

## A Study on the Variable Structure Adaptive Control Systems for a Nuclear Reactor

Sung Ha Kwon, Hee Young Chun\* and Hyun Kook Shin\*\*  
Changwon University Korea University\* Korea Advanced Energy Research Institute\*\*  
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### 가변구조 적응제어이론에 의한 원자로부하추종 출력제어에 관한 연구

권 성 하 · 천 희 영\* · 신 현 국\*\*  
창원대학교 고려대학교\* 한국에너지연구소\*\*  
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#### Abstract

This paper describes a new method for the design of variable structure model-following control systems(VSMFC). This design concept is developed using the theory of variable structure systems (VSS) and slide mode. The new results are presented on the sliding control methodology to achieve accurate tracking for a class of nonlinear, multi-input multi-output(MIMO), time varying systems in the presence of parameter variations. The design requires little computational effort. The dynamic response is insensitive to parameter variations.

The feasibility and the advantages of the method are illustrated by applying it to a 1000 MWe boiling water reactor(BWR). The control is studied in the range of 85%~90% of rated power for load-following control. A set of 12 nonlinear differential equations is used to simulate the total plant. A 6-th order linear model has been developed from these equations at 85% of rated power. The obtained controller is shown by simulations to be able to compensate for a plant parameter variation over a wide power range.

#### 요 약

本論文은 可變構造모델追從制御(VSMFC)系 設計의 새로운 方法을 고찰한 것이다. 設計 개념은 可變構造系(VSS)와 슬라이드모드 理論을 사용하여 非線型 時變多變數系가 파라미터 變動이 있을지라도 모델追從을 精確히 하게끔 制御則이 可變構造를 갖게 하는 것이다. 本論文의 方法을 실제 物理系에 적용할 때 컴퓨터 계산시간의 감소와 파라미터變動에 무관한 動的應答을 기대할 수 있다. 理論의 有效性을 밝히기 위해 VSMFC를 1000MWe의 沸騰輕水型 原子爐(BWR)에 적용하였다. 즉 原子爐의 出力要求가 定格出力의 85~90% 범위에서 變할 때 負荷追從出力制御가 원활히 이루어지는가를 컴퓨터 시뮬레이션하였다. 12개의 非線型微分方程式으로 動特性이 주어지는 原子爐에서 6次系 線型모델을 85% 定格値에서 구하고 여러범위에 걸쳐서 負荷變動이 있을때 파라미터變動을 극복하면서도 出力制御를 원활히 하는가를 研究하였다.

## 1. Introduction

Linear model-following control(LMFC) is an efficient control method that avoids difficulty of specifying a performance index, and it is usually encountered in the application of linear optimal control to multivariable control systems. However, LMFC systems are inadequate when there are large parameter variations or disturbances. This has led to the development of so-called adaptive model-following control systems(AMFC). AMFC is based on Lyapunov functions and the hyper-stability concept proposed by Popov.<sup>1),2)</sup>

Presently, various basic concepts of AMFC are applied to control system and undergoing application research activities with AMFC. In this paper, in order to develop stability and convergence, a new design method, called variable structure adaptive model following control (VSMFC), is proposed and applied to the BWR control systems.

The theory of variable structure systems(VSS) has been developed in the U.S.S.R in the last fifteen years,<sup>3),4)</sup> and nowadays the the study of the single-input single-output(SISO) system is enlivened<sup>5),6),7)</sup> but the study of MIMO system is unexhausted. VSMFC for MIMO systems by K.K.D. Young in 1978 is difficult not only to choose the gain of the variable structure control but also to assure the stability and convergence of it.<sup>8),9)</sup> In this paper, a new method to VSMFC systems has been studied and applied to BWR nuclear reactor control systems described by 12 nonlinear differential equations. The control systems consist of time varying unstable plant and reference model. The range of 85%~90% of rated power for steady state and load-following control have been studied by computer simulation.

## 2. Variable Structure Adaptive Model-Following Control Systems

The state equations of linear timevarying multivariable systems are represented by the following equations

$$\mathbf{X}_p(t) = A_p(t)\mathbf{X}_p(t) + B_p(t)\mathbf{U}_p(t) \quad (1)$$

$$\mathbf{X}_m(t) = A_m\mathbf{X}_m(t) + B_m\mathbf{U}_m(t) \quad (2)$$

where  $\mathbf{X}_p \in R^n$ ,  $\mathbf{X}_m \in R^n$ ,  $\mathbf{U}_p \in R^m$  and  $\mathbf{U}_m \in R^1$

$\mathbf{U}_m$ =input vector of reference model

$\mathbf{U}_p$ =input vector of controlled plant

It will be assumed that pairs( $A_p, B_p$ ) and ( $A_m, B_m$ ) are stabilizable and the  $A_m$  matrix is stable. The plant matrices  $A_p$  and  $B_p$  may be uncertain and timevarying. The upper and lower bounds of the elements of these matrices are assumed to be known to the designer. The state error vectors are represented by the following equations.

$$\mathbf{e}(t) = \mathbf{X}_m(t) - \mathbf{X}_p(t) \quad (4)$$

$$\begin{aligned} \dot{\mathbf{e}}(t) = & A_m\mathbf{e}(t) + [A_m - A_p(t)]\mathbf{X}_p(t) \\ & + B_m\mathbf{U}_m(t) - B_p(t)\mathbf{U}_p(t) \end{aligned} \quad (5)$$

Variable structure systems are characterised by discontinuous control which changes structure on reaching a set of switching surfaces.<sup>9)</sup>

$$\mathbf{U}_i = \begin{cases} \mathbf{U}_i^+(\mathbf{X}_p, \mathbf{e}, \mathbf{U}_m) & S_i(\mathbf{e}) > 0 \\ \mathbf{U}_i^-(\mathbf{X}_p, \mathbf{e}, \mathbf{U}_m) & S_i(\mathbf{e}) < 0 \end{cases} \quad (5)$$

where  $\mathbf{U}_i$  is the  $i$ -th component of multivariable control input  $\mathbf{U}$ . The switching hypersurfaces are chosen to be hyperplane, that is,

$$\mathbf{S} = \mathbf{G} \mathbf{e} = 0 \quad (6)$$

$$\begin{aligned} \mathbf{S} = \mathbf{G} \mathbf{e} = & \mathbf{G}[A_m \mathbf{e} + (A_m - A_p)\mathbf{X}_p \\ & + B_m\mathbf{U}_m - B_p\mathbf{U}_p] = 0 \end{aligned} \quad (7)$$

where  $\mathbf{G}$  is the switching surface matrix. If  $\mathbf{G}$  is chosen so that the roots exist on the left half plane, the state vector of the plant is following to the state vector of the model. The control system becomes less sensitive to the system parameter variations and disturbance inputs. Because the effect of the switching surface transforms the feedback structure of system, the

total system is the variable structure system. In order to satisfy the Eq. (6),  $S_i$  will enter the region of  $S_i < 0$  if  $S_i$  is in the region of  $S_i > 0$  and  $S_i$  will enter the region of  $S_i > 0$  if  $S_i$  is in the region of  $S_i < 0$ . Then control input  $U_p$  is the variable structure input. If the switching logic works infinitely fast, the control input  $U_p$  is chattering. And  $S$  is constrained to remain on the switching line  $S=0$ . Then the error between model and plant goes to zero and the model-following is obtained. So, the condition for sliding motion to occur on the  $i$ -th hyper-plane may be stated in Eq. (8). This motion occurs in the direction of the state locus, the state slides and remains for some finite time.

$$S_i \dot{S}_i < 0 \tag{8}$$

Because  $S$  is scalar in SISO systems, the dimension of  $G$  matrix is  $(1 \times m)$ . Therefore the matrix element can be designed so that the transient state error response is desirable. But  $S$  and  $U_p$  are vectors in MIMO systems, the dimension of  $S$  and  $U_p$  are  $(m \times 1)$ . In order to correspond one by one between  $S$  and  $U_p$ , the matrix  $GB_p$  in Eq. (7) should be unit matrix. So, Eq. (9) is obtained. Substituting Eq. (9) into Eq. (7), Eq. (10) is obtained. Rearranging about  $U_p$ , Eq. (11) is obtained.

$$G = (QB_p)^{-1}Q \tag{9}$$

$$\dot{S} = GA_m e + G(A_m - A_p)X_p + GB_m U_m - U_p \tag{10}$$

$$U_p = GA_m e + G(A_m - A_p)X_p + GB_m U_m - \dot{S} \tag{11}$$

Because the sign of  $S$  and  $\dot{S}$  are alternative from Eq. (8), which is the condition to occur sliding motion, Eq. (12) is obtained by taking  $\alpha S$  instead of  $-\dot{S}$ ,

$$U_p G_e e + G_p X_p + G_m U_m + \alpha S \tag{12}$$

where  $\alpha$  is a weighting value. In order that the error between model and plant goes to zero in every one sampling time,  $\alpha$  is taken as a reciprocal of computer sampling time.

The selection on matrix  $Q$  in Eq. (9) is very

important. The behaviour of the error dynamics during sliding motion needs to be considered. Substituting Eq. (12) into Eq. (5), Eq. (13) is obtained.

$$\begin{aligned} \dot{e} = & A_m e + (A_m - A_p)X_p + B_m U_m - B_p \\ & [G_e e + G_p X_p + G_m U_m + \alpha S] \end{aligned} \tag{13}$$

During sliding,  $S$  is zero, thus

$$\begin{aligned} \dot{e} = & (A_m - B_p G_e) e + (A_m - A_p - B_p G_p) X_p \\ & + (B_m - B_p G_m) U_m \end{aligned} \tag{14}$$

Perfect model-following can be achieved if

$$(A_m - A_p - B_p G_p) X_p + (B_m - B_p G_m) U_m = 0 \tag{15}$$

So that Eq. (15) holds for any  $X_p$  and  $U_m$ , Eq. (16) is obtained.

$$B_p G_p = A_m - A_p, \quad B_p G_m = B_m \tag{16}$$

So that the linear systems, Eq. (16), have a solution about  $G_p$  and  $G_m$ , the following condition is obtained.

$$\begin{aligned} \text{rank}(B_p) = & \text{rank}(B_p; A_m - A_p) \\ \text{rank}(B_p) = & \text{rank}(B_p; B_m) \end{aligned} \tag{17}$$

Eq. (17) is perfect model-following condition which is general necessary conditions to the AMFC systems.

For the perfect model-following case, Eq. (14) is reduced to Eq. (18)

$$\begin{aligned} \dot{e} = & (A_m - B_p G_e) e = (A_m - B_p G A_m) e \\ = & (I - B_p G) A_m e \\ = & [I - B_p (QB_p)^{-1} Q] A_m e \end{aligned} \tag{18}$$

The remaining unforced system Eq. (18) must be asymptotically stable, which implies that the matrix  $[I - B_p (QB_p)^{-1} Q] A_m$  must be a Hurwitz matrix, that is, all its eigenvalues have negative real parts. The eigenvalues of Eq. (18) can be placed arbitrarily in the complex plane by suitable choice of matrix  $Q$ . The matrix  $Q$  is chosen in order that the eigenvalues of Eq. (18) have negative real parts.

The matrices  $G_e$ ,  $G_p$  and  $G_m$  are represented Eq. (19) in case that the matrices are not influenced by parameter variations of  $A_p$  and  $B_p$ . However, in case that the matrices is influenced by them those matrices are represented

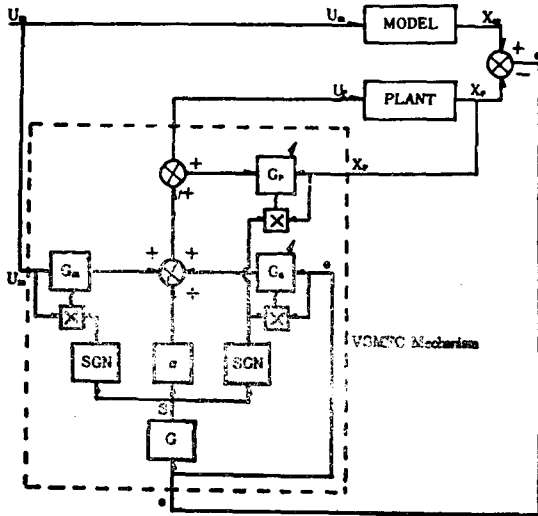


Fig. 1. Variable Structure Adaptive Model-Following Control System

Eq. (20), that is, the absolute maximum value of parameter variation range is changed by the sign of  $S$ .

$$\begin{aligned} G_e &= GA_m, \quad G_p = G(A_m - A_p), \quad G_m = GB_m \quad (19) \\ G_e &= \text{sgn}S \cdot |G_e|, \quad G_p = \text{sgn}S \cdot |G_p| \\ G_m &= \text{sgn}S \cdot |G_m| \quad (20) \end{aligned}$$

Fig.1 shows the total block diagram of variable structure model-following control systems of Eq. (12), (19) and (20).

### 3. Nuclear Reactor Model

The complicated plant like nuclear reactor should be modelled so that the important physical phenomena of the model described by some major parameters and that of the real plant coincide well and the degree of model is as low as possible. Many studies about the dynamics of BWR have been done by Linford, Carmichael, Frogner and Solberg. The Solberg's model<sup>(10), (11)</sup> is taken in this paper. Using the 12 set of nonlinear equations, it is obtained the simple form of the state equations.

$$\dot{X}(t) = f(X(t), U(t)) \quad (21)$$

The reference input vector is the control vector

which serves the steady state purpose. Because the steady state conditions are varying to the power demand of plant, by solving Eq. (22) then Eq. (23) is obtained.

$$f(X^*, U^*) = 0 \quad (22)$$

$$U^* = F_U(L), \quad X^* = F_X(L) \quad (23)$$

where  $L$  is the power demand level of plant,  $U^*$  is the reference value of control input and  $X^*$  is the reference value of state vector. Because  $U^*$  is determined by the demand  $L$  only, not the plant state, the control systems show a feed-forward quality.

The goal of the control system design is that the state vector  $X$  and the reference vector  $X^*$  are coincided when the power demand  $L$  changes. Therefore, changing the power demand, there is some difference between the control input  $U$  and reference input  $U^*$ . For the sake of compensation of this difference, a feed-back input is considered. In this paper, the input is obtained by variable structure adaptive model-following controller instead of optimal controller.

Infusing the reference input, Eq. (23), into the plant, the plant state approach the reference state. Therefore, model, Eq. (21), is linearized to Eq. (24) nearby  $X^*$  and  $U^*$ . In order to decrease the the sensitivity of state vector and control vector about the power demand, those vectors are made dimensionless type like Eq. (26) and the state perturbation equation is obtained as Eq. (25)

$$\begin{aligned} \frac{d}{dt}(X - X^*) &= \left[ \frac{\partial f}{\partial X} \right]_{X^*, U^*} (X - X^*) \\ &+ \left[ \frac{\partial f}{\partial U} \right]_{X^*, U^*} (U - U^*) \quad (24) \end{aligned}$$

$$\frac{d}{dt} \delta X = A(t) \delta X(t) + B(t) \delta U(t) \quad (25)$$

$$\left. \begin{aligned} \delta X_j &= \frac{X_j - X_j^*}{X_j^*} \quad \delta U_j = \frac{U_j - U_j^*}{U_j^*} \\ A_{ij} &= \frac{X_j^*}{X_i^*} \left[ \frac{\partial f_i}{\partial X_j} \right]_{X^*, U^*} \\ B_{ij} &= \frac{U_j^*}{X_i^*} \left[ \frac{\partial f_i}{\partial U_j} \right]_{X^*, U^*} \end{aligned} \right\} \quad (26)$$

In this paper, the state and control vector of plant are specified as follows.

- $X_1$ =Neutron power
- $X_2$ =equivalent delay neutron precursor density
- $X_3$ =fuel temperature
- $X_4$ =effect in the steam going to turbine
- $X_5$ =fraction of void
- $X_6$ =saturation temperature
- $U_1$ =jet flow of recirculation pump

$U_2$ =opening of furbine valve

The physical constraints are as follows. The maximum value of jet flow  $U_1$  permit 1.2 times rational value, 7.792[m<sup>3</sup>/sec]. The maximum value of derivative of jet flow  $\dot{U}_1$  permit two times rational value. And the maximum value of  $U_2$  permit 35,  $\dot{U}_2$  permit 3. Eq. (25) which is linearized from Solberg's model, Eq. (21) results in the following equations.

$$\dot{\delta}_{X_1} = -\frac{\beta}{l} \delta_{X_1} + \lambda \frac{X_2^*}{X_1^*} \delta_{X_2} + \frac{K_F X_3^*}{l} \delta_{X_3} + \frac{K_a X_5^*}{l} \delta_{X_5} \quad (27)$$

$$\dot{\delta}_{X_2} = \frac{\beta}{l} \frac{X_1^*}{X_2^*} \delta_{X_1} - \lambda \delta_{X_2} \quad (28)$$

$$\dot{\delta}_{X_3} = \frac{1}{H_c} \frac{X_1^*}{X_3^*} \delta_{X_1} - \frac{H_f}{H_c} \delta_{X_3} + \frac{H_f}{H_c} \frac{X_6^*}{X_3^*} \delta_{X_6} \quad (29)$$

$$\dot{\delta}_{X_4} = \frac{1}{\tau_s} \delta_{X_4} + \frac{1}{2\tau_s} \frac{1}{\Delta P} \left( \frac{\partial P_v}{\partial T_c} \right)_{\text{sat}} X_6^* \delta_{X_6} + \frac{H_{fg} \rho_g}{\tau_s} \frac{A_t}{X_4^*} \delta_{U_1} \quad (30)$$

$$\begin{aligned} \dot{\delta}_{X_5} = & \frac{H_f(1-\mu_{sc}) \left(1 - \frac{M_c}{M_{\text{tot}}}\right)}{V_c \rho_g H_{fg}} \frac{X_3^*}{X_5^*} \delta_{X_3} + \frac{X_4^*}{V_c \rho_g H_{fg} M_{\text{tot}}} \delta_{X_4} \\ & - \frac{S' U_1^*}{(1-X_5^*)^2 V_c} \delta_{X_5} - \frac{H_t(1-\mu_{sc}) \left(1 - \frac{M_c}{M_{\text{tot}}}\right)}{V_0 \rho_g H_{fg}} \frac{X_6^*}{X_5^*} \delta_{X_6} - \frac{S' U_1^*}{(1-X_5^*) V_c} \delta_{U_1} \end{aligned} \quad (31)$$

$$\dot{\delta}_{X_6} = \frac{H_f(1-\mu_{sc})}{M_{\text{tot}} C_f} \frac{X_3^*}{X_6^*} \delta_{X_3} - \frac{1}{M_{\text{tot}} C_f} \frac{X_4^*}{X_6^*} \delta_{X_4} - \frac{H_f(1-\mu_{sc})}{M_{\text{tot}} C_f} \delta_{X_6} \quad (32)$$

where

- $\beta$ =fraction of delayed neutron, 0.0065
- $l$ =average lifetime, 0.005[sec]
- $\lambda$ =average decay constant, 0.1[1/sec]
- $K_F$ =reactivity feedback coefficient from fuel, -2.0E-05
- $K_a$ =reactivity feedback coefficient from void, -0.05
- $H_c$ =heat capacity of fuel, 15.625[MWs/deg]
- $H_f$ =heat transfer coefficient of fuel to moderator, 3.125[MW/deg]
- $\tau_s$ =time constant of steam output going to turbine, 0.1[sec]
- $\Delta P$ =calibration factor for valve, 64.46[ATM]
- $T_v$ =reactor pressure
- $T_c$ =saturation temperature
- $H_{fg}$ =heat of evaporation, 1.54254[MVs/kg]
- $\rho_g$ =density of steam, 33.28[kg/m<sup>3</sup>]
- $A_t$ =max. opening of turbine valve, 3.0

- $\mu_{sc}$ =leakage ratio of thermal output, 0.1
- $V_c$ =volume of cooling channels, 13.85[m<sup>3</sup>]
- $M_c$ =mass of moderator, 4,435[kg]
- $M_{\text{tot}}$ =total mass of water in the reactor, 22175[kg]
- $S'$ =slip ratio, 1.5
- $C_f$ =specific heat of water, 0.0053[MWs/deg.kg]

#### 4. Application of VSMFC theory to BWR system

In this paper, the goal of the nuclear reactor control system is that the state vector  $X$  and the reference vector  $X^*$  coincided when the power demand  $L$  changes. Power plants usually operates at the rated power or less for load-following operation. Therefore, it is considered that the load-following control systems in power demand

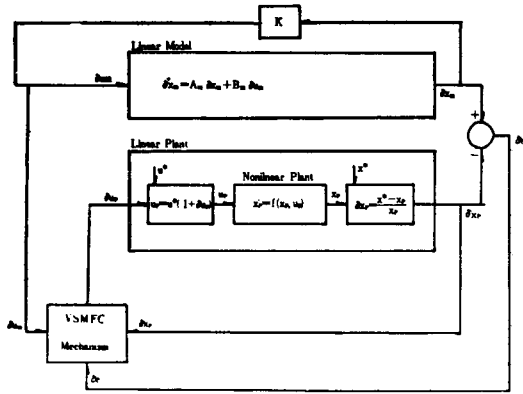


Fig. 2. Nuclear Reactor Load-Following Control System by VSMFC

are set from 85% to 90%. The 4-th term in Eq. (27) includes the void coefficient which conforms to the power demand. And the final characteristics of nuclear reactor dynamic are different from the initial state. Therefore, although the nuclear reactor has the time-varying characteristics, the reactor is followed the time-invariant model. So, a good adaptive control law and control effect are obtained.

Fig.2 shows the total systems of nuclear reactor control systems. In order to determine the control input  $U_p$  of the nonlinear physical plant, the linear plant is composed. This linear plant is

$$A_m = \begin{bmatrix} -13.0 & 13.0 & -22.08 & 0.0 & -61.92 & 0.0 \\ 0.1 & -0.1 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.099 & 0.0 & -0.2 & 0.0 & 0.0 & 0.102 \\ 0.0 & 0.0 & 0.0 & -10.0 & 0.0 & 21.415 \\ 0.0 & 0.0 & 2.821 & 0.536 & -4.56 & -1.431 \\ 0.0 & 0.0 & 0.048 & -0.002 & 0.0 & -0.024 \end{bmatrix} \quad B_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 23.489 \\ -1.738 & 0 \\ 0 & 0 \end{bmatrix} \quad (35)$$

The feed-back control input of Eq. (34) is obtained by Gopinath's Pole Assignment Method.<sup>12)</sup> That is, the eigenvalues are  $-4.05, -10, -13.1, -0.1, -0.66$  then the feed-back gain  $K$  are same as Eq. (36).

$$K = \begin{bmatrix} -0.66E-3 & -0.66E-1 & -0.26E-2 & 0.28E-2 & 0.48E-2 & -0.50E0 \\ -0.66E-3 & -0.66E-1 & -0.26E-2 & 0.28E-2 & 0.48E-2 & -0.50E0 \end{bmatrix} \quad (36)$$

The VSMFC mechanism is already described in section 2. Some matrices which had been used for computer simulation are same as Eq. (37)-(41).

$$Q = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (37)$$

controlled so as to follow the time-invariant linear model.

The control input of nonlinear plant is  $U_p$ ;  $U_{p1}$  is the jet flow of recirculation pump and  $U_{p2}$  is opening of turbine valve. The control input vector  $U_p$  depends on the reference input vector  $U^*$  and the input perturbation vector  $\delta U_p$ . The reference input vector  $U^*$  is obtained by substituting power demand  $L (=X_1^*=900MWe)$  into Eq. (23). And the input perturbation vector  $\delta U_p$  is obtained by VSMFC Mechanism.

The output of nonlinear plant is the neutron power  $X_{p1}$ . The reference state vector  $X^*$  is obtained by substituting power demand  $L$  into Eq. (23), too. The state perturbation vector  $\delta X_p$  depends on the reference state vector  $X^*$  and the nonlinear plant state vector  $X_p$ . The error perturbation vector  $\delta e$  is the difference of the state perturbation vector between the linear model and the linear plant. The linear model, Eq. (2), is represented by the state perturbation vector as Eq. (33).

$$\delta \dot{X}_m = A_m \delta X_m + B_m \delta U_m \quad (33)$$

$$\delta U_m = -K \delta X_m \quad (34)$$

Calculating at 85% power level and using the system constants given in section 3,  $A_m$  and  $B_m$  matrices are resulted in Eq. (35).

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & -0.58E-1 & 0 \\ 0.43E-1 & 0 & 0 & 0.43E-1 & 0 & 0 \end{bmatrix} \quad (38)$$

$$G_e = \begin{bmatrix} 0 & 0 & -0.16E1 & -0.31E0 & 0.26E1 & 0.82E0 \\ -0.55E0 & 0.55E0 & -0.94E0 & -0.43E0 & -0.26E1 & 0.91E0 \end{bmatrix} \quad (39)$$

$$G_p = \begin{bmatrix} -0.61E-7 & -0.46E-7 & 0.18E0 & -0.10E0 & 0.12E0 & -0.95E-1 \\ -0.81E-7 & -0.67E-7 & -0.21E-4 & -0.12E-6 & 0.18E0 & 0.17E-4 \end{bmatrix} \quad (40)$$

$$G_m = \begin{bmatrix} 0.10E1 & -0.31E-6 \\ 0.97E-8 & 0.10E1 \end{bmatrix} \quad (41)$$

### 5. Results

It is difficult to perform the actual experiment for the nuclear reactor. In this paper, a digital simulation BWR of 1000MWe has been performed with Data General Nova Computer. The purpose of control is that the maximum overshoot of BWR restrain under 105% power level and the power of BWR transforms to the desire level smoothly. The reactor power was varied from 850 MWe level to 900 MWe, step increase or ramp increase. Then the plant parameters are varied. The solution of dynamic equations are resolved by Runge-Kutta-Merson Algorithm. The sampling time is 0.025[sec]. The results of simulation are plotted during 10 seconds.

Fig. 3 shows nuclear power transients following a 5% step increase in power demand from 85% to 90%. The overshoot is 5 percent at 0.7 second and the response converge to the steady state well. The optimal control theory was applied to the same BWR systems. This results agreed approximately with that overshoot. But

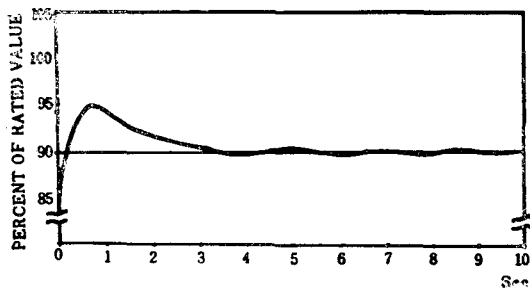


Fig. 3. Nuclear Power Transients following a 5% Step Increase in Power Demand from 85% to 90%

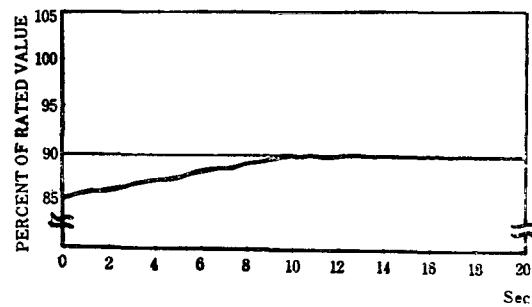


Fig. 4. Nuclear Power Transients following a 0.5%/sec Ramp Increase in Power Demand and from 85% to 90%

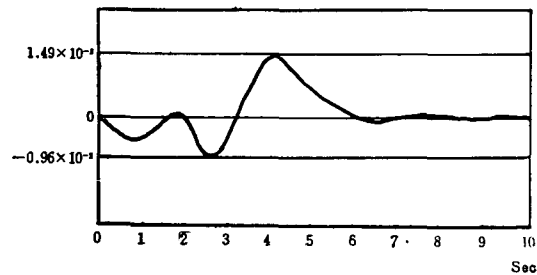


Fig. 5. Model and Plant State Perturbation Error of Reactor Power

for convergence, the VSMFC theory superior to the optimal control theory.

Fig. 4 shows nuclear power transients following a 0.5%/sec ramp increase in power demand from 85% to 90%. The changes of the reactor power demand are followed.

$$L = \begin{cases} 0.85 + 0.05t & 0 \leq t \leq 10 \text{ sec} \\ 0.60 & t > 10 \text{ sec} \end{cases}$$

In this case, there is no the overshoot such as Fig. 3. The reason may be ramp increase in power demand.

Fig. 5 shows the model and plant state perturbation error in case of step increase. The

maximum value of the error is  $1.49 \times 10^{-2}$ . It seems to be a good results. In case of ramp increase, the error grows less and less.

## 6. Conclusion

This paper describes a new method to the design of variable structure model-following control systems in order to develop the stability and convergence for nonlinear time-varying multivariable systems. The VSMFC theory was applied to BWR and simulated. A summary of the results is shown below.

(a) In the presence of parameter variation or time varying systems, only the range of parameter variation is known, the VSMFC theory is applied to the design of adaptive controllers. The system becomes less sensitive to system parameter variations and noise disturbances. If the reactor power level increase from 85% to 90%, void coefficient, the element of  $A_p(1,5)$ , varies from  $-63.77$  to  $-51.39$ . However, the state perturbation error goes to zero rapidly.

(b) Although the reactor power demand turn into another level, the same results are expected. If the reactor power demand increase from 95% to 100%, nonlinear plant is linearized nearby the demand  $L(=950\text{MWe})$  and time-invariant linear model  $(A_m, B_m)$  is specified. The reference values  $X^*$  and  $U^*$  are determined by the demand  $L(=1000\text{MWe})$ . The same process is performed and the same results are expected.

(c) The gain matrices are obtained simply using the signum function, therefore this method requires little computational effort in comparison to conventional method. Most of adaptive control requires many computational effort. In the approach called Indirect Adaptive Control, the plant parameter are estimated and the control parameters are adjusted based on these estimates so that the overall plant transfer function matches that of the reference model. In the approach

called Direct Adaptive Control, no effort is made to identify the plant parameter but the control parameters are directly adjusted to minimize the error between plant and model outputs. In this paper, the design technique is easy and the control structure is simple, therefore, the design can be carried out with a little computational effort. So, the real time control is possible. The implementation of the controller is easily carried out with the aid of microprocessors.

(d) Taking the switching surface matrix  $G$  such as Eq. (9), the algorithm can be easily applied to the multivariable control systems.

(e) Such as Eq. (12), taking  $\alpha S$  instead of  $-S$ , the algorithm is simple. A satisfactory results are obtained in comparison to Young's Algorithm.

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