〈研究論文〉

Journal of the Korean Institute
of Industrial Engineers
Vol. 11, No. 1, June 1985

Order-level Inventory Policy with Two Suppliers

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Abstract

This paper studies the two supplier inventory system in which order-level inventory policy with constant leadtimes is adopted. An optimal ordering policy to achieve the expected minimum total inventory cost is found by utilizing the concepts of the equivalence relation. Sensitivity analysis of the system parameters, the replenishment cost and the unit price, is done through a numerical example.

1. Introduction

Most of the inventory models in the literature have implicitly assumed that each stock item is replensihed from one supplier only. However, in practice, it is not uncommon to have more than one supplier and split a replenishment order quantity into several portions, one for each supplier and place the orders at the same time. By maintaining multiple supplier system, some benefits can be expected from the viewpoint of the purchasing manager. That is, through inducing competetiveness between the

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suppliers, quality of the purchased item can be improved and unit price lowered. Also, risk of having shortages due to the incompetence of keeping due date on the part of supplier can be decreased.

Previous work by Sculli and Wu [3] showed through the simulation of an example problem that the reorder level of two suppliers inventory system (hereafter called TSIS) is lower than that of one supplier inventory system (hereafter called OSIS) and observed that TSIS decreases in holding and shortage cost but increases in replenishment cost compared to OSIS.

The objective of this paper is to investigate analytically the effects of having more than one supplier on the purchasing inventory system. The system analyzed is the order-level, (t_p, Z), system in which the scheduling period t_p is prescribed and a replenishment raises the inventory position, the sum of what is currently on hand and already on order, at the beginning of each scheduling period to order-level Z.

Two suppliers are assumed and an optimal order-level of Z as well as a policy to split the replenishment order into the optimum portion for each supplier is determined.

2. Development of the Model

Notations :

c_1	:	Holding cost per unit per unit time
c_2	:	Shortage cost per unit per unit time
c_3	:	Replenishment cost
t_p	:	Prescribed scheduling period
L_1	:	Leadtime of supplier A
L_2	:	Leadtime of supplier B $(L_2 > L_1)$
d	:	$L_1 - L_2$
x(t)	:	Demand during time t
f(x;t)	:	p.d.f. of x(t)
u	:	Demand during leadtime L_1
$H_1(u)$	:	p.d.f. of u
v	:	Demand during leadtime L_2
$h_2(v)$	:	p.d.f. of v
H	:	Inventory on hand at the beginning of a scheduling period
Z	:	Order-level
Z_1	:	Suborder-level
q	:	Ordering quantity from suppliers and $q = Z - H$
q_1	:	Ordering quantity from supplier A
q_2	:	Ordering quantity from supplier B and $q = q_1 + q_2$
R_i	:	Proportion of ordering from ith supplier and $R_1 + R_2 = 1$

Assumtions :

(1) Order-level inventory policy is adopted.

(2) There is never more than one order outstanding, i.e., $t_p > L_2$.
(3) Leadtimes, L_1 and L_2, are deterministic.
(4) Demand is probabilistic and its probability density function is known.

Under TSIS, we consider two types of ordering policies, policy I and II.

In policy I, the suborder-level Z_1 and the order-level Z ($Z > Z_1$) are determined. At the beginning of each scheduling period if the inventory on hand H is less than Z_1, the quantity of Z_1-H is ordered from the supplier with the shorter leadtime (hereafter called supplier A) and the quantity of Z-Z_1, from the other supplier (hereafter called supplier B). On the other hand, when H is greater than or equal to Z_1, the quantity of Z-H is ordered from supplier B only.

In policy II, at the beginning of each scheduling period we split the order quantity according to the predetermined ratio R_i and order R_i(Z–H) from ith supplier.

Cost Model with Policy I

Let t_p be the prescribed scheduling period. Then t_p can be divided into two parts, d ($d = L_2 - L_1$) and t_p–d. Now, two order-levels have to be decided, i.e., Z_1 which is related to the cost incurred during the time interval d and Z related to the time interval t_p-d.

Since the demand x during t_p is random, at the reordering point two cases can occur in terms of x and $Z-Z_1$, either x is greater than $Z-Z_1$, i.e., H is less than Z_1 (hereafter called Case 1) or x is less than or equal to $Z-Z_1$, i.e., H is greater than or equal to Z_1 (hereafter called Case 2). Figure 1 and 2 show the inventory fluctuations of each case.

The inventory cost during d can be shown as

$$\mathrm{TC}_1(Z_1)=c_1\int_0^{Z_1}(Z_1-w)\,k_1(w,d)\,dw+c_2\int_{Z_1}^{\infty}(w-Z_1)\,k_1(w,d)\,dw \tag{1}$$

where $k_1(w,d)=\int_0^{w}\int_{w-u}^{\infty}\frac{f(x;d)}{x}h_1(u)\,dx\,du$

Similarly, the inventory cost during t_p–d becomes

$$\mathrm{TC}_2(Z)=c_1\int_0^{Z}(Z-w)\,k_2(w,t_p-d)\,dw+c_2\int_{Z}^{\infty}(w-Z)\,k_2(w,t_p-d)\,dw \tag{2}$$

where $k_2(w,t_p-d)=\int_0^{w}\int_{w-v}^{\infty}\frac{f(x;t_p-d)}{x}\,h_2(v)\,dx\,dv$

Therefore, the total cost equation during t_p is represented as

$$\begin{aligned}\mathrm{TC}(Z_1,Z)=&\int_{Z-Z_1}^{\infty}\left\{\frac{d}{t_p}\mathrm{TC}_1(Z_1)+\frac{t_p-d}{t_p}\mathrm{TC}_2(Z)\right\}f(x;t_p)\,dx \quad \text{(Case 1)}\\ &+\int_0^{Z-Z_1}\left\{\frac{d}{t_p}\mathrm{TC}_1(Z-x)+\frac{t_p-d}{t_p}\mathrm{TC}_2(Z)\right\}f(x;t_p)\,dx \quad \text{(Case 2)}\\ &+c_3/t_p\end{aligned} \tag{3}$$

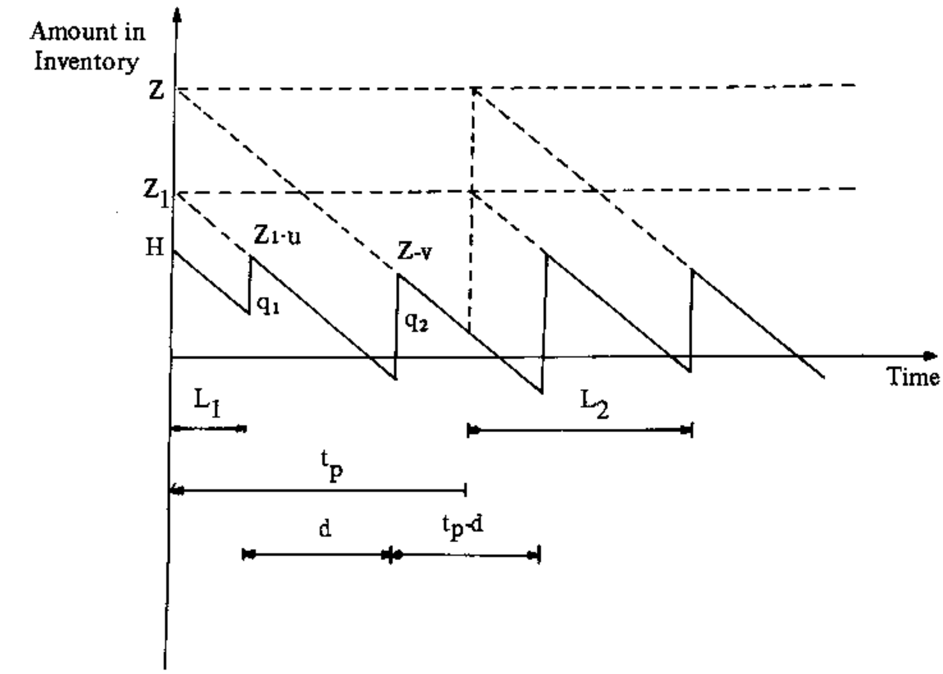

Figure 1. The inventory fluctuation of Case 1 in policy I

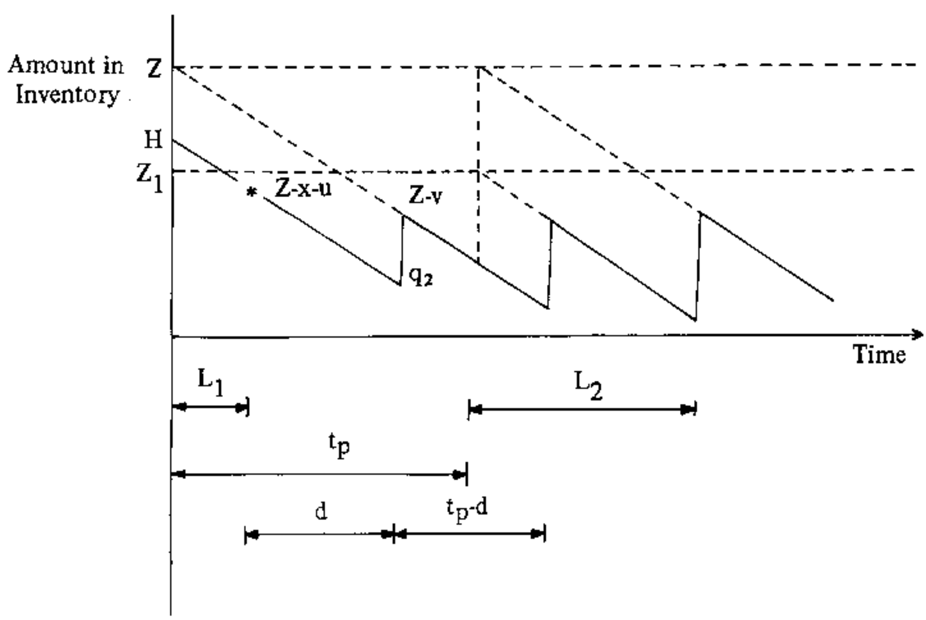

Figure 2. The inventory fluctuation of Case 2 in policy I

We want to find the optimal Z_1^0 and Z^0 which minimize $TC(Z_1, Z)$. By taking the partial derivatives of TC with respect to Z_1 and Z and substituting the equations (1) and (2) into TC_1 and TC_2, respectively, the following equations are derived.

$$\int_0^{Z_1^0} k_1(w, d)\, dw = \frac{c_2}{c_1+c_2} \tag{4}$$

$$\frac{\dfrac{t_p-d}{t_p}\displaystyle\int_0^{Z^0} k_2(w, t_p-d)\, dw + \frac{d}{t_p}\int_0^{Z^0-Z_1^0}\int_0^{Z^0-x} k_1(w, d)\, f(x; t_p)\, dw\, dx}{\dfrac{t_p-d}{t_p} + \dfrac{d}{t_p}\displaystyle\int_0^{Z^0-Z_1^0} f(x; t_p)\, dx} = \frac{c_2}{c_1+c_2} \tag{5}$$

With Z_1^0, Z^0 from equations (4) and (5), the expected minimum total cost TC (Z_1^0, Z^0) can be computed from equation (3).

The expected ordering quantity of each supplier can be obtained as follows: Let q_1 and q_2 be the ordering quantities from supplier A and supplier B, respectively. Then $q_1 = x - Z^0 + Z_1^0$ and $q_2 = Z^0 - Z_1^0$ in Case 1 while $q_1 = 0$ and $q_2 = Z^0 - Z_1^0$ in Case 2. Thus, $E(q_1)$ and $E(q_2)$ are

$$E(q_1) = \int_0^{Z^0-Z_1^0} o \cdot f(x; t_p)\, dx + \int_{Z^0-Z_1^0}^{\infty} (x - Z^0 + Z_1^0)\, f(x; t_p)\, dx$$

$$E(q_2) = \int_0^{Z^0-Z_1^0} x \cdot f(x; t_p)\, dx + \int_{Z^0-Z_1^0}^{\infty} (Z^0 - Z_1^0)\, f(x; t_p)\, dx \tag{6}$$

Cost Model with Policy II

In policy II, we split the order quantity Z–H according to the predetermined ratio, R_1 and R_2 (where $R_1 + R_2 = 1$ and $0 \leq R_1, R_2 \leq 1$) and order $R_1(Z - H)$ from supplier A and $R_2(Z - H)$ from supplier B. The inventory fluctuation of this policy is shown in Figure 3.

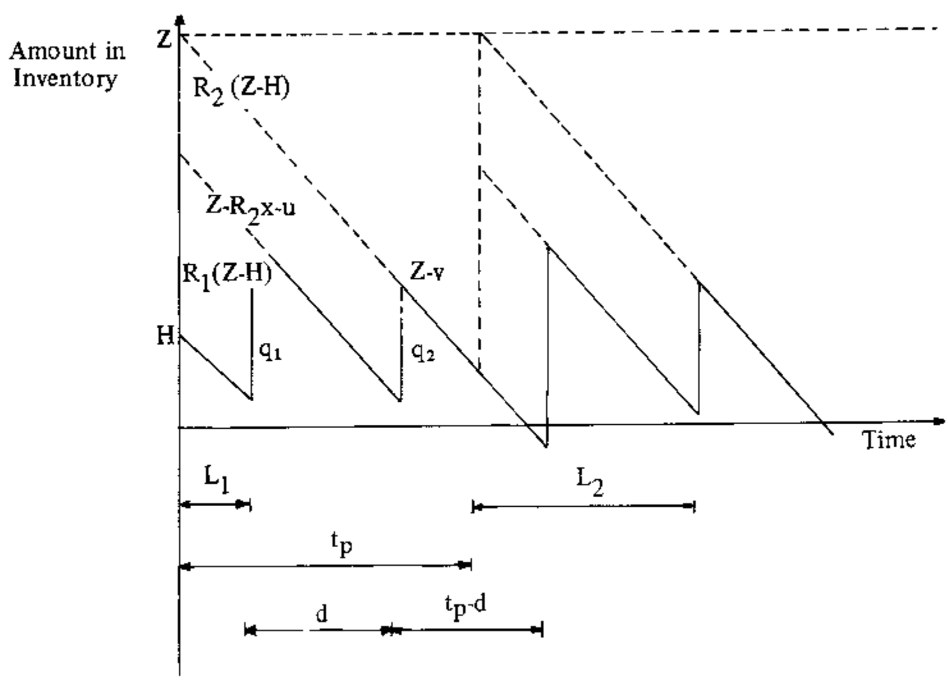

Figure 3. The inventory fluctuation in policy II

During the time interval t_p, the inventory cost becomes

$$TC(Z, R_2) = \int_0^\infty \{\frac{d}{t_p} TC_1(Z-R_2 x) + \frac{t_p-d}{t_p} TC_2(Z)\} f(x;t_p)dx + c_3/t_p$$
$$= \frac{d}{t_p}\int_0^\infty TC_1(Z-R_2 x) f(x;t_p)dx + \frac{t_p-d}{t_p} TC_2(Z) + c_3/t_p \tag{7}$$

In order to find the optimal Z^0, and $R_2{}^0$, we take the partial derivatives of TC with respect to Z and R_2 and substitute (1) and (2) for TC_1 and TC_2, respectively. Then, the following equations are obtained.

$$\frac{\int_0^\infty x\{\int_0^{Z^0-R_2^0 x} k_1(w,d)dw\} f(x;t_p)dx}{\int_0^\infty x f(x;t_p)dx} = \frac{c_2}{c_1+c_2} \tag{8}$$

$$\frac{d}{t_p}\int_0^\infty \{\int_0^{Z^0-R_2^0 x} k_1(w,d)dw\} f(x;t_p)dx + \frac{t_p-d}{t_p}\int_0^{Z_0} k_2(w, t_p-d)dw = \frac{c_2}{c_1+c_2} \tag{9}$$

Substitution of Z^0 and $R_2{}^0$ from equations (8) and (9) into equation (7) gives the expected minimum total cost.

The expected ordering quantitites can be written as

$$E(q_1) = R_1 \int_0^\infty x f(x;t_p)dx$$
$$E(q_2) = R_2 \int_0^\infty x f(x;t_p)dx \tag{10}$$

Numerical Example

In an inventory system with (t_p, Z), t_p is 4 weeks and the demand x during one week has the probability density function $f(x;1) = \exp(-x)$. Two suppliers are used to replenish an item, and the leadtime

Table 1. The results of the example

	OSIS		TSIS	
	Policy A	Policy B	Policy I	Policy II
order level	Z = 5.6	Z= 8.2	Z_1 = 3.89 Z = 6.53	Z = 5.98 R_1 = 0.74
*total cost	4.0380	4.8980	3.6328	3.9134
holding cost	2.7438	3.3698	2.4448	2.6052
shortage cost	1.2932	1.5282	1.1880	1.3082
ordering quantity	q = 4	q = 4	q_1 = 1.566 q_2 = 2.434	q_1 = 2.96 q_2 = 1.04

*Replenishment cost is excluded from the total cost.

of supplier A and supplier B are 1 week and 3 weeks, respectively. The unit cost parameters are c_1 = \$1 per unit per week, c_2 = \$9 per unit per week, and c_3 = \$5.

Table 1 shows the results of each policy in TSIS along with those of OSIS for the comparison purpose. In policy A of OSIS. supplier A is utilized while in policy B, order is placed with supplier B only.

3. Sensitivity Analysis

In section 2, we implicitly assumed that among each ordering policy the replenishment cost is identical. But in real world situation, substantial differences might exist in the replenishment cost on the total cost is in need. For brievity, policy I and policy A are chosen for this analysis. Let c_{3I} and c_{3A} be the replenishment costs of the policies I and A, respectively. Then from Table 1, the expected minimum total costs per week become

$TC\ (\text{policy I}) = 3.6328 + c_{3I}/4$

and

$TC\ (\text{policy A}) = 4.0380 + c_{3A}/4.$

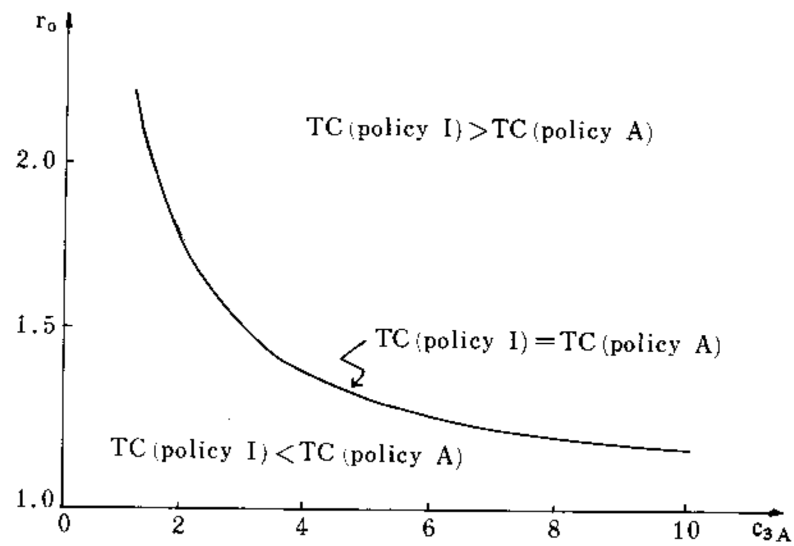

Figure 4. The effect of the replenishment cost on the total cost

Let $r = c_{3I}/c_{3A}$. The value, r_0, of r which makes TC (policy I) = TC (policy A) is $r_0 = 1 + 1.6328/c_{3A}$. Depending on the values of c_{3I} and c_{3A}, there are three cases as follows:

1) $r < r_0$, then TC (policy I) < TC (policy A)
2) $r = r_0$, then TC (policy I) = TC (policy A)
3) $r > r_0$, then TC (policy I) > TC (polich A)

The above results are shown in Figure 4 graphically.

4. Concluding Remarks

The cost models for two suppliers system were proposed from which a procedure to find an optimal solution is developed. We observed the decrease in the holding and shortage costs in policy I compared with policy II which was also observed by Sculli and Wu [3].

The results of the sensitivity analysis indicated that the effects of the replenishment cost are substantial on the total cost. In reality, suppliers with a shorter lead time might demand higher unit price compared to those with a longer lead time and this kind of inventory related costs should be also studied before selecting an optimal policy.

As a further study based on this paper, there are several topics deserving to be explored such as

1) Two suppliers with the reorder point lot size system with (Z, q) system,
2) The system with probabilistic leadtimes,
3) Three or more suppliers system.

APPENDIX

It will be proved that each of the total cost equations of TSIS, TC (Z_1, Z) and TC (Z, R_2) is the convex functions.

1. On the convexity of TC (Z_1, Z)

Let $a = d/t_p$ and $b = (t_p - d)/t_p$.
The first derivatives of TC (Z_1, Z) become

$$\frac{\partial TC}{\partial Z_1} = a\,\frac{\partial TC_1(Z_1)}{\partial Z_1} \tag{1}$$

$$\frac{\partial TC}{\partial Z} = b\,\frac{\partial TC_2(Z)}{\partial Z} + a\int_0^{Z-Z_1} \frac{\partial TC_1(Z-x)}{\partial Z} f(x;t_p)\,dx \tag{2}$$

Since $\frac{\partial^2 TC_i}{\partial Z^2} = (c_1+c_2)\,k_i(Z) \geqq 0$, both $TC_1(Z)$ and $TC_2(Z)$ are the convex functions. The second derivatives of TC (Z_1, Z) are

$$\frac{\partial^2 TC}{\partial Z_1^2} = a\,\frac{\partial^2 TC_1(Z_1)}{\partial Z_1^2} \geqq 0 \tag{3}$$

$$\frac{\partial^2 TC}{\partial Z_1 \partial Z} = 0 = -a\left[\frac{\partial TC_1(Z-x)}{\partial Z}\right]_{x=Z-Z_1} \cdot f(Z-Z_1;t_p) \tag{4}$$

$$\begin{aligned}\frac{\partial^2 TC}{\partial Z^2} &= b\,\frac{\partial^2 TC_2(Z)}{\partial Z^2} + a\left[\frac{\partial TC_1(Z-x)}{\partial Z}\right]_{x=Z-Z_1} \cdot f(Z-Z_1;t_p) \\ &\quad + a\int_0^{Z-Z_1} \frac{\partial^2 TC_1(Z-x)}{\partial Z^2} f(x;t_p)\,dx \\ &= b\,\frac{\partial^2 TC_2(Z)}{\partial Z^2} + a\int_0^{Z-Z_1} \frac{\partial^2 TC_1(Z-x)}{\partial Z} f(x;t_p)\,dx \geqq 0\end{aligned} \tag{5}$$

The above results show that the Hessian matrix of TC (Z_1, Z) is positive semi-definite. Hence, TC (Z_1, Z) is a convex function of Z_1 and Z.

2. On the convexity of TC (Z, R_2)

The first derivatives of TC (Z, R_2) are

$$\frac{\partial TC}{\partial R_2} = a\int_0^\infty \left\{\frac{\partial}{\partial R_2} TC_1(Z-R_2x)\right\} f(x;t_p)dx \tag{6}$$

$$\frac{\partial TC}{\partial Z} = a\int_0^\infty \left\{\frac{\partial}{\partial Z} TC_1(Z-R_2x)\right\} f(x;t_p)dx + b\frac{\partial}{\partial Z} TC_2(Z) \tag{7}$$

The second derivatives of TC (Z, R_2) become

$$\frac{\partial^2 TC}{\partial Z^2} = a\int_0^\infty \left\{\frac{\partial^2}{\partial Z^2} TC_1(Z-R_2x) f(x;t_p)dx + b\frac{\partial^2}{\partial Z^2} TC_2(Z)\right.$$

$$= (c_1+c_2)\left[a\int_0^\infty k_1(Z-R_2x) f(x;t_p)dx + bk_2(Z)\right] \geqq 0 \tag{8}$$

$$\frac{\partial^2 TC}{\partial Z\partial R_2} = a\frac{\partial}{\partial Z}\int_0^\infty \left\{\frac{\partial}{\partial R_2} TC_1(Z-R_2x)\right\} f(x;t_p)dx$$

$$= -a(c_1+c_2)\int_0^\infty x\, k_1(Z-R_2x) f(x;t_p)dx \tag{9}$$

$$\frac{\partial^2 TC}{\partial R_2^2} = a(c_1+c_2)\int_0^\infty x^2 k_1(Z-R_2x) f(x;t_p)dx \tag{10}$$

From the Schwartz inequality,

$$\frac{\partial^2 TC}{\partial Z^2}\cdot\frac{\partial^2 TC}{\partial R_2^2} - \left[\frac{\partial^2 TC}{\partial Z\partial R_2}\right]^2 \geqq 0 \tag{11}$$

With (11) it is observed that the Hessian matrix of TC (Z, R_2) is positive semi-definite. Therefore, TC (Z, R_2) is a convex function of Z and R_2.

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