최소 자승법을 이용한 마이크로스트립선로의 수치해석에 관한 연구

Least Square Method for Analysis of Microstrip Line

* 최 성 진 (Choi, Seong Jin)
** 나 극 환 (Ra, Kuk Hwan)
*** 최 호 열 (Choi, Ho Yul)

요 약

본 논문에서는 마이크로스트립선로의 특성파라메타 분석을 위한 새로운 방법이 제안 되었다. 마이크로스트립선로의 특성파라메타들은 최소자승법에 의해서 계산하였고, 이 방법에 의해 마이크로스트립선로의 구조와 차원에 따른 결과를 보였다.

이들 결과를 차분법에 의해서 얻어진 결과와 비교 검토 하였다.

ABSTRACT

In this paper, a new method for the analysis of the characteristic parameters of microstrip line is proposed. The characteristic parameters of microstrip line bounded by a shielding wall are computed by using least square method with iterative method for optimization.

The results by this method depend on the dimensions and the structure of microstrip line. We compare the present results with those obtained by finite difference method.

I. INTRODUCTION

In recent years microstrip lines and its modifications have been extensively studied because of their compatibility with integrated circuits, [1]–[14]. The computation of the circuits, teristic impedance of various microstrip lines supporting TEM modes is a problem of considerable importance for the design of microwave circuits. [2] The impedance of such lines can be computed by using conformal transformation technique, variation method, and finite difference method. [1] [9] [11] Because a limited number of the transformations are applicable to microstrip lines which occur in practice, it is understandable that considerable work has been spent on numerical techniques for computing the characteristic impedance of several microstrip lines. [5] [7] The purpose of this paper is to show that least square method is

* 광운대학 전자공학과 강사
** 대유공업전문대학 교수
*** 대유공업전문대학 교수

particularly suited for the evaluation of the characteristic impedance of microstrip lines by machine computation. The accurary of the solution is demonstrated by comparing the present results using matrices of the order of 30 with those derived by other authors using finite difference method. [11] [12]

II. LEAST SQUARE METHOD

Suppose a domain D in which the electromagnetic fields are defined by

$$X = \sum_n X_n \phi_n$$

The Maxwell's equations at any point M_i in the domain D can be denoted by the sum of the components,

$$\sum_n a_{in} X_n = 0$$

The points M_i is limited by N, and so there are N equations. We find the approximate solution by discomposition on the N_0 base vectors ϕ_n. In general, the N equations wont's be able to be verified simultaneously because of the cutted sections, so we are going to search for the solution which minimizes the value of the function

$$f(X_1, X_2, \cdots, X_n) = \sum_{i=1}^{N} \left| \sum_{n=0}^{N_0} a_{in} X_n \right|^2$$

The discrete sum for i can be expanded to the integral form of the variables z

$$a_{in} = a_n(z_i)$$

Then,

$$f(X_1, X_2, \cdots, X_n) = \int_D \left| \sum_n a_n(z) X_n \right|^2 dz$$

If we rewrite f ,

$$f(X_1, X_2, \cdots, X_n) = \sum_{mn} A_{mn} X_n X_n^*$$

where

$$A_{mn} = \int_D a_m(z) a_m^*(z) dz$$

Now we want to find the vector with the minimum eigenvalue which has N_0 base vectors. The A_{mn} can be considered like a matrix representation of an operator A. If we suppose the functions ϕ_n are normalized, the problem becomes the minimization of $< XAX >$ with $< XAX >$ = constant.

Then,

$$AX = \lambda X$$

We will choose the smallest one λ_{min} among the possible eigenvalues λ_{min} because the minimum value of $< XAX >$ coincides with λ_{min}.

$$\langle XAX \rangle = \lambda_{min} \langle X \ X \rangle$$

III. ITERATION METHOD

A basic iterative operation which involves the replacement of a trial vector by an improved vector was used. The iterative procedure consists in continually transforming successive transforms into itself. If we take the eigenvalue problem in the form,

$$AX = \lambda X$$

The sequence of vectors Y_1, Y_2, Y_3, $\cdots$, Y_k, $\cdots$ is constructed from the initial vector Y_0 by making

$$Y_{k+1} = AY_k$$

Now we can expand Y_0 in terms of eigenvectors X_i,

$$Y_0 = \sum_{i=1}^{n} C_i X_i$$

Then,

$$Y_1 = AY_0 = \sum_{i=1}^{n} C_i \lambda_i X_i$$

$$Y_2 = AY_1 = \sum_{i=1}^{n} C_i (\lambda_i)^2 X_i$$

$$\vdots \qquad \vdots$$

$$Y_k = AY_{k+1} = \sum_{i=1}^{n} C_i (\lambda_i)^k X_i$$

$$\vdots \qquad \vdots$$

If we suppose that λ_n is the eigenvalue with the largest absolute value and $C_n \neq 0$, we have

$$Y_k = C_n(\lambda_n)^k \{ X_n + \sum_{i=1}^{n-1} \frac{C_i}{C_n} (\frac{\lambda_i}{\lambda_n})^k X_i \}$$

The summation in this equation becomes negligible compared with X_n, as K sets large, because $|\lambda_i/\lambda_n| < 1 (i \neq n)$. And hence Y_k approaches a multiple of X_n. Thus the iterative process yields convergence to the mode corresponding to eigenvalue of the largest absolute eigenvalue. Since we want to obtain the vector with the smallest eigenvalue by this iterative method, the matrix must be transformed

$$B = \lambda_{max} - A$$

Then

$$Y'_{k+1} = BY'_k = (\lambda_{max} - A) Y'_k$$

$$Y'_0 = \sum_{j=1}^{m} d_j X_j$$

$$Y'_1 = AY'_0 = \sum_{j=1}^{m} d_j (\lambda_{max} - \lambda_j) X_j$$

$$Y'_2 = AY'_1 = \sum_{j=1}^{m} d_j (\lambda_{max} - \lambda_j)^2 X_j$$

$$\vdots \qquad \vdots$$

$$Y'_k = AY_{k+1} = \sum_{j=1}^{m} d_j (\lambda_{max} - \lambda_j)^k X_j$$

$$\vdots \qquad \vdots$$

If we suppose again that $(\lambda_{max} - \lambda_m)$ is the new largest eigenvalue, λ_m is the smallest eigenvalue of the matrix A. Finally we can obtain the vector X_m corresponding to λ_m.

IV. FORMULATION

Since the microstrip structure is an open structure, the electric field region is essentially semi-infinite. Although the voltage functions can be solved numerically for such a semi-infinite region, it is more convenient to consider the microstrip line to be enclosed in a box as shown in Fig. 1. Since a similar enclosing structure is invariably used in most of the microstrip circuits, the configuration shown in Fig. 1-a is quite realstic.

In fact, it is an advantage of numerical method that the effect of enclosing box is taken into account. We can consider only the left-half side as shown in Fig. 1-b because microstrip line is symmetric.

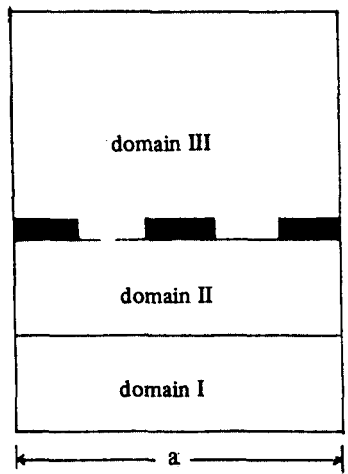

Fig. 1a. Enclosed microstrip.

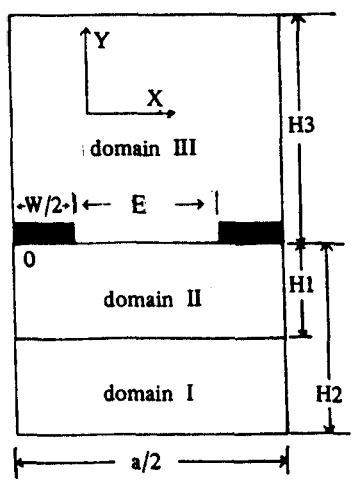

Fig. 1b. 2-dimensional structure for analysis.

Let us assume that the cross section of the microstrip line is defined by domains Ⅰ, Ⅱ, and Ⅲ shown in Fig. 1. We consider the forms of voltage functions in the three domains,

$$V_1=\sum_{n\neq 0} A_n \sinh \alpha_n(H_2+y) \cos \alpha_n x \quad (1)$$

$$V_2=\sum_{n\neq 0} (B_n \sinh \alpha_n y+C_n \cosh \alpha_n y) \cos \alpha_n x \quad (2)$$

$$V_3=\sum_{n\neq 0} D_n \sinh \alpha_n (H_3-y) \cos \alpha_n x \quad (3)$$

with the following boundary conditions

〔1〕 $y=-H_1$ (1) $V_1=V_2$ (4-a)

(2) $D_{y1}=D_{y2}$ (4-b)

〔2〕 $y=0$ (1) $V_2=V_3$ (5-a)

(2)-1) conductor; $E_{x2}=E_{x3}=0$ (5-b)

-2) dielectric; $D_{y2}=D_{y3}$ (5-c)

where

$$\alpha_n=(2_n-1)\pi/a, \quad (n=1,2,3\cdots)$$

The coefficients A_n, B_n, C_n and D_n are as yet unknown.

Using boundary condition (4-a), (4-b), we have, after some arrangement

$$A_n \sinh \alpha_n(H_2-H_1) = C_n \cosh \alpha_n H_1 - B_n \sinh \alpha_n H \quad (6)$$

$$\epsilon_1 A_n \cosh \alpha_n (H_2-H_1) = \epsilon_2(B_n \cosh\alpha_n H_1 - C_n \sinh \alpha_n H_1) \quad (7)$$

We devide equation (7) by equation (6)

$$\epsilon_2 \tanh \alpha_n(H_2-H_1)=\epsilon_1 \left(\frac{C_n-B_n \tanh \alpha_n H_1}{B_n-C_n \tanh \alpha_n H_1}\right) \quad (8)$$

If we suppose $C_n = K_n B_n$, we can obtain equation (8) as follows;

$$K_n=\frac{\epsilon_2 \tanh \alpha_n(H_2-H_1)+\epsilon_1 \tanh \alpha_n H_1}{\epsilon_1+\epsilon_2 \tanh \alpha_n H_1 \cdot \tanh \alpha_n (H_2-H_1)} \quad (9)$$

Using boundary condition (5-a), (5-b) and (5-c), we have, after some arrangement

$$D_n=\frac{C_n}{\sinh \alpha_n H_3} = \frac{K_n B_n}{\sinh \alpha_n H_3} \quad (10)$$

$$E_{x2}(=E_{x3}) = \sum_{n\neq 0} \alpha_n K_n B_n \sin \alpha_n x \quad (11)$$

$$D_{y2}-D_{y3} = \sum_{n\neq 0} \left(\epsilon_2+\frac{\epsilon_3 K_n}{\tanh \alpha_n H_3}\right) \alpha_n B_n \cos \alpha_n x \quad (12)$$

If we suppose

$$\gamma_{n1}=\alpha_n K_n, \quad \gamma_{n2}=\left(\epsilon_2+\frac{\epsilon_3 K_n}{\tanh \alpha_n H_3}\right) \alpha_n$$

The equation (11), (12) can be rewritten as follows;

$$\sum_{n\neq 0} \gamma_{n1} B_n \sin \alpha_n x=0 \;:\; 0\leq x\leq \frac{\omega}{2},\ \frac{\omega}{2}+E\leq x\leq \frac{a}{2} \quad (13)$$

$$\sum_{n\neq 0} \gamma_{n2} B_n \cos \alpha_n x=0 \;:\; \frac{\omega}{2}\leq x\leq \frac{\omega}{2}+E \quad (14)$$

The approximate solution for this problem can be obtained by least square method because n is finite. Therefore we consider the minimum condition of the function

$$\int_D \left(\sum_{n\neq 0} \gamma_n B_n\right)^2 dx$$

where $\gamma_n=\gamma_{n1} \sin \alpha_n x (=\gamma_{n2} \cos \alpha_n x)$

In the end, we can obtain the matrix

$$\begin{aligned} A_{mn}&=\int_D \gamma_m \cdot \gamma_n^* \, dx \\ &= \int_0^{\frac{\omega}{2}} \gamma_{m1} \sin \alpha_m x \cdot \gamma_{n1}^* \sin \alpha_n x \, dx \\ &+ \int_{\frac{\omega}{2}}^{\frac{\omega}{2}+E} \gamma_{m2} \cos \alpha_m x \cdot \gamma_{n2}^* \cos \alpha_n x \, dx \\ &+ \int_{\frac{\omega}{2}+E}^{\frac{a}{2}} \gamma_{m1} \sin \alpha_m x \cdot \gamma_{n1}^* \sin \alpha_n x \, dx \end{aligned} \quad (15)$$

where $\gamma_{n1}^*, \gamma_{n2}^*$ is the conjugate forms γ_{m1}, γ_{m2}.

If we define

$$S_{mn} = \alpha_m + \alpha_n, \quad D_{mn} = \alpha_m - \alpha_n \tag{16}$$

equation (15) can be rewritten as follows;

a) m = n case

$$A_{mn} = \frac{1}{4} |\gamma_{m1}|^2 a + \frac{1}{2} (|\gamma_{m2}|^2 - |\gamma_{m1}|^2) E + \frac{1}{4\alpha_m} (|\gamma_{m1}|^2 + |\gamma_{m2}|^2)[\sin \alpha_m(\omega + 2E) - \sin \alpha_m \omega]$$

b) m ≠ n case

$$A_{mn} = \frac{1}{2D_{mn}} (\gamma_{m1} \gamma_{n1}^* - \gamma_{m2} \gamma_{n2}^*)[\sin \frac{D_{mn}\omega}{2} - \sin \frac{D_{mn}(\omega+E)}{2}] + \frac{1}{2 S_{mn}} (\gamma_{m1} \gamma_{n1}^* + \gamma_{m2} \gamma_{n2}^*) [\sin \frac{S_{mn}(\omega+E)}{2} - \sin \frac{S_{mn}\omega}{2}]$$

V. CALCULATION OF CHARACERISTIC IMPEDANCE

The capacitance of microstrip line is computed from the line integral of the electric field normal to the boundary.

By Gauss's Law

$$Q_n = \int_{-\frac{\omega}{2}}^{\frac{\omega}{2}} D_{y2}\, dx - \int_{-\frac{\omega}{2}}^{\frac{\omega}{2}} D_{y3}\, dx \tag{17}$$

substituting equation (12) into equation (17)

$$Q_n = 2B_n [\epsilon_2 + \frac{\epsilon_3 K_n}{\tanh \alpha_n H_3}] [\sin \alpha_n (\frac{\omega}{2})] \tag{18}$$

The voltage is obtained by using equation (2)

$$V = K_n B_n \cos(\alpha_n x) \tag{19}$$

So the capacitance of the microstrip line can be determined

$$C = \frac{2}{K_n} [\epsilon_2 + \frac{\epsilon_3 K_n}{\tanh \alpha_n H_3}] [\frac{\sin \alpha_n (\frac{\omega}{2})}{\cos \alpha_n x}] \tag{20}$$

The characteristic impedance for the microstrip line in approximate quasi-TEM is

$$Z_0 = \sqrt{\frac{L}{C}} = Z_{0m} (\frac{C_a}{C})^{\frac{1}{2}} \tag{21}$$

where $Z_{0m} = 1/C_a C'$, C_a is the capacitance the case where the dielectric in the microstrip is replaced by air, C is the capacitance the case where the dielectric in the microstrip is dielecric material, C' is the velocity of light.

VI. DISCUSSION OF NUMERICAL RESULTS

The computer program is outlined in flow graph form in Fig. 2. We consider 30 harmonics for the numerical analysis using least square method. The time spent for calculation of the characteristic impedances was 110 seconds for the iterative method. These results are presented in Fig. 3, 4, and 5.

In Fig. 3, we obtained the 50-ohm curve of the characteristic impedances which depend on the width of the microstrip line W and S in the case of HA/H=0.5.

In Fig. 4, characteristic impedances for the microstrip line is plotted against W/H, the ratio of width of the line to thickness of dielectric substrate in the case of HA/H=0, and HA/H=0.5 with S=0. We are understable that characteristic impedances decrease as the width of the line increases, increase in the line width increases the capacitance per unit length, which decreases the characteristic impedances. We compared the results obtained by least square method with those obtained by finite difference method.

In Fig. 5, lower characteristic impedances are obtained for higher values of dielectric constant of the substrate material because of the same reason. Also, there was a little difference with the results of the finite difference method, but calculation can be improved by increasing the number of harmonics.

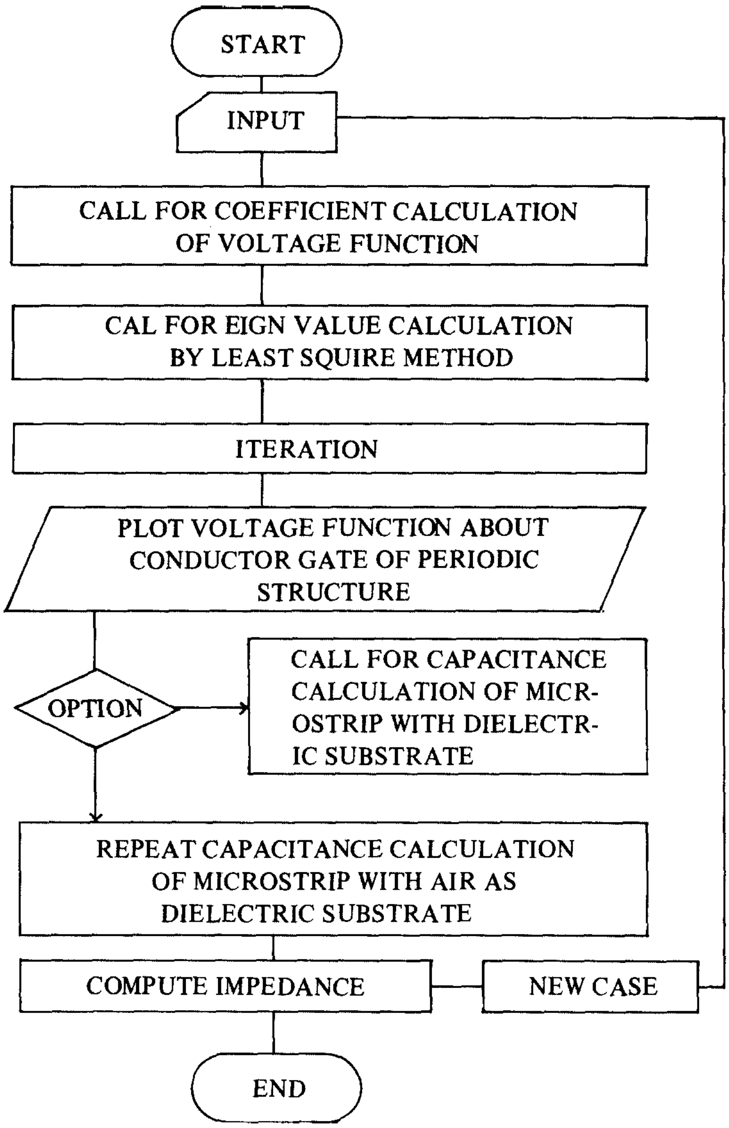

Fig. 2. Flow chart for analysis.

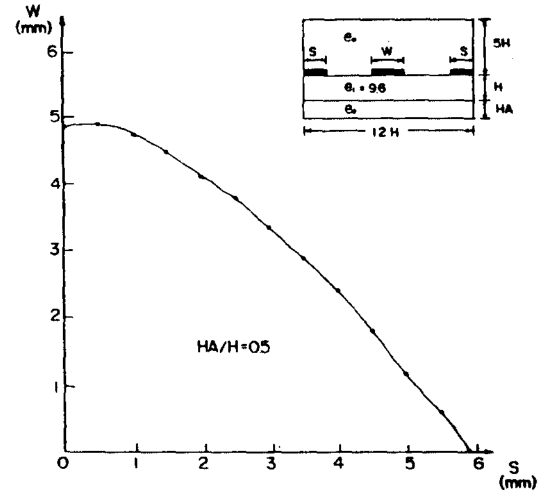

Fig. 3. 50-ohm curve of the characteristic impedances for microstrip line in the case of HA/H=0.5.

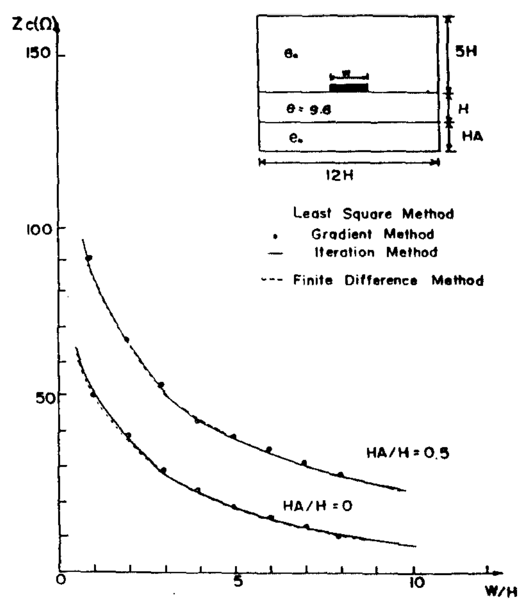

Fig. 4. Characteristic impedances of microstrip line as a function of W/H in the case of HA/H=0.5 and HA/H=0.

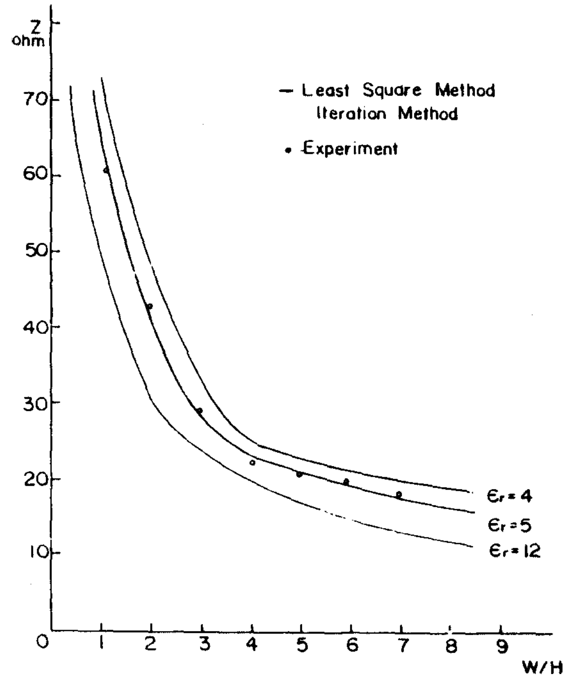

Fig. 5. Characteristic impedances of microstrip line as a of function W/H.

VII. CONCLUSION

The least square method and iterative method for optimization are a simple and accurate method of computing the general transmission parameters of a microstrip line and have shown the dependence of the characteristic impedance on the structure of microstrip line.

These results are verified by measurements and compared with finite difference method.

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