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Stability Analysis of One-Multiplier Lattice Digital Filter Using a Constructive Algorithm

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單一係數 格子型 디지탈 필터의 安定度 解析

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國 文 要 略

마이크로 컴퓨터로 實現되는 디지탈 필터는 有限語張效果에 基因한 量子化(quantization)와 잉여현상(overflow) 때문에, 理想的인 線型필터도 非線型 特性을 나타내어 Limit Cycle과 같은 오차의 원인이 된다. 本 論文에서는 수학行列의 安定성질을 이용한 Norm—Lyapunov 함수를 이용하여, 디지탈 필터의 안정도를 해석하였다. 잉여현상이 없는 경우에는 Jury—Lee 판별법을 적용하여 前者와 비교하였다.

I. INTRODUCTION

Due to recent advances in semiconductor technology, there has been increasing interest in the digital processing of signals. The basic element of almost every digital system is a digital filter. These digital filters are often implemented using a microprocessor with fixedpoint arithmetic. Due to finiteness of the signal wordlength, digital filters become nonlinear[1], and for this reason the output of the digital filter deviates from what is actually desired. This is particularly true in the case of recursive structures, i.e., structures whose signal flow diagram involves directed loops. It follows that in digital filter design there are stability problems which may have to be considered due to these effects.

In papers [2] and [3], Brayton and Tong established some significant results which make it possible to construct computer-generated Lyapunov functions to analyze the stability of nonlinear systems by means of a constructive algorithm. We find the regions in the parameter plane where a given second-order fixed-point one-multiplier lattice digital filter is globally asymptotically stable, using the constructive algorithm of Brayton and Tong. In these regions, the absence of limit cycles due to the quantization and overflow nonlinearities is ensured. This specific digital filter was suggested in[4] for further study.

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II. BACKGROUND MATERIALS

A. Notation

Let U and V be arbitrary sets. If u is an element of U, we write $u \in U$. If U is a subset of V, we write $U \subset V$. Let U x V denote the cartesian product of U and V. The boundary of U is denoted by ∂U. If W is a convex polyhedral region, the elements of the set $E[W]$ denote its extreme vertices and $H[W] = W \cup \partial W$ denotes its convex hull.

Let R denote the real line, and let R^n denote the set of real-valued n-tuples. The symbol $|\cdot|$ denotes a vector norm on R^n. If f is a function of a set X into a set Y, we write $f: X \to Y$. The function $f(\cdot)$ is said to belong to the sector $[k_1, k_2]$, if $f(0)=0$ and $k_1 \leqq f(x)/x \leqq k_2$, $x \neq 0$, for all $x \in R$. And $B(r)=\{x \in R^n: |x| < r\}$.

Matrices are usually assumed to be real and we denote them by upper case letters. If $A=[a_{ij}]$ is an arbitrary n x n matrix, then A^T denotes the transpose of A. Also, $||A||$ is used to denote the matrix norm of A induced by some vector norm. A set of matrices is denoted by an underlined upper case letters, e.g., $\underline{A}$. The set of extreme matrices of a convex set of matrices $\underline{A}$ is denoted by $E[\underline{A}]$.

Let $p \in I = \{t_0 + k\}$, $k=0,1,2,\cdots$, with $t_0 \in R^+$. In a digital filter structure block diagram, z^{-1} represents a unit time delay.

B. Stability of Systems Described by Difference Equations

In the present subsection, we consider systems described by ordinary autonomous difference equations of the form

$$x(p+1)=g[x(p)] \qquad (E)$$

where $x \in R^n$, $g: R^n \to R^n$ and $p \in I$. We denote unique solutions of (E) by $x(p; x_0, p_0)$, where $x_0 = x(p_0; x_0, p_0)$. Since we are dealing with autonomous equations we shall assume without loss of generality that $t_0 = 0$. Any point $x_e \in R^n$ for which it is true that $x_e = g(x_e)$ is called an equilibrium point of (E). We shall henceforth assume that $x=0$ is an isolated equilibrium of (E). Thus, we have in particular $g(0) = 0$.

We will call any nontrivial periodic solution of (E) a limit cycle. It is customary in the study of digital filters to include nonzero equilibrium points as limit cycles. We will follow this practice.

Since (E) is a system of nonlinear equations, it is in general not possible to generate a closed-form solution for (E). For this reason, the qualitative analysis of the equilibrium $x=0$ of (E) is of great importance, especially the stability analysis of $x=0$ in the sense of Lyapunov.

The principal Lyapunov results which yield conditions for stabiliy, asymptotic stability or instability, involve the existence of functions, $v: R^n \to R$. For those definitions and their related terms, refer to [5].

The first forward difference of a function $v: R^n \to R$ along the solution of (E) is given by

$$Dv_{(E)}(x) = v[g(x)] - v(x). \qquad (1)$$

Furthermore, we shall assume that v is continuous and that it satisfies a Lipschitz condition with respect to x. We now state a Lyapunov theorem which will be of interest to us.

Theorem 1. The equilibrium $x=0$ of (E) is globally asymptotically stable if there exists a function $v: R^n \to R$ such that (i) v is radially unbounded and (ii) $Dv_{(E)}(x)$ is negative definite for all $x \in R^n$.

Note that if it is possible to find a v-function for (E) which satisfies the conditions of Theorem 1, then

i) system (E) has only one equilibrium ;

ii) this equilibrium will be $x=0$;

iii) no limit cycles will exist for system (E).

C. Extreme Matrices of a Convex Set of Matrices

In this subsection, we introduce the concepts of a convex set of matrices, an extreme subset and an extreme matrix. We phrase our definitions in terms of a linear vector space of real n x n matrices over R. For general definitions of these concepts, see [6].

Definition 1. Let($R^{n\times n}$, R) denote the linear space of real n x n matrices over R. A set $\underline{A} \in R^{n\times n}$ is convex if X, $Y \in \underline{A}$, $k \in R$, and $0 \leqq k \leqq 1$, imply $kX+(1-k)Y \in \underline{A}$.

Definition 2. Let A_1, $A_2 \in \underline{A}$ and $k \in R$. A nonvoid subset $B \subseteq \underline{A}$ is said to be an extreme subset of $\underline{A}$ if a proper convex combination $kA_1+(1-k)A_2$, $0<k<1$, is in $\underline{B}$ only if A_1, $A_2 \in \underline{B}$. An extreme subset of $\underline{A}$ consisting of just one matrix is called an extreme matrix of $\underline{A}$. The set of extreme matrices of $\underline{A}$ is denoted by $E(\underline{A})$.

In order to apply the above definitions to a second-order digital filter, we consider matrices of the form

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where the elements of A satisfy the inequalities

$$a_1 \leqq a \leqq a_2$$
$$b_1 \leqq b \leqq b_2$$
$$c_1 \leqq c \leqq c_2$$
$$d_1 \leqq d \leqq d_2$$

where a_i, b_i, c_i and d_i, $i=1,2$ are constants. Let $\underline{A}$ be the set of all matrices obtained by varying a, b, c and d over all allowable values. The set of extreme matrices may be obtained as,

$$E(\underline{A})=\left\{\begin{bmatrix} a_i & b_j \\ c_k & d_m \end{bmatrix},\ i,j,k,m=1,2\right\}. \tag{2}$$

D. Constructive Stability Algorithm

In two papers[2] and[3], Brayton and Tong present an algorithm to construct a Lyapunov function to establish the stability and global asymptotic stability of the equilibrium $x=0$ of dynamical systems described by ordinary differential equations and also by difference equations.

To utilize this constructive stability algorithm, we rewrite the given system equation,

$$x(k+1)=g[x(k)] \tag{3}$$

as

$$x(k+1)=M[x(k)]x(k) \tag{4}$$

where $M[x(k)]$ is chosen so that $M[x(k)]x(k)=g[x(k)]$. For every $x(k) \in R^n$, $M[x(k)]$ will be a real n x n matrix. If we let $\underline{M}$ denote the set of all matrices obtained by varying x in M(x) over all allowable values, then we can rewrite (4) equivalently as

$$x(k+1)=M_K\, x(k),\quad M_K \in \underline{M}. \tag{5}$$

In[2]and [3], it is shown that the equilibrium $x=0$ of (3) is stable (globally asymptotically stable) if the set of matrices $\underline{M}$ is stable(asymptotically stable). The precise definitions of these terms are given next. A summary of the results of Brayton and Tong is presented next. Refer to[2], [3] and[7] for further details.

We call a set $\underline{A}$ of n x n real matrices stable if for every neighborhood of the origin $U \subset R^n$, there exists another neighborhood of the origin $V \subset R^n$ such that for every $M \in \underline{A}'$ we have $MV \subseteq U$. Here $\underline{A}'$ denotes the multiplicative semigroup generated by $\underline{A}$ and $MV=\{u \in R^n: v=Mv,\ v \in V\}$.

In[2], it is shown that the following statements which characterize the properties of a class of stable matrices are equivalent.

a) $\underline{A}$ is stable.

b) $\underline{A}'$ is bounded.

c) There exists a bounded neighborhood of the origin $W \subset R^n$ such that $MW \subseteq W$ for every $M \in \underline{A}$. Furthermore, W can be chosen to be convex and balanced.

d) There exists a vector norm $|\cdot|_W$ such that $|Mx|_W \leqq |x|_W$ for all $M \in \underline{A}$ and for all $x \in R^n$.

Now let $rW = \{u \in R^n : u = rw,\ w \in W\}$, where $r \varepsilon R$ and $W \subset R^n$. Since statements c) and d) above are related by

$$|x|_W = \inf\{r : r \geqq 0,\ x \in rW\} \tag{6}$$

it follows that $|x|_W$ defines a Lyapunov function for $\underline{A}$, i.e., it defines a function v with the property

$$v(Mx) \leqq v(x) \quad \text{for all } M \in \underline{A} \text{ and } x \in R^n. \tag{7}$$

Next, we call a set of matrices $\underline{A}$ <u>asymptotically stable</u> if there exists a number $\rho > 1$ such that $\rho\underline{A}$ is stable. In [3], it is shown that the following statements which characterize the properties of a class of asymptotically stable matrices are equivalent.

a) $\underline{A}$ is asymptotically stable.

b) There exists a convex, blanced and polyhedral neighborhood of the origin W and a positive number $r < 1$ such that for each $M \varepsilon \underline{A}$ we have $MW \subseteq rW$.

c) $\underline{A}$ is stable and there exists a positive constant K such that for all $M \in \underline{A}'$, $|\lambda_i(M)| \leqq k < 1$, $i = 1, \cdots\cdots, n$, where $\lambda_i(M)$ denotes the i-th eigenvalue of M.

In[2] and[3], a constructive algorithm is given to determine whether a set of m n x n real matrices $\underline{A} = \{M_0, \cdots\cdots M_{m-1}\}$ is stable by starting with an initial polyhedral neighborhood of the origin W_0 and by defining a sequence of regions W_{k+1} by

$$W_{k+1} = H[\bigcup_{j=0}^{\infty} M_{k_1}^{j} W_k], \text{ where } k_1 = (k-1) \bmod m. \tag{8}$$

Now $\underline{A}$ is stable if and only if

$$W^* = \bigcup_{k=0}^{\infty} W_k \tag{9}$$

is bounded. Since all extreme points z of W_{k+1} are of the form $z = M_1^j u$, where u is an extreme point of W_k, we need only deal with the exteme points of W_k in order to obtain

$$W_{k+1} = H[M_{k_1}^{j} u : u \in E(W_k)] \cdot \tag{10}$$

If $|\lambda(M_{k_1})| < 1$ for all $M_{k_1} \in \underline{A}$, then there exists an integer J such that

$$H[\bigcup_{j=0}^{\infty} M_{k_1}^{j} W_k] = H[\bigcup_{j=0}^{J} M_{k_1}^{j} W_k] \tag{11}$$

since W_k is a bounded neighborhood of the origin. Thus, W_{k+1} will be formed in a finite number of steps, since W_k is expressed as the convex hull of a finite set of points.

Ⅲ. NONLINEARITIES IN DIGITAL FILTERS

In fixed-point arithmetic, each number is represented by a sign bit and a magnitude. Thus, the magnitude of any number is represented by a string of binary digits of fixed length B. When two B-bit numbers are multiplied, the result is a 2B-bit number. A quantization nonlinearity is produced when the 2B-bit number is reduced in wordlength to B bits. Addition also poses a problem when sum of two numbers falls outside the representable range. An overflow nonlinearity results when this number is modified so that it falls back within the representable range. Quantization only affects the least significant bits. In general, the overflow nonlinearty changes the most significant bits as well as the least significant bits of a fixed-point number. These two types of nonlinearities are well described in the literature(e.g., [1] and[8]).

Quantization can be performed by substituting the nearest possible number that can be represented by the limited number of bits. This type of nonlinear operation is called a roundoff quantizer. Another possibility consists of discarding the least significant bits in the number. If the signals are represented by sign and magnitude then we have a magnitude truncation quantization characteristic.

If an overflow occurs, a number of different actions may be taken. If the number that caused the overflow is replaced by a number having the same sign, but with a magnitude corresponding to the overflow level, a saturation overflow is obtained. Zeroing overflow substitutes the number zero in case of an overflow. In two's complement arithmetic, the most significant bits that caused the overflow are discarded. Overflows in intermediate results do not cause errors, as long as the final result does not have overflow. In this case, we use two's-complement overflow. Another way of dealing with overflow is the triangular overflow as proposed by Eckhardt and Winkelnkemper(see [1]).

It is possible to have different wordlengths for the various signals in the filter, resuling in different quantization stepsizes and/or different overflow levels. We will assume throughout this paper that all quantizers in a filter have the same quantization stepsize, q, and are the same type, e.g., roundoff or truncation. Similarly, we will assume that all overflow nonlinearities in a filter have the same overflow level, p, and are the same type.

The above nonlinearities will be viewed as belonging to a sector $[k_m, k_M]$, where

$$k_m \leqq f(w)/w \leqq k_M \text{ for all } w \in R, w \neq 0. \quad (12)$$

Under the above assumptions, we view the quantization nonlinearities as belonging to the sector $[0, k_q]$ where

$$k_q = \begin{cases} 1 & \text{for truncation} \\ 2 & \text{for roundoff.} \end{cases} \quad (13)$$

Henceforth, k_q will represent the upper slope of the sector that contains the quantization nonlinearity. The overflow nonlinearities are represented as belonging to the sector $[k_o, 1]$ where

$$k_o = \begin{cases} 0 & \text{saturation or zeroing} \\ -1/3 & \text{triangular} \\ -1 & \text{two's-complement.} \end{cases} \quad (14)$$

Henceforth, k_o will represent the lower slope of the sector that contains the overflow nonlinearity.

Ⅳ. ONE-MULTIPLIER LATTICE DIGITAL FILTER

A. Constructive Stability Results

We apply the contructive algorithm to the stability analysis of one-multiplier lattice digital filter. Our presnt results are closely related to the interesting work of [4]. They suggested this specific digital filter for further study.

Since their introduction by Itakura and Saito [9], lattice digital filters have been used extensively in the area of speech and signal processing[10]. The particular lattice structure we consider is the one-multiplier lattice filter[11]. There are two types of structures, positive and negative. In this paper the negative type of the second order will be used.

Quantization is assumed to take place after each multiplication and overflow is placed after each addition. This structure is a realistic implementation of the actual filter using a fixed-point microprocessor, and is shown in Figure 1.

Gary and Markel[11] have shown that the linear digital filter (of infinite wordlength) will have all of its poles within the unit circle, and thus will be globally asymptotically stable, if and only if all of the k_i parameters satisfy

$$|k_i|<1, \quad i=1,2. \tag{15}$$

The state equations for the structure shown in Figure 1 are

$$\begin{aligned} x_1(k+1) &= P_1[-Q_1[k_1P_3[x_1(k) \\ &\quad -Q_2(k_2x_2(k))]]-Q_2[k_2x_2(k)]] \\ x_2(k+1 &= P_2[x_1(k)+Q_1[k_1P_3[x_1(k) \\ &\quad -Q_2(k_2x_2(k))]]] \end{aligned} \tag{16}$$

with Q_i, $i=1,2$ representing the quantizers and P_i, $i=1,2,3$ representing the overflow nonlinearities. We next develop the set of extreme matrices for the structure.

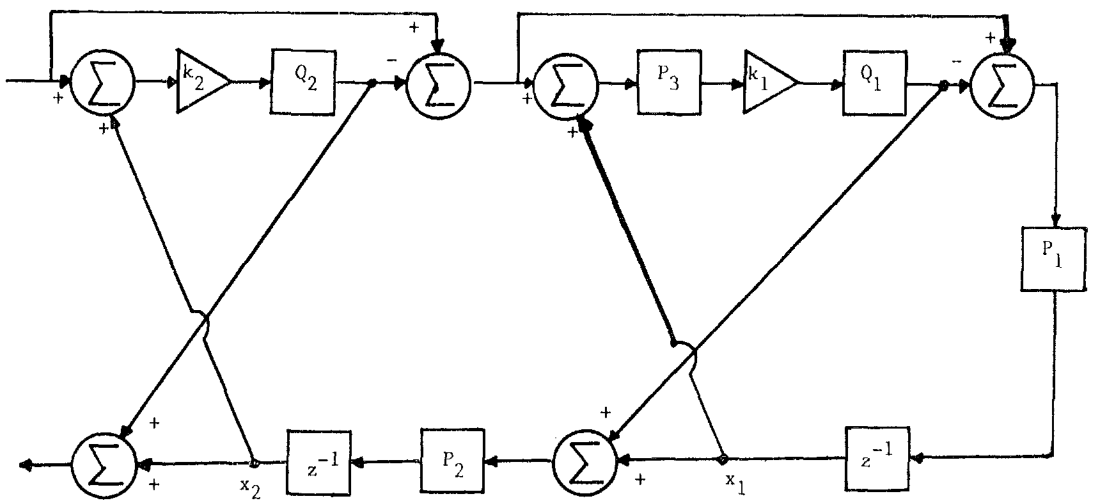

Fig. 1. One-multiplier Lattice Digital Filter : (−) type with two quantizers and three overflow nonlinearities.

Following the technique outlined in Section II—D, the state equations are expressed as

$$x(k+1)=M[x(k)]x(k) \tag{17}$$

where $M[x(k)]$ is given by

$$M(x)=\begin{pmatrix} -k_1\phi_6(x) & k_2\phi_7(x) \\ \phi_8(x) & -k_1k_2\phi_9(x) \end{pmatrix} \tag{18}$$

and

$$\begin{aligned} &\phi_1(x)=P_1(x)/x \qquad \phi_2(x)=Q_1(x)/x \\ &\phi_3(x)=Q_2(x)/x \qquad \phi_4(x)=P_3(x)/x \\ &\phi_5(x)=P_2(x)/x \end{aligned} \tag{19}$$

$$\begin{aligned} &\phi_6(x)=\phi_1(x)\phi_2(x)\phi_4(x) \\ &\phi_7(x)=\phi_1(x)\phi_3(x)(k_1\phi_2(x)\phi_4(x)-1) \\ &\phi_8(x)=\phi_5(x)(1+k_1\phi_2(x)\phi_4(x)) \\ &\phi_9(x)=\phi_2(x)\phi_3(x)\phi_4(x)\phi_5(x). \end{aligned}$$

The functions $\phi_i(x)$, $i=1$ to 5, are bounded as follows :

$$\begin{aligned} &r_1 \leqq \phi_i(x) \leqq r_2 \text{ for } i=1,4,5 \\ &s_1 \leqq \phi_i(x) \leqq s_2 \text{ for } i=2,3 \end{aligned} \tag{20}$$

where $r_1=k_o$, $r_2=1$, $s_1=0$ and $s_2=k_q$.

The functions $\phi_6(x)$, $\phi_1(x)\phi_3(x)$, $\phi_2(x)\phi_4(x)$ and $\phi_9(x)$ are also bounded by constants,

$$\begin{aligned} &a_1 \leqq \phi_6(x) \leqq a_2 \\ &b_1 \leqq \phi_1(x)\phi_3(x) \leqq b_2 \\ &c_1 \leqq \phi_2(x)\phi_4(x) \leqq c_2 \\ &d_1 \leqq \phi_9(x) \leqq d_2 \end{aligned} \tag{21}$$

where,

$$\begin{aligned} &a_1=b_1=c_1=k_ok_q \\ &a_2=b_2=c_2=k_q \\ &d_1=k_ok_q^2 \text{ and } d_2=k_q^2. \end{aligned}$$

The functions $\phi_7(x)$ and $\phi_8(x)$ are also bounded by constants,

$$e_1 \leqq \phi_7(x) \leqq e_2$$
$$f_1 \leqq \phi_8(x) \leqq f_2 \qquad (22)$$

where $e_1 = \min\{b_i(k_1c_j - 1),\ i, j = 1, 2\}$

$e_2 = \max\{b_i(k_1c_j - 1),\ i, j = 1, 2\}$

$f_1 = \min\{r_i(1 + k_1c_j),\ i, j = 1, 2\}$

$f_2 = \max\{r_i(1 + k_1c_j),\ i, j = 1, 2\}$.

Thus the extreme matrices of the set $\underline{M}$ are

$$E(\underline{M}) = \left\{\begin{bmatrix} -k_1a_i & k_2e_j \\ f_m & -k_1k_2d_n \end{bmatrix}, i, j, m, n = 1, 2\right\}. \quad (23)$$

In this case, the constructive algorithm uses sixteen extreme matrices for every point in the k_1-k_2 parameter plane.

If the overflow nonlinearites are absent, then the set of extreme matrices is :

$$E(\underline{M}) = \left\{\begin{bmatrix} -k_1a_i & k_2d_j \\ e_m & -k_1k_2c_n \end{bmatrix}, i, j, m, n = 1, 2\right\}, \quad (24)$$

where $a_1 \leqq \phi_2(x) \leqq a_2$

$b_1 \leqq \phi_3(x) \leqq b_2$

$c_1 \leqq \phi_2(x)\phi_3(x) \leqq c_2$

$d_1 \leqq \phi_3(x)[k_1\phi_2(x) - 1] \leqq d_2$

$e_1 \leqq 1 + k_1\phi_2(x) \leqq e_2 \qquad (25)$

with $a_1 = b_1 = c_1 = 0$

$a_2 = b_2 = k_q$

$c_2 = k_q^2$

$d_1 = \min\{b_i(k_1a_j - 1), i, j = 1, 2\}$

$d_2 = \max\{b_i(k_1a_j - 1), i, j = 1, 2\}$

$e_1 = \min\{1 + k_1a_i, i = 1, 2\}$

$e_2 = \max\{1 + k_1a_i, i = 1, 2\}$.

Using these extreme matrices, we apply the constructive algorithm to get the stability results. Some of them are shown in Figures 2—5. Only half of these regions are shown, since they are symmetric about the k_2-axis. We used the value of $\rho = 1.001$ to show that A is asymptotically stable for a stable matrix A.

The analysis by the constructive algorithm yields sufficient conditions for global asymptotic stability in terms of the parameters of a given filter under zero-input condition. These results constitute also sufficient conditions for the absence of zero-input limit cycles.

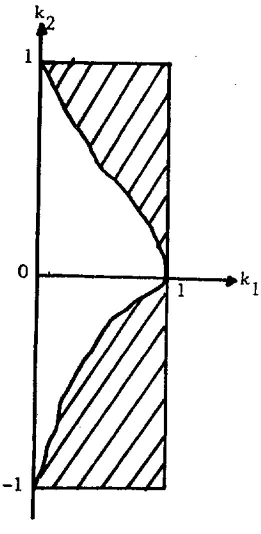

Fig. 2. Magnitude truncation quantizer and triangular overflow

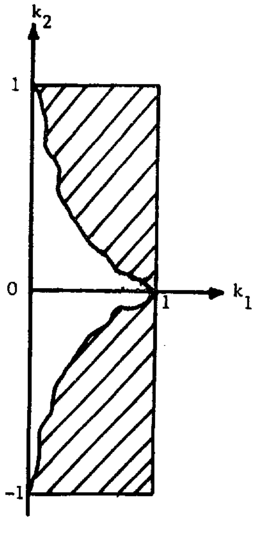

Fig. 3. Magnitude truncation quantizer and two's-complement overflow

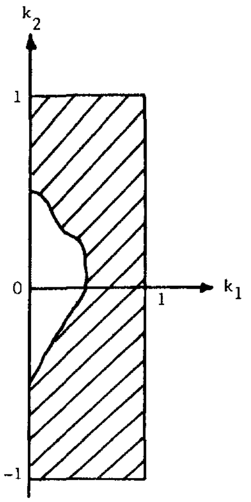

Fig. 4. **Roundoff quantizer and saturation or zeroing overflow**

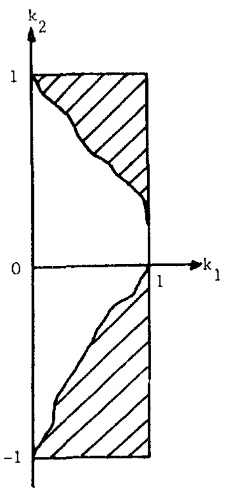

Fig. 5. **Magnitude truncation quantizer and no overflow**

B. Jury and Lee Stability Results

For digital filters with quantizers and without overflow nonlinearities, an absolute stability criterion by Jury and Lee[12] can be used to determine sufficient conditions for the global asymptotic stability of the equilibrium of the system.

A system with several nonlinearities is represented by the system shown in Figure 6. The m nonlinerar elements are represented by the vector-valued function $f(w)$ where $f_i(w_i)$ is the output of the i-th nonlinear element. The input of this element is the i-th component of the vector $w^T=[w_1, \cdots\cdots, w_m]$.

The inputs and outputs of the nonlinear elements are interconnected by linear filters with transfer functions, $g_{ij}(z)$, assumed to be controllable and observable[13], that are the elements of the m x m transfer matrix $G(z)$. The linear filter $g_{ij}(z)$ connects the output of the j-th nonilinear element and the input of the i-th nonlinear element. We assume that each element $g_{ij}(z)$ has all of its poles within the unit circle except possibly one pole at $z=1$. We assume that the nonlinearities $f_i(w_i)$ satisfy the following conditions :

i) $f_i(0)=0,\ i=1,2,\cdots\cdots,m$

ii) $0<f_i(w_i)/w_i<k_{ii}$, for all $w_i\neq 0$

iii) $w(k)\rightarrow 0$ implies $y(k)\rightarrow 0$

iv) $$-\infty<\frac{df_i(w_i)}{dw_i}<\infty \qquad (26)$$

where k_{ii} is the i-th diagonal element of the m x m matrix K.

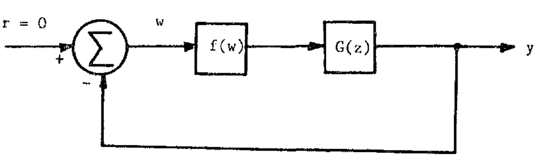

Fig. 6. **A general discrete-time system with many nonlinearities**

Theorem 2. [12] : The system of Figur 6 satisfying the above conditions for $G(z)$ with nonlinearities described by (26) is globally asymptotically stable if

$$H(z)=2K^{-1}+G(z)+G^*(z) \qquad (27)$$

is positive definite for all $z : |z|=1$, where $G^*(z)$ denotes the complex conjugate transpose of $G(z)$.

For the second-order one-multiplier lattice digital filter with two quantization nonlinearities and no overflows in Figure 1, the matrix $G(z)$ may be written as

$$G(z)=\begin{pmatrix} k_1 z^{-1} & k_1(1+z^{-1}) \\ k_2(z^{-2}-z^{-1}) & k_2 z^{-2} \end{pmatrix} \qquad (28)$$

The matrix $H(z)$, given by

$$H(z)=\begin{pmatrix} 2/k_{11}+k_1(z+z^{-1}) & k_1(1+z^{-1})+k_2(z^2-z) \\ k_1(1+z)+k_2(z^{-2}-z^{-1}) & 2/k_{22}+k_2(z^2+z^{-2}) \end{pmatrix} \qquad (29)$$

must be positive definite for all $z : |z|=1$. For magnitude truncation, $k_{11}=k_{22}=1$, and for roundoff, $k_{11}=k_{22}=2$. One of two cases is shown in Figure 7 for magnitude truncation. Only half of the region is shown, since it is symmetric about the k_2-axis.

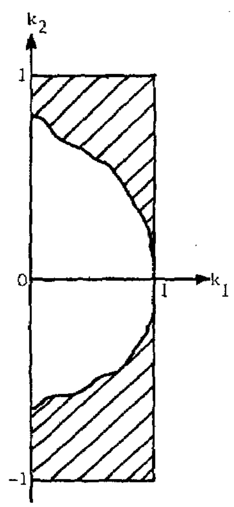

Fig. 7. **Magnitude truncation quantizer and no overflow by Jury-Lee stability criterion**

V. CONCLUSION

Using the constructive stability algorithm due to Brayton and Tong, we analyzed the stability of the equilibrium $x=0$ of the one-multiplier lattice digital filter of the second order. All the results are new. We used the Jury and Lee absolute stability criterion for comparison with the constructive results.

While existing methods of stability analysis [1] are generally different for each particular structure, the constructive algorithm allows us to use one method to study the stability of nonlinear digital structures, and moreover it may be applied to higher-order filters with decomposition and aggregation method.

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References

1. T. A. C. Claasen, W. F. G. Mecklenbrauker and J. B. H. Peek, "Effects of Quantization and Overflow in Recursive Digital Filters," IEEE Trans. on Acoustics, Speech, and Signal Processing, vol. ASSP—24, pp. 517—529, 1976.
2. R. K. Brayton and C. H. Tong, "Stability of dynamical systems : A constructive approach," IEEE Trans. Circuits Syst., vol. CAS—26, pp. 224—234, 1979.
3. ______________, "Constructive stability and asymptotic stability of dynamical systems," IEEE Trans. Circuits Syst, vol. CAS—27, pp. 1121—1130, 1980.
4. A. N. Michel and K. T. Erickson, "Stability analysis of fixed-point digital filters using computer generated Lyapunov functions-Part II : Wave digital filters and Lattice digital filters," IEEE Trans. Circuits Syst. vol. CAS—32, pp. 132—142, 1985.
5. A. N. Michel and R. K. Miller, Qualitative Analysis of Large Scale Dynamical Systems. New York : Academic, 1977.
6. N. Dunford and J. T. Schwartz, Linear Operators-Part I : General Theory. New York : Interscience, 1958.
7. J. H. Tsitsiklis, "Conditions for finiteness of a constructive algorithm for determining stability," IEEE Trans. on Circuits Syst. vol. CAS—29,

pp. 400—402, 1982.
8. B. Liu, "Effects of finite wordlength on the accuracy of digital filter-A review," IEEE Trans. on Circuit Theory, vol. CT—18, pp. 670—677, 1971.
9. F. Itakura and S. Saito, "Digital filtering techniques for speech analysis and synthesis," Proceedings of the Seventh International Congress on Acoustics 3, pp. 261—264, 1971.
10. B. Friedlander, "Lattice filters for adaptive processing," Proceedings of the IEEE 70, pp. 829—967, 1982.
11. A.H. Gray and J.D. Markel, "Digital lattice and ladder filter synthesis," IEEE Trans. Audio Electroacoust., vol. AU—21, pp. 491—500, 1973.
12. E.I. Jury and B.W. Lee, "The absolute stability of systems with many nonlinearities," Automat. Remote Contr., vol. 26, pp. 943—961, 1965.
13. G.F. Franklin and J.D. Powell, Digital Control of Dynamical Systems. Reading, MA : Addison-Wesley, 1980.