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Optimal Screening Procedures for Improving Outgoing Quality Based on Correlated Normal Variables

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ABSTRACT

Optimal screening procedures for improving outgoing product quality based on correlated normal variables are presented. The performance variable and the screening variables are assumed to be jointly normally distributed. These procedures do not require specialized tables, and closed-form solutions are obtained for the case of one-sided specification. Methods for finding optimal solutions for the case of two-sided specifications are also considered.

1. Introduction

Consider two correlated random variables X and Y coming from a bivariate normal distribution. Y is the performance variable representing the quality characteristic of a product and X is a screening variable. Suppose that the probability that a measurement on Y meets a certain specification is γ.

We wish to improve the outgoing product quality through a screening process using X. Such a screening procedure would be appropriate if Y is based on a measurement that is more difficult or expensive to make than X. For example, the measurement of Y may require destructive or costly testing.

Attempts to improve the outgoing quality using screening variables have been made by several researchers; Owen, McIntire and Seymour (1975) studied the screening method for increasing the proportion of units within specifications from γ to a specified higher proportion δ after screening; Owen and Boddie (1976) and Owen and Su (1977)

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considered screening methods with some parameters unknown; Li and Owen (1979) extended these results to the case of two-sided specifications; Owen, Li and Chou (1981) and Madsen (1982) considered selection procedures which give a specified degree of confidence that a guaranteed proportion of screened items is conforming. Much of the work done in this area requires specialized tables to implement the screening procedures. Odeh and Owen (1980) give a collection of tables for screening based on the bivariate normal distribution. Some case examples are given in Neal (1981) for applying bivariate normal distributions including the screening procedure.

Two types of misclassification errors are involved in these screening procedures; a conforming item may be classified as nonconforming (type I error), or a nonconforming item may be classified as conforming (type II error). In fact, screening with screening variables may be performed with less inspection cost but with less accuracy compared with the screening with performance variables.

In this paper we present screening methods which minimize the expected cost; the cost of screening inspection plus the costs due to misclassification errors. The screening procedure using a single screening variable under one-sided specification limit is presented in Section 2. The case of two-sided specification limits is dealt with in Section 3. Approximate solutions having closed forms are presented and a numerical example is given for illustrative purpose and for comparison between approximate and exact solutions. The case of two or more screening variables is considered in Section 4.

2. One-sided Specification Limit

Suppose there exists a lower specification limit L; that is, items with Y values at or above L are conforming and those with Y values below L are nonconforming. Assume that X and Y have a bivariate normal distribution with known means μ_x and μ_y, known standard deviations σ_x and σ_y and known correlation coefficient ρ, $-1<\rho<1$ and $\rho\neq 0$. Let γ be defined as

$$\gamma=P[Y\geq L]=P[Y\geq\mu_y-U_\gamma\sigma_y],$$

where U_γ is the 100γ percent point of the standard normal distribution.

Assume that ρ is positive. Then an appropriate screening procedure is to accept any item whose X value is at least $\mu_x-K\sigma_x$, where K is the cutoff score to be determined. Let $\alpha(K)$ be the probability that an item is accepted and $\beta(K)$ be the joint probability that an item is conforming and it is accepted. Then they can be expressed as

$$\alpha(K)=P[X\geq\mu_x-K\sigma_x]=\Phi(K) \tag{1}$$

and

$$\beta(K)=P[X\geq\mu_x-K\sigma_x \text{ and } Y\geq L]=\Psi(K, U_r;\ \rho), \tag{2}$$

where

$$\Phi(K)=\int_{-\infty}^{K}\phi(z)\,dz, \tag{3}$$

$$\Psi(K, U_r;\ \rho)=\int_{-\infty}^{K}\int_{-\infty}^{U_r}\phi(z_1, z_2;\ \rho)\,dz_1dz_2 \tag{4}$$

and $\phi(z)$ and $\phi(z_1, z_2;\ \rho)$ represent, respectively, the standard normal probability density function and the standardized bivariate normal probability density function with correlation coefficient ρ.

Let the cost of accepting an item not meeting the specification, for convenience, be the economic unit, 1. Let C_r be the relative cost of scrapping (or reprocessing) an item and C_s be the relative cost of screening an item as compared with the economic unit, 1.

Then the expected cost per item with screening becomes

$$f(K)=\alpha(K)-\beta(K)+C_r\{1-\alpha(K)\}+C_s. \tag{5}$$

Theorem 1. $f(K)$ has a minimum at $K=K^*$ where,

$$K^*=(U_r+U_{c_r}\sqrt{1-\rho^2})/\rho. \tag{6}$$

Proof. Since

$$\frac{d}{dK}\Phi(K)=\phi(K)$$

and

$$\frac{d}{dK}\Psi(K, z;\ \rho)=\phi(K)\Phi\left(\frac{z-\rho K}{\sqrt{1-\rho^2}}\right),$$

we have

$$\frac{d}{dK}f(K)=\phi(K)\left[\Phi\left(\frac{-U_r+\rho K}{\sqrt{1-\rho^2}}\right)-C_r\right]. \tag{7}$$

It is seen that (7) has zero at $K=K^*$. Since $\phi(K)>0$ for any K and $\Phi(z)$ is a nondecreasing function of z, K^* is the minimum point of $f(K)$. ∎

Optimal screening procedures for situations where ρ is negative or when there is an upper specification limit U on Y can be similarly derived. The results are shown in Table 1.

Once the optimal screening procedure is found, we have to decide which strategy to

Table 1. Optimal Screening Procedures

	Lower Specification Limit (L)	Upper Specification Limit (U)
$\rho>0$	Accept an item if $X\geq\mu_x-K^*\sigma_x$	Accept an item if $X\leq\mu_x+K^*\sigma_x$
$\rho<0$	Accept an item if $X\leq\mu_x-K^*\sigma_x$	Accept an item if $X\geq\mu_x+K^*\sigma_x$

choose among the three (screening, acceptance without screening and scrapping without screening) by comparing their associated costs. Note that the expected costs per item for the cases of acceptance without screening and scrapping without screening are $1-\gamma$ and C_r, respectively.

Example 1. Let Y be the performance variable for some device which is expensive to measure or when it is measured the device is destroyed. A measurement X that is correlated with Y can be taken. Suppose that X and Y have a bivariate normal distribution with $\rho=0.8$, $\mu_x=3$, $\mu_y=2$, and $\sigma_x=\sigma_y=1$. We assume that the specification on Y is $Y\geq 0.80$, so that in the unscreened population 88.49 percent of the items are conforming. Suppose that C_s and C_r are 0.03 and 0.08, respectively. Then we have, from (6),

$$K^*=(1.20-1.4053\sqrt{1-0.8^2})/0.8=0.4460.$$

Hence the optimal screening procedure is to accept all items for which $X\geq 2.5540$. In this case $\alpha=\Phi(0.4460)=0.6723$ and $\beta=\Psi(0.4460,\ 1.20;\ 0.80)=0.6619$. Thus the proportion of conforming items among the accepted ones is $\beta/\alpha=0.9846$. The expected cost per item with screening is, from (5), 0.0666. The cost reduction is significant compared with the cost for acceptance without screening which is 0.1151 or the cost for scrapping without screening which is 0.08. As the value of ρ gets higher, the cost reduction becomes more significant. For example, when $\rho=0.95$, the expected cost per item with screening is 0.0507.

3. Two-sided Specification Limits

In this section we consider the case where there exist both L and U, that is, all items with Y values between L and U are conforming and those with Y values below L or above U are nonconforming.

Let $P[Y\geq L]=\gamma_1$ and $P[Y\leq U]=\gamma_2$. Thus before screening, the proportion of conforming items is $\gamma_1+\gamma_2-1$. Then an appropriate screening procedure is to accept all items for which

$$\mu_x-K_1\sigma_x\leq X\leq\mu_x+K_2\sigma_x, \tag{8}$$

where K_1 and K_2, $-K_1\leq K_2$, are to be determined so as to minimize the expected cost per item with screening as given by

$$f(K_1, K_2)=\alpha-\beta+C_r(1-\alpha)+C_s, \tag{9}$$

where

$$\alpha\equiv\alpha(K_1, K_2)=P[\mu_x-K_1\sigma_x\leq X\leq\mu_x+K_2\sigma_x] \tag{10}$$

and

$$\beta\equiv\beta(K_1, K_2)=P[\mu_x-K_1\sigma_x\leq X\leq\mu_x K_2\sigma_x \text{ and } L\leq Y\leq U]. \tag{11}$$

Note that the costs associated with acceptance without screening and scrapping without screening are $2-\gamma_1-\gamma_2$ and C_r, respectively.

(10) and (11) can, respectively, be rewritten as

$$\alpha(K_1, K_2)=\Phi(K_1)+\Phi(K_2)-1 \tag{12}$$

and

$$\beta(K_1, K_2)=\Psi(K_2, U_{\gamma_2};\rho)-\Psi(K_2, -U_{\gamma_1};\rho)-\Psi(-K_1, U_{\gamma_2};\rho)+\Psi(-K_1, -U_{\gamma_1};\rho). \tag{13}$$

Differentiation of (9) with respect to K_i yields

$$\frac{\partial}{\partial K_i}f(K_1, K_2)=\phi(K_i)g_i(K_i),\quad i=1,2, \tag{14}$$

where

$$g_i(K)=\Phi\left(\frac{-U_{\gamma_i}+\rho K}{\sqrt{1-\rho^2}}\right)+\Phi\left(-\frac{U_{\gamma_{3-i}}+\rho K}{\sqrt{1-\rho^2}}\right)-C_r. \tag{15}$$

It can be easily seen that, for each $i=1,2$, (15) has a minimum at $K=K_i^\circ$ with a common minimum value

$$g^\circ\equiv g_i(K_i^\circ)=2\Phi\left(-\frac{U_{\gamma_1}+U_{\gamma_2}}{2\sqrt{1-\rho^2}}\right)-C_r, \tag{16}$$

where

$$K_i^\circ=(-1)^i(U_{\gamma_2}-U_{\gamma_1})/2\rho. \tag{17}$$

Note that, from (17),

$$K_1^\circ+K_2^\circ=0, \tag{18}$$

and, from (15)

$$g_1(K)=g_2(-K) \tag{19}$$

and

$$g_i(K)=g_i(2K_i^\circ-K),\quad i=1,2. \tag{20}$$

We know that if (16) is negative, the equation $g_i(K_i)=0$ has two distinct roots for each i. Let K_i^* be the larger of the two. In the following a point (K_1, K_2) satisfying $K_1+K_2=0$ will be referred to as a boundary point. When $K_1+K_2=0$, $\alpha=\beta=0$ from (10) and (11). Thus the value of $f(K_1, K_2)$ at any boundary point is C_r+C_s.

Theorem 2. (a) If $g^\circ\geq 0$, $f(K_1, K_2)$ is minimized at any boundary point. (b) If $g^\circ<0$, $f(K_1, K_2)$ is minimized at either (K_1^*, K_2^*) or a boundary point.

Proof. Consider the following constrained optimization problem:

$$\underset{K_1, K_2}{\text{minimize}}\ f(K_1, K_2) \tag{21}$$

subject to $K_1+K_2\geq 0$.

A necessary condition for $(\overline{K}_1, \overline{K}_2)$ to be optimal is that there exists $\overline{\lambda}\geq 0$ such that

$$\phi(\overline{K}_i)g(\overline{K}_i)-\overline{\lambda}=0,\quad i=1,2, \tag{22}$$

$$\overline{K}_1+\overline{K}_2\geq 0 \tag{23}$$

and

$$\overline{\lambda}(\overline{K}_1+\overline{K}_2)=0. \tag{24}$$

See Bazaraa and Shetty [2], pp. 137—139.

Note that either $\overline{K}_1+\overline{K}_2=0$ or $\overline{\lambda}=0$. We immediately see that any boundary point for which $g_i(K_i)\geq 0$, $i=1,2$, satisfies the necessary condition. Thus from now on we consider the non-boundary point. Then from (22) and (24), $(\overline{K}_1, \overline{K}_2)$ must satisfy

$$g_i(\overline{K}_i)=0,\quad i=1,2. \tag{25}$$

If $g^\circ\geq 0$, only a boundary point satisfies the necessary condition. For, the only root of equation (25) is $\overline{K}_i=K_i^\circ$, $i=1,2$, but then $\overline{K}_1+\overline{K}_2=0$ from (18). Thus the optimal solution is any boundary point.

If $g^\circ<0$, (K_1^*, K_2^*) is the only non-boundary point which satisfies the necessary condition. For, there are three points other than (K_1^*, K_2^*) satisfying (25). From (19), these are $(K_1^*, -K_1^*)$, $(-K_2^*, K_2^*)$ and $(-K_2^*, -K_1^*)$. Since $K_i^\circ<K_i^*$, $i=1,2$, $-(K_1^*+K_2^*)$ is negative. Thus (23) does not hold for $(-K_2^*, -K_1^*)$. The other two points are indeed boundary points. Thus in this case the optimal solution is either (K_1^*, K_2^*) or a boundary point.∎

Note that a boundary point leads to scrapping of all items. Hence from Theorem 2, the optimal screening procedure is as follows:

i) If $g^\circ<0$ and $f(K_1, K_2)<C_r+C_s$; accept all items for which

$\mu_x - K_1\sigma_x \leq X \leq \mu_x + K_2\sigma_x$

and scrap all other items.

ii) Otherwise; scrap all items.

Theorm 3. $K_1^* - K_2^* = 2K_1^\circ$. (26)

Proof. From (20), $2K_i^\circ - K_i^*$ is the other root of equation $g_i(K_i) = 0$. We see from (19) that $-K_i^*$ and $K_i^* - 2K_i^\circ$ are the two roots of equation $g_{3-i}(K_{3-i}) = 0$. Since $K_i^* > 2K_i^\circ - K_i^*$, $K_i^* - 2K_i^\circ$ is the larger of the two roots of equation $g_{3-i}(K_{3-i}) = 0$. Hence by definition $K_i^* - 2K_i^\circ = K_{3-i}^*$. ▮

From Theorem 3, it suffices to find either one of K_1^* or K_2^*. A closed-form solution for K_i^* does not exist, but K_i^* can easily be obtained by using numerical methods, such as regula falsi.

We now consider a method for finding approximate solutions. If $|\rho|$ is relatively large, the first term of (15) is negligible for $\rho < 0$, and the second term of (15) is negligible for $\rho > 0$. Thus we can approximate (15) by $g_i^a(K)$ where, for $i = 1, 2$,

$$g_i^a(K) = \begin{cases} \Phi\left(\dfrac{-U_{\gamma_i} + \rho K}{\sqrt{1-\rho^2}}\right) - C_r & \text{for } \rho > 0, \\ \Phi\left(-\dfrac{U_{\gamma_{3-i}} + \rho K}{\sqrt{1-\rho^2}}\right) - C_r & \text{for } \rho < 0. \end{cases} \tag{27}$$

By equating $g_i^a(K_i) = 0$, the approximate solution K_i^a becomes

$$K_i^a = \begin{cases} (U_{\gamma_i} + U_{C_r}\sqrt{1-\rho^2})/\rho & \text{for } \rho > 0, \\ -(U_{\gamma_{3-i}} + U_{C_r}\sqrt{1-\rho^2})/\rho & \text{for } \rho < 0. \end{cases} \tag{28}$$

From (17) and (27) it is seen that

$$g_i^a(K) = g_{3-i}^a(K - 2K_i^\circ), \quad i = 1, 2, \tag{29}$$

from which it follows that

$$K_1^a - K_2^a = 2K_1^\circ. \tag{30}$$

Consider the difference

$$g_i(K_i^a) - g_i(K_i^*) = 1 - \Phi\left(\frac{U_{\gamma_1} + U_{\gamma_2}}{\sqrt{1-\rho^2}} + U_{C_r}\right). \tag{31}$$

This implies that $K_i^\circ < K_i^* < K_i^a$. Note that $(U_{\gamma_1} + U_{\gamma_2})$ is positive. Thus when $|\rho|$ is relatively large, K_i^a is a good approximation to K_i^* as will be shown in the following example. K_i^a can also be used as an initial value in obtaining K_i^*.

Example 2. There exist both L and U on Y, $\gamma_1 = 0.97$ and $\gamma_2 = 0.94$. C_r, C_s and ρ are the same as those given in Example 1. From (17),

$K_1^\circ = (U_{0.97} - U_{0.94})/(2\rho) = 0.4075.$

Table 2. Comparison of Approximate and Exact Solutions

ρ	Approximate Solution			Exact Solution		
	K_1^a	K_2^a	Cost	K_1^*	K_2^*	Cost
0.6	0.5034	1.2366	0.2580	0.3735	1.1067	0.2571
0.7	0.5142	1.1427	0.2377	0.4755	1.1040	0.2376
0.8	0.5462	1.0961	0.2120	0.5411	1.0910	0.2120
0.9	0.6085	1.0973	0.1781	0.6084	1.0973	0.1781

Using this value in (16) we see that g° is negative. Thus from (28),

$$K_1^a=(U_{0.97}+U_{0.08}\sqrt{1-0.80^2})/0.80=1.2972.$$

Using this as an initial value we have $K_1^*=1.2971$ by regula falsi method. Thus we obtain from (26) and (30)

$$K_2^a=K_1^a-2K_1^\circ=0.8897$$

and

$$K_2^*=K_1^*-2K_1^\circ=0.8896.$$

Hence $f(K_1^*, K_2^*)=f(1.2971, 0.8896)=0.0681$, which is significantly less than $C_s+C_r=0.11$. Therefore the optimal screening procedure is to accept only those items for which

$$\mu_x-1.2971\sigma_x\leq X\leq\mu_x+0.8896\sigma_x.$$

For different values of ρ, approximate and exact solutions and their associated costs are shown in Table 2. We can see that as ρ increases the difference in the two solutions decreases. In this example, the cost of acceptance without screening and the cost of scrapping without screening are 0.09 and 0.08, respectively. Note that when $\rho=0.6$, the expected cost per item with screening is, from Table 2, 0.0877, which is greater than the cost of scrapping without screening. In this case the optimal strategy is therefore to scrap all items without screening.

4. The Case of Two or More Screening Variables

Suppose there exist p screening variables, $X_1, X_2, \cdots, X_p$, and assume that $(Y, X_1, X_2, \cdots, X_p)$ are jointly normally distributed with known means $\mu_y, \mu_1, \cdots, \mu_p$, and known standard deviations $\sigma_y, \sigma_1, \cdots, \sigma_p$, respectively. Assume also that the correlations are known, and are ρ_{yi} between Y and X_i, and ρ_{ij} between X_i and X_j, $i, j=1, \cdots, p$.

Owen, McIntire and Seymour [9] considered two screening variables and showed that the procedure based on the linear combinations of the screening variables is more efficient than the one dealing with the screening variables separately although the latter is somewhat simpler to calculate. In this section we consider the procedure based on the linear combination of the screening variables $V=a_1X_1+a_2X_2+\cdots+a_pX_p$.

Then the screening procedure is to accept only those items for which

$$V\geq\mu_v-K\sigma_v \text{ when there exists } L,$$

and

$$V\leq\mu_v+K\sigma_v \text{ when there exists } U,$$

where

$$\mu_v=\sum_{i=1}^{p}a_i\mu_i$$

and

$$\sigma_v^2=\sum_{i\neq j}^{p}\sum^{p}a_ia_j\rho_{ij}\sigma_i\sigma_j+\sum_{i=1}^{p}a_i^2\sigma_i^2.$$

Since the probabilities of misclassification errors, which have direct influence on the screening cost, decrease as the correlation between the performance variable and the screening variables increases if the other conditions are held constant, $a_1, a_2, \cdots, a_p$ are to be selected to maximize the correlation between Y and V and to minimize the variance of $(Y-V)$. Therefore, we choose a_i to be σ_y/σ_i times i-th element of $\boldsymbol{\rho}_y'\boldsymbol{R}^{-1}$, where $\boldsymbol{\rho}_y$ is a $p\times 1$ vector with i-th element equal to ρ_{yi} and $\boldsymbol{R}$ is a $p\times p$ matrix with (i,j)-th element equal to ρ_{ij}. The correlation between Y and V is the multiple correlation coefficient and its value ρ_{yv} is $(\boldsymbol{\rho}_y'\boldsymbol{R}^{-1}\boldsymbol{\rho}_y)^{1/2}$, see Anderson (1958).

The problem in this manner is reduced to one with a single screening variable, V, and the procedures of Sections 2 and 3 are directly applicable.

Example 3. There exists L $(=0.8)$ on Y and μ_y is 2. Two screening variables, X_1 and X_2, are available, with $\sigma_y=\sigma_1=1$, $\sigma_2=2$, $\rho_{y1}=0.70$, $\rho_{y2}=-0.60$ and $\rho_{12}=-0.20$. The costs of screening inspection for X_1 and X_2 are, respectively, 0.025 and 0.015, and C_r is 0.08. The values of a_1 and a_2 are

$$a_1=\frac{\sigma_y(\rho_{y1}-\rho_{y2}\rho_{12})}{\sigma_1(1-\rho_{12}^2)}=0.7083$$

and

$$a_2=\frac{\sigma_y(\rho_{y2}-\rho_{y1}\rho_{12})}{\sigma_2(1-\rho_{12}^2)}=-0.2292.$$

Since

Table 3. Optimal Solutions and Associated Costs

	strategy	K^*	cost
without screening	acceptance	—	0.2000
	rejection	—	0.2500
screening	use X_1 only	0.5142	0.1617
	use X_2 only	−0.5034	0.1664
	use X_1 and X_2	0.5679	0.1568

$$\rho_{yv}=\left[\frac{\rho_{y1}^2-2\rho_{y1}\rho_{y2}\rho_{12}+\rho_{y2}^2}{1-\rho_{12}^2}\right]^{1/2}=0.8898,$$

we have

$$K^*=(1.20+1.4053\sqrt{1-0.8898^2})/0.8898=0.6278.$$

Thus if $\mu_1=1$ and $\mu_2=2$, the optimal screening procedure is to accept only those items for which

$$0.7083X_1-0.2292X_2\geq-0.2787.$$

The corresponding expected cost per item with screening is 0.0579. The optimal solutions and their associated costs for all possible combinations of strategies are shown in Table 3.

In the above example, as one might expect, the screening procedure using X_1 and X_2 is the most economical. In general, however, as the number of screening variables increases, reduction in costs due to the increase in the correlation between Y and V may not offset the increase in the cost of screening inspection. Thus when several screening variables are available, the problem as to how to select the best subset of screening variables may arise. In this case it would be possible to apply the variables selection principles of multiple regression or to utilize the branch and bound techniques.

5. Concluding Remarks

We have given optimal screening procedures for one-sided and two-sided specifications based on correlated variables assuming a multivariate normal structure among the variables.

These procedures do not need specialized tables, and in cases of one-sided specification the optimal cutoff scores have closed-form solutions. Approximate solutions having closed forms are presented for the case of two-sided specifications, and these approximate solutions are close to exact ones when the correlation between the performance variable and the screening variable is relatively high.

Methods presented in this paper are developed under the assumption that all parameters are known. The theory for cases where some parameters are not known is currently under development.

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