Jour. Korean Inst. Mining Geol.
Vol. 18, No. 1, p. 41-47, 1985

A Comparison between J_0 and J_1 Digital Linear Filters in Resistivity Soundings

Hee Joon Kim*

Abstract: The filtering ability of J_0 and J_1 digital linear filters is compared by means of an adaptive linear filter. Any J_0 domain Hankel transform integral can be transformed mathematically into its corresponding J_1 domain integral. The apparent resistivities for any electrode configuration employed in resistivity soundings can be evaluated with a single J_1 filter. The J_1 filter usually has similar accuracy to, but shorter length than, the corresponding J_0 filter. The domain transformation from J_0 to J_1 enables us to use effective expressions of apparent resistivity, involving J_1 alone, not only for Schlumberger but also for dipole-dipole array.

INTRODUCTION

Since a digital linear filter method was introduced by Ghosh (1971a, b) in order to estimate Hankel transform integrals, many applications of it was found in geophysical problems. The evaluation of digital filtering or numerical convolution using predetermined coefficients is approximately an order of magnitude faster than the direct numerical integration of Hankel transform. In fact, the convolution technique completely avoids Bessel function evaluations which are requires in the direct numerical integration.

Many integral transforms encounterd in resistivity and electromagnetic (EM) soundings are the Hankel transforms of order 0 and 1. Higher integer orders can be expressed in terms of orders 0 and 1 by recursion (Watson, 1962). Koefoed and Dirk (1979) firstly pointed out the resemblance between the convolution filter and the Wiener filter. Following their algorithm, one can design an optimum length filter with desired accuracy. Anderson (1979) proposed an adaptive filtering technique which automatically truncates the summation within given tolerance. The Anderson's filter is especially useful in evaluating EM mutual coupling ratios for various loop-loop configurations over a layered earth model because the filter avoids re-evaluations for Hankel transforms of the same order. On the other hand, Das (1982) showed that there exist two basic filter spectra for the design of any filter function in computing resistivity and EM soundings. Once the spectra are stored, one can derive other filter coefficients and avoid extensive computations. All these procedures, however, involve two kinds of Hankel transform of orders 0 and 1.

A mehod of using a single J_0 filter in evaluating Wenner, Schlumberger and dipole-dipole apparent resistivity curves was given by Davis et al. (1980). The method eliminates the need for J_1 filter by transforming the apparent resistivity expressions into the form of potential expression, involving J_0 alone, through a series of substitutions. On the other hand, Das (1984) showed that a simple mathematical manipulation transforms a J_0 domain integral into its corresponding J_1 domain integral. This enables us to use a single J_1 filter for a variety of computa-

* Department of Applied Geology, National Fisheries University of Pusan

tions in electrical methods.

In this paper the filtering ability of J_0 and J_1 filters is compared by using the Anderson's adaptive filter (Anderson, 1979). The Anderson's filter was designed for Hankel transforms of orders 0 and 1, where both J_0 and J_1 filters have identical abscissa. The abscissa range is sufficiently long that an adaptive or variable cut-off algorithm can be applied to a variety of kernel functions. The adaptive cut-off algorithm enables us to estimate the filtering ability of J_0 and J_1 filters. It will be shown that the J_1 filter is more efficient than the J_0 filter in resistivity sounding problems. All numerical results shown in this paper are computed in single precision arithmetic.

DIGITAL LINEAR FILTER

A general procedure of designing digital linear filter coefficients for Hankel transform integrals is reviewed briefly in this section. We define the Hankel transform of the kernel $k(\lambda)$ of integer order n as

$$H(b)=\int_0^\infty k(\lambda)\lambda J_n(b\lambda)d\lambda,\ \ b>0, \tag{1}$$

where J_n is the Bessel function of the first kind of order n. In electrical methods, the order n is usually 0 or 1. The transform argument $b(b>0)$ is real, but $k(\lambda)$ and $H(b)$ may be complex functions of a real variable. By using the transformations $x=\ln(b)$ and $y=\ln(1/\lambda)$ on Eq. (1) and multiplying both sides by $\exp(x)$ to obtain symmetry in both abscissa directions, we get

$$e^x H(e^x)=\int_{-\infty}^{\infty}[e^{-y}k(e^{-y})]\ [e^{x-y}J_n(e^{x-y})]dy. \tag{2}$$

Eq. (2) has the form of linear convolution integral, where the term in the first and the second brackets in the integrand are the input and the filter response functions, respectively, and the left-hand side of the equation is the output function. Using the convolution theorem, the filter response function can be determined by using known input-output function pairs. Details on the design technique of digital linear filter can be found in Koefoed et al. (1972), Verma (1977), etc.

DOMAIN TRANSFORM

According to Das (1984), any J_0 domain Hankel transform integral can be transformed into its corresponding J_1 domain. Consider Eq. (1) with zero-order Bessel function:

$$H(b)=\int_0^\infty k(\lambda)\lambda J_0(b\lambda)d\lambda. \tag{3}$$

Applying Hankel transformation (Watson, 1962) to Eq. (3), we get

$$k(\lambda)=\int_0^\infty H(b)bJ_0(b\lambda)db. \tag{4}$$

Differentiate both sides of Eq. (4) with respect to λ, i.e.,

$$\frac{dk(\lambda)}{d\lambda}=-\int_0^\infty H(b)b^2J_1(b\lambda)db, \tag{5}$$

and apply Hankel inversion to Eq. (5) to obtain

$$H(b)=\int_0^\infty\left[-\frac{1}{b}\frac{dk(\lambda)}{d\lambda}\right]\lambda J_1(b\lambda)d\lambda. \tag{6}$$

A comparison between Eqs. (3) and (6) shows that the J_0 domain Hankel integral can be transformed into its corresponding J_1 domain Hankel integral.

In order to confirm the above approach numerically, an analytical Hankel transform pair with elementary kernel function is chosen in Gradshteyn and Ryzhik (1965). That is,

$$H(b)=\int_0^\infty[\exp(-a\lambda^2)]\lambda J_0(b\lambda)d\lambda, \tag{7}$$

where a is a positive constant. The solution of (7) is analytically obtained as

$$H(b)=\exp[-b^2/(4a)]/(2a). \tag{8}$$

By using the domain transform considered above, Eq. (7) is also represented as

$$H(b)=(2a/b)\int_0^\infty[\lambda\exp(-a\lambda^2)]\lambda J_1(b\lambda)d\lambda. \tag{9}$$

Eqs. (7) and (9) are evaluated numerically by the Anderson's filter (Anderson, 1979) as

Table 1 Comparison of computations of the analytical Hankel transform pair with elementary kernel function, i.e., Eqs. (7), (8) and (9), where $a=0.5$.

b	Exact	Filtered		Error		Length	
		J_0	J_1	J_0	J_1	J_0	J_1
.100E−02	.1000E+01	.1000E+01	.1000E+01	−.1192E−06	−.2384E−06	150	150
.100E−01	.1000E+01	.9999E+00	.1000E+01	.2384E−06	−.2980E−06	150	150
.500E−01	.9988E+00	.9988E+00	.9988E+00	.5960E−07	−.1192E−06	150	150
.100E+00	.9950E+00	.9950E+00	.9950E+00	.0000E+01	.0000E+01	150	91
.200E+00	.9802E+00	.9802E+00	.9802E+00	.4172E−06	.4172E−06	86	53
.500E+00	.8825E+00	.8825E+00	.8825E+00	−.5960E−07	.1073E−05	65	43
.700E+00	.7827E+00	.7827E+00	.7827E+00	−.1788E−06	.2027E−05	63	41
.100E+01	.6065E+00	.6065E+00	.6065E+00	−.7749E−06	.5364E−06	62	40
.150E+01	.3247E+00	.3247E+00	.3247E+00	.7451E−06	.2086E−06	61	38
.200E+01	.1353E+00	.1353E+00	.1353E+00	.6184E−05	.5379E−05	60	38

shown in Tabel 1. Here the constant a is 0.5 and the truncation tolerance of filter is fixed to 1.0E-07. From the table, we see that the accuracy of J_1 filter is similar to that of J_0 filter, but the J_1 filter is more efficient than the corresponding J_0 filter in $b>0.5$. This efficiency results from the fact that the J_1 filter function converges at a slightly higher rate than the J_0 filter function.

APPARENT RESISTIVITY

Fig. 1 shows the three typical sounding arrays: Wenner, Schlumberger and dipole-dipole. As usual, A and B indicate the current electrodes, and M and N the potential electrodes. For these four terminal arrays, the apparent resistivity, ρ_a, can be expressed as

$$\rho_a = K\Delta U/I, \tag{10}$$

where K is the geometric factor:

$$K=2\pi\left(\frac{1}{AM}-\frac{1}{BM}-\frac{1}{AN}+\frac{1}{BN}\right)^{-1}, \tag{11}$$

and ΔU is the potential difference between potential electrodes M and N:

$$\Delta U = U_M - U_N, \tag{12}$$

and I is the induced current. The potential U at a distance r from a point current source I on the surface of a layered earth (Fig. 2) is given by

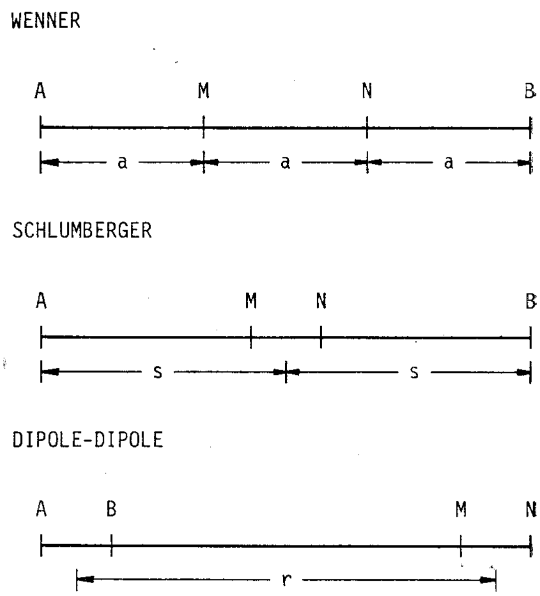

Fig. 1 Common resistivity sounding arrays. A and B indicate the current electrodes, and M and N the potential electrodes.

$$U(r) = (I/2\pi)\int_0^\infty T(\lambda)J_0(\lambda r)\,d\lambda, \tag{13}$$

where $T(\lambda)\equiv T_1(\lambda)$ is the resistivity transform or associated kernel obtained from the following recurrence formula (Koefoed, 1970; Kim, 1981)

$$T_i(\lambda)=\rho_i \times \frac{(\rho_i+T_{i+1})-(\rho_i-T_{i+1})\exp(-2\lambda d_i)}{(\rho_i+T_{i+1})+(\rho_i-T_{i+1})\exp(-2\lambda d_i)},$$
$$i=n-2,\ n-3,\cdots,\ 1, \tag{14}$$

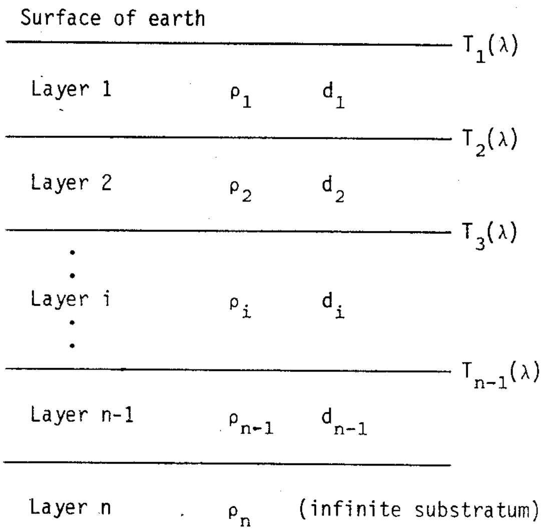

Fig. 2 A horizontally layered earth model. The definitions of symbols are explained in the text.

and

$$T_{n-1}(\lambda)=\rho_n \times \frac{(\rho_{n-1}+\rho_n)-(\rho_{n-1}-\rho_n)\exp(-2\lambda d_{n-1})}{(\rho_{n-1}+\rho_n)+(\rho_{n-1}-\rho_n)\exp(-2\lambda d_{n-1})}, \tag{15}$$

where ρ_i and d_i are the resistivity and thickness of the i-th layer, respectively.

The Wenner apparent resistivity, ρ_{aW}, for example, can be expressed as

$$\rho_{aW}=(K/I)[U(r_1)-U(r_2)], \tag{16}$$

where

$$r_1=AB/2-MN/2,$$
$$r_2=AB/2+MN/2,$$

and

$$AM=MN=NB=a.$$

By using Eqs. (11) and (13), Eq. (16) is rewritten by

$$\rho_{aW}(a)=(I/2\pi)\int_0^\infty T(\lambda)[J_0(\lambda a)-J_0(2\lambda a)]d\lambda. \tag{17}$$

From Eq. (17), we see that the Wenner apparent resistivity can be computed by the J_0 filter. Similar to the Wenner apparent resistivity, the Schlumberger and dipole-dipole apparent resistivities are also expressed with the potential difference. Hence all apparent resistivity for the layered earth can be evaluated numerically by the J_0 filter.

Here let's transform the potential of J_0 domain (13) into that of J_1 domain. Rearranging (13) as

$$U(r)=(I/2\pi)\int_0^\infty [T(\lambda)/\lambda]\lambda J_0(\lambda r)d\lambda, \tag{18}$$

and using the domain transform considered in the last section, we get

$$U(r)=\frac{I}{2\pi r}\int_0^\infty \left[\frac{T(\lambda)}{\lambda}-\frac{dT(\lambda)}{d\lambda}\right] \times J_1(\lambda r)d\lambda, \tag{19}$$

where $dT(\lambda)/d\lambda \equiv dT_1(\lambda)/d\lambda$ is the derivative of resistivity transform $T(\lambda)$:

$$\frac{dT_i}{d\lambda}=4\rho_i\exp(-2\lambda d_i) \times \frac{\rho_i\frac{dT_{i+1}}{d\lambda}+d_i(\rho_i^2-T_{i+1}^2)}{[(\rho_i+T_{i+1})+(\rho_i-T_{i+1})\exp(-2\lambda d_i)]^2}$$
$$i=n-2, n-3, \ldots, 1, \tag{20}$$

and

$$\frac{dT_{n-1}}{d\lambda}=4\rho_{n-1}d_{n-1} \times \frac{(\rho_{n-1}^2-\rho_n^2)\exp(-2\lambda d_{n-1})}{[(\rho_{n-1}+\rho_n)+(\rho_{n-1}-\rho_n)\exp(-2\lambda d_{n-1})]^2} \tag{21}$$

Eq. (19) shows that the potential can be also computed by the J_1 filter.

Table 2 compares the potentials evaluated respectively by J_0 and J_1 filters for a three-layer earth model. The earth model is the same as in Inman et al. (1973). These potentials are also computed by the Anderson's adaptive filter (Anderson, 1979). From the table, we see that both filters have similar accuracy, but the length of J_1 filter is shorter than that of J_0 filter for all r's. Noting that the additional time for computing the derivative in (19) is almost negligible compared with digital filtering, the potential and therefore the apparent resistivity for a layered earth are computed by the J_1 filter more efficiently than by the J_0 filter.

The familiar expressions of apparent resisti-

Table 2 Comparison of computations of the potentials in J_0 and J_1 domains over a layered earth having parameters: $\rho_1=20\Omega\cdot m$, $\rho_2=500\,\Omega\cdot m$ $\rho_3=100\Omega\cdot m$, $d_1=10m$ and $d_2=40m$.

r	Potential		Difference	Length	
	J_0	J_1		J_0	J_1
2.0	2.179	2.179	.1669E-05	171	127
3.0	1.647	1.647	.3576E-06	171	127
4.0	1.378	1.378	.3576E-06	171	127
5.0	1.216	1.216	.2384E-06	171	127
7.0	1.025	1.025	.7153E-06	171	127
10.0	.871	.871	.4172E-06	171	126
15.0	.730	.730	.5960E-07	171	125
20.0	.641	.641	.2980E-06	171	124
30.0	.523	.523	.5960E-07	171	123
40.0	.443	.443	.2384E-06	168	123
50.0	.383	.383	-.8941E-07	166	122
70.0	.300	.300	.2980E-07	161	122
100.0	.221	.221	.7451E-07	158	121
150.0	.147	.147	.7451E-07	156	121
200.0	.106	.106	.2980E-07	151	121

vity for Schlumberger and dipole-dipole arrays on the surface of a layered earth are different from (10). The Schlumberger apparent resistivity, $\rho_{aS}(s)$, is usually expressed as

$$\rho_{aS}(s)=s^2\int_0^\infty T(\lambda)\lambda J_1(\lambda s)\,d\lambda, \tag{22}$$

where $s=AB/2$ is the half electrode spacing (see Fig. 1). Here the potential electrode separation MN is assumed to be so small compared with the current electrode separation AB, then the potential difference divided by MN represents the electric field at the midpoint between A and B. Eq. (22) contains only one integral and thus is more time-saving to evaluate than Eq. (10). Besides, Eq. (22) can be computed by the J_1 filter.

A general expression of dipole-dipole apparent resistivity, ρ_{aD}, is

$$\rho_{aD}(r)=(1-c)r^2\int_0^\infty T(\lambda)\lambda J_1(\lambda r)\,d\lambda - cr^3\int_0^\infty T(\lambda)\lambda^2 J_0(\lambda r)\,d\lambda, \tag{23}$$

where r is the spacing between current and potential dipoles (see Fig. 1) and c is a constant depending upon the type of dipole-dipole array used. Here both current and potential dipole separations (AB and MN) are assumed to be small compared with the dipole spacing r. For the polar dipole-dipole system as shown in Fig. 1, the constant c is taken to be 0.5 (Das and Ghosh, 1973). By transforming the J_0 domain integral of (23) into its corresponding J_1 domain integral, we get

$$\rho_{aD}(r)=r^2\int_0^\infty\left[T(\lambda)+c\lambda\frac{dT(\lambda)}{d\lambda}\right]\lambda J_1(\lambda r)\,d\lambda. \tag{24}$$

This equation contains only one integral of J_1 domain and thus is more efficient to evaluate than Eq. (10).

DISCUSSION AND CONCLUSIONS

In electrical methods, the Hankel transform integrals of interest usually involve the Bessel functions of order 0 and 1. Occasionally, we must evaluate the Hankel integrals involving both J_0 and J_1 Bessel functions, e.g., in the dipole-dipole apparent resistivity represented by (23) or the mutual coupling ratio for vertical coaxial loops. If we eliminate the need for either J_0 or J_1 filter, then many advantages will be expected especially in computer storage.

By using a potential expression, the apparent resistivity for any electrode configuration can be evaluated with a single J_0 filter as shown in Eq. (10). This approach always requires twice evaluations of Hankel integral to get an apparent resistivity for any array. For the Schlumberger array, however, the well-known expression of apparent resistivity, i.e., Eq. (22), involves only one integral of J_1 domain. Hence the method of using a single J_0 filter for computing apparent resistivities is a relatively time-consuming technique.

The alternative unification is also possible by means of the domain transform described in this paper. This approach enable us to use the more time-saving expressions (22) for the Schlumberger array and (24) for the dipole-dipole array. These expressions require only once evaluation of Hankel integral involving J_1 Bessel function. Besides, from Tables 1 and 2 we can find that the J_1 filter usually has similar accuracy to, but shorter length than, the corresponding J_0 filter.

The eqi-spaced electrode configurations, i.e., Wenner and Eltran arrays, have no time-saving expression of apparent resistivity, because the potential electrode spacing is relatively wide compared with the separation between current and potential electrodes. Thus the apparent resistivities for Wenner and Eltran arrays should be represented with potential expressions involving two Hankel integrals of J_0 domain. Even in these cases, the transformation from J_0 to J_1 may be useful in saving the computer time as shown in Table 2.

The domain transform is not necessarily restricted in resistivity soundings but it can be applied to the simultaneous evaluations of Hankel integrals involving J_0 and J_1 Bessel functions, e.g., in EM soundings or three-dimensional modelings. The domain transform may unify the concept of designing different digital linear filters for various computations in electrical methods. A single J_1 filter with sufficiently long coefficients can be used to evaluate different quantities measured by any electrode or coil configurations employed in electrical methods. Although the Anderson's adaptive filter (Anderson, 1979) was used in this paper, one can design alternatively an optimum length J_1 filter with desired accuracy by following the Wiener filter techniques (Koefoed and Dirks, 1979; Murakami and Uchida, 1982).

ACKNOWLEDGMENTS

I wish to thank Dr. Y.Q. Kang, Nat. Fish. Univ. Pusan, for his helpful comments on this work.

REFERENCES

Anderson, W.L. (1979) Numerical integration of related Hankel transforms of orders 0 and 1 by adaptive digital filtering. Geophysics, v. 44, p. 1287-1305.

Das, U.C. (1982) Designing digital linear filters for computing resistivity and electromagnetic sounding curves. Geophysics, v. 47, p. 1456-1459.

Das, U.C. (1984) A single digital linear filter for computations in electrical methods: A unifying approach. Geophysics, v. 49, p. 1115-1118.

Das, U.C. and Ghosh, D.P. (1973) A study on the direct interpretation of dipole sounding resistivity measurements over layered earth. Geophys. Prosp., v. 21, p. 371-400.

Davis, P.A., Greenhalgh, S.A. and Merrick, N.P. (1982) Resistivity sounding computations with any array using a single digital filter. Bull. Austr. Soc. Expl. Geophys., v. 11, p. 54-62.

Ghosh, D.P. (1971a) The application of linear filter theory to the direct interpretation of geoelectrical resistivity sounding measurements. Geophys. Prosp., v. 19, p. 192-217.

Ghosh, D.P. (1971b) Inverse filter coefficients for the computation of apparent resistivity standard curves for a horizontally stratified earth. Geophys. Prosp., v. 19, p. 769-775.

Gradshteyn, I.S. and Ryzhik, I.M. (1965) Tables of Integrals, Series, and Products. New York, Academic Press, 1086pp.

Inman, J.R., Ryu, J. and Ward, S.H. (1973) Resistivity inversion. Geophysics, v. 38, p. 1088-1108.

Kim, H.J. (1981) Studies on the resistivity inversions: 1. Automatic interpretation of electrical resistivity sounding data. J. Korean Inst. Mining Geol., v. 14, p. 193-201.

Koefoed, O. (1970) A fast method for determining the layer distribution from the raised kernel function. Geophys. Prosp., v. 18, p. 564-570.

Koefoed, O. and Dirks, F.J.H. (1979) Determination of resistivity sounding filters by the Wiener-Hopf least-squares method. Geophys. Prosp., v. 27, p. 245-250.

Murakami, Y. and Uchida, T. (1982) Accuracy of the linear filter coefficients determined by the iteration of the least-squares method. Geophysics, v. 47, p. 244-256.
Verma, R.K. (1977) Detectability by electromagnetic sounding systems. IEEE Trans. Geosci. Electron, v. GE-15, p. 232-251.
Watson, G.N. (1962) A Treatise on the Theory of Bessel Functions, 2nd Ed. Cambridge Univ. Press, 804pp.

비저항탐사에서 J_0 및 J_1 디지탈 선형필터의 비교

김 희 준*

요약 : Hankel변환적분의 계산에 사용되는 J_0 및 J_1 디지탈 선형필터의 성능을 adaptive 필터를 사용함으로써 비교 하였다. J_0 영역의 어떠한 Hankel 변환적분도 수학적으로 이에 대응하는 J_1 영역으로 바꿀 수 있으며, 비저항탐사에서 이용되는 모든 전극배치에 대한 겉보기비저항은 J_1 필터만으로 계산이 가능하다. 보통 J_1 필터는 이와 대응하는 J_0필터에 비하여 정확성은 비슷하지만, 계산효율은 좀 더 높다. 또한 J_0에서 J_1영역으로 변환시키면, Schlumberger배치 뿐만 아니라 쌍극자배치 사용시에도 J_1함수만을 포함하는 효율적인 겉보기비저항 계산식을 이용할 수 있다.

* 부산수산대학 응용지질학과