(X, \tau)$ (resp. τ^α). It was observed in [15] that τ^α is a topology on X and that $\tau \subset \tau^\alpha \subset SO(X, \tau)$. The topology τ^α is, in fact, the finest topology in the class of topologies on X which yield the same semi-open sets as τ and was denoted by $F(\tau)$ in [6]. A bijection $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called a *semihomeomorphism* [6] if $f(U) \in SO(Y, \sigma)$ for each $U \in SO(X, \tau)$ and $f^{-1}(V) \in SO(X, \tau)$ for each $V \in SO(Y, \sigma)$. A property P is called *semitopological* [6] if it is preserved under semihomeomorphisms.

The following results, which will be very useful in the sequal, are established in [11].

THEOREM 2.6 *A function* $f : (X, \tau) \longrightarrow (y, \sigma)$ *is a semihomeomorphism if and only if* $f : (X, \tau^\alpha) \longrightarrow (y, \sigma^\alpha)$ *is a homeomorphism.*

COROLLARY 2.7 *A topological property is semitopological if and only if it is shared by a space* (X, τ) *and the space* (X, τ^α).

THEOREM 2.8 *Semihomeomorphisms are* θ*-homeomorphisms.*

COROLLARY 2.9 *Semiregular properties are semitopological.*

3. Invertible and semi-invertible spaces.

Let $I(J)$ be a class of topological properties such that $P \in I\,(P \in J)$ if and only if for any invertible space (X, τ) if a nonempty set $U \in \tau$ is such that $U(cl(U))$ has P, as a subspace of (X, τ), then (X, τ) has P. It was shown in [7], [13], and [14] that the following properties are in I(resp. J) : T_0, T_1, T_2, separability, first countability, second countability, regularity normality, and T_1 complete regularity (resp. compactness and T_2 paracompactness).

We say that a topological property P is *finite unionable* (resp. *open finite unionable, closed finite unionable*) if a space (X, τ) has P when a subspace $A_i\,(i=1, 2, \ldots, n)$ of (X, τ) has P and $X = \bigcup_{i=1}^{n} A_i$.

THEOREM 3.1 *If a topological property P is open finite unionable (resp. closed finite unionable), then $P \in I$ (resp. $P \in J$).*

Proof. Let P be an open finite unionable topological property, let (X, τ) be an invertible space and let U be a nonempty open subset which has P as a subspace of (X, τ). Then there exists a homeomorphism $h : (X, \tau) \longrightarrow (X, \tau)$ such that $h(X-U) \subset U$. It is not difficult to see that this implies that $X = U \cup h(U)$. Clearly, $h(U)$ is open in (X, τ) and $h(U)$ has P. Since P is open finite unionable, (X, τ) has P. Therefore $P \in I$.

The proof of the second part of the theorem is paralell to that of the first part when it is observed that $X = cl(U) \cup h^{-1}(cl(U))$.

We remark that compactness, countable compactness, Lindelofness and separability are finite unionable properties and that T_0, T_1, first countability and second countability are open finite unionable. In this section we prove that two more properties are in the class I.

THEOREM 3.2 *A property of spaces being Urysohn is in the class I.*

Proof. Let (X, τ) be an invertible space and let U be a nonempty open set which is, as a subspace, Urysohn. Then U is T_2 and since T_2 is in the class I, (X, τ) is T_2. To show that (X, τ) is Urysohn, let $x_1 \neq x_2$ belong to X. Then, because (X, τ) is T_2, there exist disjoint open sets V_1 and V_2 in (X, τ) containing x_1 and x_2, respectively. Furthermore, V_1 and V_2 may be chosen so that $cl(V_1) \cup cl(V_2) \neq X$. Otherwise, X consists of exactly two points and the proof is complete. Since (X, τ) is invertible, there exists a homeomorphism $h : (X, \tau) \longrightarrow (X, \tau)$ such that

$$h(cl(V_1) \cup cl(V_2)) = cl(h(V_1)) \cup cl(h(V_2)) \subset U$$

([7], Theorem 6). Clearly, $h(V_1)$ and $h(V_2)$ are open in (X, τ) and hence open in U. Since $h(x_1) \neq h(x_2)$ and U is Urysohn, there exist open sets W_1 and W_2 in U containing $h(x_1)$ and $h(x_2)$, respectively, such that $cl_U(W_1) \cap cl_U(W_2) = \phi$. Denote $W_1 \cap h(V_1)$ by H_i $(i=1, 2)$. It is clear that $cl(H_i) \subset cl(h(V_i)) \subset U (i=1, 2)$. Therefore $cl_U(H_i) = U \cap cl(H_i) = cl(H_i)$ $(i=1, 2)$. Since $H_1, H_2 \in \tau$, $x_i \in cl(h^{-1}(H_i))$ $(i=1, 2)$ and $cl(h^{-1}(H_1)) \cap cl(h^{-1}(H_2)) = \phi$, (X, τ) is Urysohn.

THEOREM 3.3 *Extremal disconnectedness is in the class I.*

Proof. Let F be a proper regular-closed subset of an invertible space (X, τ) and let U be a nonempty open subset which is extremally disconnected as a subspace of (X, τ).

Case 1: $U-F\neq\phi$. Then $V=U-F$ is open in (X, τ) and hence is open in U. Since extremal disconnectedness is open hereditary, V is extremally disconnected as a subspace of (X, τ). Let h be an inverting homeomorphism for V. Then $h(F)\subset V$ and $h(F)\in RC(X, \tau)$. By Lemma 2.4(c), $h(F)$ is regular-closed in V. Since V is extremally disconnected, $h(F)$ is open in V and hence is open in (X, τ). Therefore F is open in (X, τ).

Case 2: $U-F=\phi(U\subset F)$. Denote by V the regular-open subset $X-F$ of (X, τ). Let h be an inverting homeomorphism for U. Then $h(V)\subset h(X-U)\subset U$ and $h(V)\in RO(X, \tau)$. By Lemma 2.4(b), $h(V)$ is regular-open in U and since U is extremally disconnected, $h(V)$ is closed in U. Noting that $h(X-U)$ is closed in (X, τ) and that $h(V)\subset h(X-U)\subset U$, we have that $cl(h(V))\subset U$ and hence $h(V)$ is closed in (X, τ). This implies that V is closed in (X, τ) and consequently, F is open in (X, τ).

In Theorem 2.3 of [5] it was shown that if a space (X, τ) is semi-invertible, then (X, τ^α) is invertible. The converse of this result is also true.

THEOREM 3.4 *A space (X, τ) is semi-invertible if and only if (X, τ^α) is invertible.*

Proof. Assume that (X, τ^α) is invertible. By Theorem 1.1 of [5] it follows that for each nonempty $U\in SO(X, \tau^\alpha)$, there exists a homeomorphism $h:(X, \tau^\alpha)\longrightarrow(X, \tau^\alpha)$ such that $h(X-U)\subset U$. Since $SO(X, \tau^\alpha)=SO(X, \tau)$ and since by Theorem 2.6 $h:(X, \tau)\longrightarrow(X, \tau)$ is a semi-homeomorphism, (X, τ) is semi-invertible.

COROLLARY 3.5 *Let P be a semitopological property belonging to the class I and let (X, τ) be a semi-invertible space. If a nonempty set $U\in\tau^\alpha$ has P, as a subspace of (X, τ), then (X, τ) has P.*

Proof. Since $(U, \tau|U)$ has P and P is semitopological, $(U(\tau|U)^\alpha)$ has P by Corollary 2.7. Noting that $(\tau|U)^\alpha=\tau^\alpha|U$, $(U, \tau^\alpha|U)$ has P. By Theorem 3.4, (X, τ^α) is invertible and since $(U, \tau^\alpha|U)$ has P and $P\in I$, (X, τ^α) has P. Since P is semitopological, (X, τ) has P by

Corollary 2. 7.

In [6] it was shown that separability and T_2 are semitopological properties. Since they belong to the class I, Corollary 3. 5 gives us Theorem 2. 7 and Theorem 2. 6 of [5]. Also, Theorem 3. 2 and Theorem 3. 3 can be slightly improved by use of Corollary 3. 5 since extremal disconnectedness and a property of spaces being Urysohn are semiregular and hence are semitopological by Corollary 2. 9.

COROLLARY 3. 6 *If (X, τ) is a semi-invertible space and contains a nonempty set $U \in \tau^\alpha$ which is Uryohn (resp. extremally disconnected), as a subspace of (X, τ), then (X, τ) is Urysohn (resp. extremally disconnected).*

4. Almost-invertible spaces

DEFINITION 4. 1 *A space (X, τ) is almost-invertible if for every nonempty regular-open subset U of (X, τ) there exists a θ-homeomorphism $h : (X, \tau) \longrightarrow (X, \tau)$ such that $h(X-U) \subset U$.*

By Theorem 2. 2 and Theorem 2. 8 we obtain the following basic characterization of almost-invertible spaces.

THEOREM 4. 2 *A space (X, τ) is almost-invertible if and only if (X, τ_s) is invertible.*

As we mentioned earlier, the class of invertible spaces is contained in the class of semi-invertible spaces. By means of Theorem 2. 2, Theorem 2. 8 and Theorem 3. 4 we see that semi-invertible spaces are almost-invertible. The following example shows that there are almost-invertible spaces which are not semi-invertible.

EXAMPLE 4. 3 Let the plane R^2 have the topology generated by the subbase consisting of the usual open sets together with $A = \{(0, y) \in R^2 :$ y is rational and $0 < y < 1\}$ and call this topology τ. Now let $X = I \times I - \{(x, y) \in R^2 : 0 < x < 1,\ 0 < y < 1\}$ $(I = [0, 1])$ and give X the subspace topology inherited from (R^2, τ). Now observe that the semiregularization topology τ_s associated with (X, τ) is the subspace topology on X given from R^2 with the usual topology. It follows that (X, τ) is almost-invertible since (X, τ_s) is invertible. To show that (X, τ) is not semi-invertible, note that the set $B = \{(0, y) : y$ is irrational and $0 < y < 1\}$ is a nowhere dense set in (X, τ). According to [15], $C=(0,1)-B$ is open in (X, τ^{α}). But for the open set C in (X, τ^{α}) there is no semihomeomorphism h from (X, τ) onto (X, τ) such that $h(X-C)\subset C$ which shows that (X, τ) is not semi-invertible.

THEOREM 4.4 *If a space (X, τ) is semi-invertible and nowhere dense subsets of (X, τ) are closed, then (X, τ) is invertible.*

Proof. This follows form Theorem 3.4 and the fact that $\tau=\tau^{\alpha}$ if and only if the nowhere dense subsets of (X, τ) are closed [15].

THEOREM 4.5 *If a space (X, τ) is almost-invertible and semiregular, then (X, τ) is invertible.*

THEOREM 4.5 *If a space (X, τ) is almost-invertible and dense subsets of (X, τ_s) are dense in (X, τ), then (X, τ) is semi-invertible.*

Proof. If (X, τ) and (X, τ_s) have the same dense sets, they are semihomeomorphic [11] Therefore $\tau^{\alpha}=(\tau_s)^{\alpha}$. Since (X, τ_s) is invertible, $(X, (\tau_s)^{\alpha})$ is invertible and consequently (X, τ^{α}) is invertible. By Theorem 3.4, (X, τ) is semi-invertible.

THEOREM 4.7 *If (X, τ) is almost-invertible and contains a nonempty open connected set U, then (X, τ) consists of at most two componenets. If (X, τ) is not connected, then $V=int(cl(U))$ and $X-V$ are the components of (X, τ) and they are θ-homeomorphic.*

Proof. Noting that connectedness is a semiregular property and that $V=int(cl(U))$ is connected, we have by Lemma 2.5 that V is nonempty open connected set in (X, τ_s). If (X, τ) is not connected, then (X, τ_s) is not connected and since (X, τ_s) is invertible, V and $X-V$ are the components of (X, τ_s) and they are homeomorphic by Theorem 2 of [7]. This implies that V and $X-V$ are θ-homeomorphic components of (X, τ).

A pace (X, τ) is called *weakly Hausdorff* [17] if each $x\in X$ has the form $\{x\}=\cap\{F : x\in F \text{ and } F\in RC(X, \tau)\}$. It is clear that a space (X, τ) is weakly Hausdorff if and only if (X, τ_s) is T_1 and that weak Hausdorfness is a semiregular property. Combining the previous facts, Theorem 4.6, Theorem 4.7 and Theorem 3 of [7] we obtain the following two results.

THEOREM 4.8 *If an almost-invertible space (X, τ) is weakly Hausdorff and contains a nondegenerate open connected subset, then (X, τ) is connected.*

THEOREM 4.9 *A locally connected weakly Hausdorff almost-invertible space (X, τ) is either connected or the zero-sphere.*

Similar to Corollary 3.5 we have the following general result for almost-invertible spaces.

THEOREM 4.10 *Let P be a semiregular property belonging to the class I and let (X, τ) be an almost-invertible space. If a nonempty set $U \in \tau_s$ has P, as a subspace of (X, τ), then (X, τ) has P.*

Proof. Since $(U, \tau | U)$ has P and P is semiregular, $(U, (\tau | U)_s)$ has P. By Lemma 2.4(d) it follows that $(U, \tau_s | U)$ has P. Since (X, τ_s) is invertible by Theorem 4.2 and $P \in I$, (X, τ_s) has P. The semiregularity of P implies that (X, τ) has P.

COROLLARY 4.11 *Let (X, τ) be an almost-invertible space and suppose that there exists a $U \in \tau_s$ which is, as a subspace of (X, τ), T_2 (resp. weakly Hausdorff, Urysohn, extremally disconnected). Then (X, τ) is T_2 (rep. weakly Hausdorff, Urysohn, extremally disconnected).*

Proof. This follows from the fact that these properties are semiregular and in the class I along with use of Theorem 4.10.

Recall that a topological property P is called *contagious* [4] if a space (X, τ) has P when a dense subset of (X, τ) has P.

THEOREM 4.12 *Let P be a semiregular, contagious and open finite unionable topological property and (X, τ) be an almost-invertible space. If a nonempty open set U has P, as a subspace of (X, τ), then (X, τ) has P.*

Proof. Denote $int(cl(U))$ by V. Since U has the contagious property P and U is dense in V, V has P. The almost-invertibility of (X, τ) implies hat there exists a θ-homeomorphism $h : (X, \tau) \longrightarrow (X, \tau)$ such that $h(X - V) \subset V$. This gives that $X = V \cup h(V)$. From Theorem 2.5 (a) it follows that $h | V : V \longrightarrow h(V)$ is a θ-homeomorphism. Since P is semiregular, $h(V)$ has P by Corollary 2.3. Since P is open finite unionable and $X = V \cup h(V)$, (X, τ) has P.

Recall that a space (X, τ) is said to be *quasi H-closed* (resp. *lightly compact*) if every open (resp. countable open) cover of (X, τ) has a finite subfamily whose closures cover the space. A space is *H-closed* if it is Hausdorff and quasi H-closed. According to [2], quasi H-closedness, lightly compactness and pseudocompactness are semiregular, contagious and finite unionable. In [3] it was shown that S-closedness is semiregular, contagious and clopen finite unionable. Since a space (X, τ) is *S-closed* [3] if every regular-closed cover of (X, τ) has a finite subcover, it is not difficult to show, by use of Lemma 2.4(c), that S-closedness is open finite unionable. Combining the previous facts and Theorem 4.12 we obtain the following corollary.

COROLLARY 4.13 *Let (X, τ) be an almost-invertible space and suppose that there exists a nonempty open set U which is, as a subspace of (X, τ), quasi H-closed (resp. lightly compact, pseudocompact, S-closed). Then (X, τ) is quasi H-closed (resp. lightly compact, pseudocompact, S-closed).*

COROLLARY 4.14 *H-closedness, quasi H-closedness, lightly compactness, pseudo-compactness and S-closedness are in the class I.*

THEOREM 4.15 *Let P be a semiregular and closed finite unionable topological property, let (X, τ) be an almost-invertible space and suppose that there exists a nonempty open subset U of (X, τ) such that $cl(U)$, as a subspace, has P. Then (X, τ) has P.*

Proof. If $cl(U)=X$, the proof is complete. Assume that $cl(U)\neq X$ and let $h:(X, \tau)\longrightarrow(X, \tau)$ be an inverting θ-homeomorphism for $int(cl(U))$. Then we have that $X=cl(U)\cup h^{-1}(cl(U))$. By Lemma 2.5 (b), $h^{-1}|cl(U):cl(U)\longrightarrow h^{-1}(cl(U))$ is a θ-homeomorphism. Since P is semiregular, by Corollary 2.3 it follows that $h^{-1}(cl(U))$ has P. Since P is closed finite unionable and $X=cl(U)\cup h^{-1}(cl(U))$, (X, τ) has P.

COROLLARY 4.16 *Let (X, τ) be an almost-invertible space and suppose that there exists a nonempty open subset U of (X, τ) such that $cl(U)$, as a subspace, is H-closed (resp. quasi H-closed, lightly compact, pseudocompact). Then (X, τ) is H-closed (resp. quasi H-closed, lightly compact, pseudocompact).*

A Hausdorff space (X, τ) is *locally H-closed* [16] if for each $x \in X$ there exists an open set U containing x such that $cl(U)$ is H-closed. The following result is an immediate consequence of Corollary 4.16.

COROLLARY 4.17 *Locally H-closed almost-invertible spaces are H-closed.*

A Urysohn (resp. completely Hausdorff) space is called *Urysohn-closed* [1] (resp. *completely Hausdorff-closed* [1]) provided it is closed in every Urysohn (resp. completely Hausdorff) space in which it can be embeded.

LEMMA 4.18 *If* $A_i (i=1, 2, \dots, n)$ *is a Urysohn-closed (resp. completely Hausdorff-closed) subspace of a Urysohn (resp. completely Hausdorff) space* (X, τ) *and* $X = \bigcup_{i=1}^{n} A_i$, *then* (X, τ) *is Urysohn-closed (resp. completely Hausdorff-closed).*

Proof. We prove only the first part of the lemma since the proof of the second part is analogous. Suppose that there is a homeomorphism $h : (X, \tau) \longrightarrow Y$ embedding (X, τ) in a Urysohn space Y. Clearly $h| A_i : A_i \longrightarrow h(A_i)$ is a homeomorphism and since A_i is Urysohn-closed, $h(A_i)$ is closed in Y. This implies that $h(X) = \bigcup_{i=1}^{n} h(A_i)$ is closed in Y. Therefore (X, τ) is Urysohn-closed.

Combining Theorem 3.1, Theorem 3.2 and Lemma 4.18 it is not difficult to show the following result.

COROLLARY 4.19 *Urysohn-closedness is in the class* $I \cap J$.

COROLLARY 4.20 *Let* (X, τ) *be an invertible completely Hausdorff-closed space and suppose that there exists a nonempty set U which is completely-Hausdorff-closed as a subspace of* (X, τ). *Then* (X, τ) *is completely Hausdorff-closed.*

LEMMA 4.21 ([9], [10]). *Urysohn-closedness and complete Hausdorff-closedness are semiregular.*

THEOREM 4.22 *Let* (X, τ) *be an almost-invertible space and suppose that there exists a nonempty open subset U of* (X, τ) *such that* $cl(U)$, *as a subspace, is Urysohn-closed. Then* (X, τ) *is Urysohn-closed.*

Proof. Since a property of spaces being Urysohn is hereditary, $int(cl(U))$ is Urysohn. By Corollary 4.11, (X, τ) is Urysohn. From Theorem 4.2, Lemma 4.21 and Theorem 4.15 it follows that (X, τ) is Urysohn-closed.

THEOREM 4.23 *Let* (X, τ) *be an almost-invertible completely Hausdorff space and suppose that there exists a nonempty open set* U *of* (X, τ) *such that* $cl(U)$*, as a subspace, is completely Hausdorff-closed. Then* (X, τ) *is completely Hausdorff-closed.*

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Department of Mathematical Sciences, University of Arkansas, Fayetteville, AR 72701, USA.
Department of Mathematics, LSU, Alexandria, LA 71301, USA.
Department of Mathematics, Faculty of Civil Engineering, University of Belgrade, 11000 Belgrade, Yugoslavia.