J. Astron. Space Sci., Vol. 2, No. 2, pp. 101-115, 1985.

Physical Parameters of Late Type Spiral Galaxies I-Mass and Luminosity of NGC 6946*

Sug-Whan Kim

Korean National Astronomical Observatory

and

Mun-Suk Chun

Department of Astronomy & Meteorology, Yonsei University

(Received December 1, 1985)

Abstract

Using Brandt model the mass distribution of the late type spiral galaxy NGC 6946 was derived, and the total mass was reestimated to understand the M/L ratio of this galaxy. Two kinds of the rotation curve with shape parameter n = 1 and 3.3 were examined.

The followings are the main results;

(1) The total masses of NGC 6946 are 3.1×10^{11} M (n=1) and 2.8×10^{11} M (n=3.3) respectively, and the corresponding M/L are about 17 and 16 for both cases.

(2) The optical image in the blue light, whose radius is 9.6 kpc, has 8×10^{10} $M_\odot$ and 1.4×10^{11} $M_\odot$. These give the value of M/L about 5 and 8 respectively.

(3) The masses and M/L of the nuclear region within 1.2 kpc are 4.0×10^9 $M_\odot$, 4.7×10^9 $M_\odot$ and 3, 4 for both cases. Those of the disk from 1.2 kpc to 9.6 kpc are $7.6 \times 10^{10} M_\odot$, 1.4×10^{11} $M_\odot$, and 5, 8.

(4) The masses of the outer halo extended to few hundreds kiloparsecs are 2.3×10^{11} $M_\odot$ and 1.4×10^{11} $M_\odot$. The corresponding M/L are about 62 and 37.

*Yonsei University Observatory Contribution No. 35.

I. Introduction

NGC 6946 is an isolated spiral galaxy of luminosity class I (van den Bergh 1960) and of morphological type SAB(rs)cd (de Vaucouleurs 1976, hearafter RC2), which is located near the galactic plane ($b=11.^{\circ}7$). The photometric study for this galaxy has been made previously by Ables (1971). But, more recently, Kim and Chun (1984) have made the detailed surface photometry using the PDS microdensitometer, and derived the several physical parameters. Among them, the simply derived mass to luminosity ratio M/L contains some roughness due to the mass determinations of the early studies (Gordon *et al.* 1968, Rogstadt and Shostak 1973), and the mass distribution in NGC 6946 is not known till now.

These stimulated us to begin the work to calculate the mass and its distribution of NGC 6946. They will let us to know the M/L structure, which means the total M/L, the radial variation of local M/L and the integrated M/L. The M/L is one of the most important parameters to understand the structure and the evolution of disk galaxy. Especially it is a fundamental data of population synthesis with colour and spectral index, and a base to discuss the massive halo problem. The main interests of this work are to construct the mass and M/L distributions of NGC 6946, to find the clear evidence of unluminous massive halo, and, if it is, to discuss the characteristic properties.

The radial distribution of M/L of spiral galaxy has been studied for the past twenty years. But the characteristic patterns have not been conclusive. For example, a negative gradient of local M/L was found by Fish (1961), whereas the positive gradient was also found by de Vaucouleurs (1958, 1960). The recent studies (Blackman, 1979a, b, c) do not show any definite patterns of the variation of local M/L. But the significant portion of the sample galaxies has a middle dip in the distribution of local M/L, and Blackman(1979a) tried to interpret it by using the desnity wave theory. In the mean time, the radial variations of integrated M/L show the positive gradient or constant one (Blackman and van Moorsel 1984).

The distance to NGC 6946 has been provided by Hodge (1966) as 4.1 Mpc, and Sandage and Tammann(1974) as 10.5 Mpc. Gordon *et al.*(1968) and Rogstadt and Shostak(1973), who made the rotation curves used in this study, used 4.2 Mpc and 10.1 Mpc respectively. But we will use 5.5 Mpc suggested by Buta(1982) in the following discussion; at this distance, $1' = 1.60$ kpc.

II. Mass and Luminosity

1. Total Mass and Mass to Luminosity Ratio

From the neutral hydrogen observation Gordon *et al.*(1968), and Rogstadt and Shostak(1973) obtained the rotation curve of NGC 6946. The results of their studies are listed in Table 1.

Table 1. Rotation Properties and Mass of NGC 6946

	Rogstadt and Shostak(1973)	Gorden *et al.* (1968)
Shape parameter n	1	3.3
Maximum radial velocity Vm.rad.	104 kms^{-1}	110 kms^{-1}
Maximum rotation velocity Vm.rot.	208 kms^{-1}	294 kms^{-1}
Location of maximum Vm	5'.63	6'
Inclination i	30°	22°
Distance	10.1 Mpc	4.2 Mpc
Total Mass	1.8×10^{11} $M_\odot$	2.32×10^{11} $M_\odot$
Mass model	Schmidt (1965)	Brandt (1960)

The rotation curve of Brandt model (Brandt 1960, Brandt and Belton 1962, Brandt and Scheer 1965) is defined as

$$V(R) = \frac{AR}{[1 + B^n R^n]^{3/2n}} \quad \cdots\cdots (1)$$

where,

$$A = \frac{V_m}{R_m} 3^{3/2n} \quad \cdots\cdots (2)$$

and,

$$B = \frac{2^{1/n}}{R_m} \quad \cdots\cdots (3)$$

Using equation (1) and parameters listed in Table 1, we can reproduce the rotation curves of NGC 6946 as in Figure 1. The curve of shape parameter 1 (hereafter case 1) shows that the rotation velocities increase rapidly to the maximum, and keep rather constant. However the curve of shape parameter 3.3 (hereafter case 3.3) has steep rise to the maximum, and then decreases. The case 1 is similar to that of M83 (Bosma 1981), which has the similar structure and we assume that NGC 6946 has a similar rotation curve as in M83.

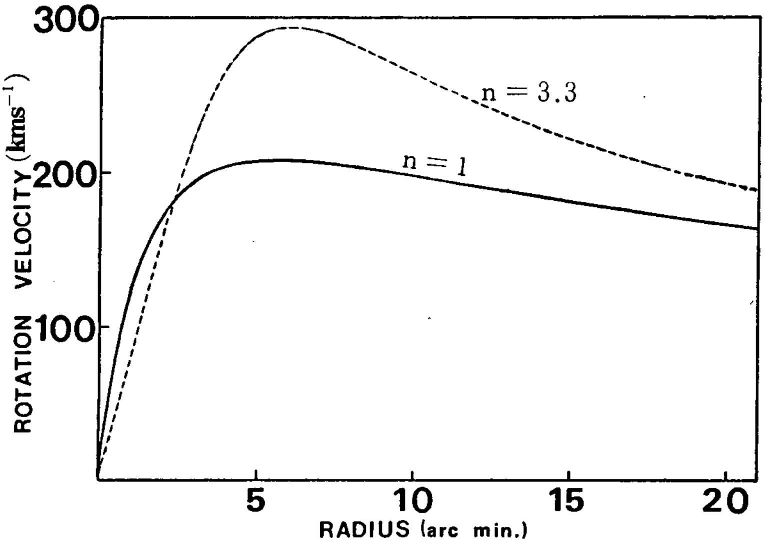

Figure 1. Rotation curve of NGC 6946 with shape parameters 1 and 3.3.

Even there exists the difference of the shape between these two rotation curves, it is important to recognize that the observed maximum radial velocities and radii of the maximum are similar for both cases (see Table 1). This means that the mass represented by equation (4) will be dependent on the shape parameter and the inclination. For the inclination, we use the inclinations of above two cases respectively. Because there is not criterior to distinguish which inclination is correct.

The total mass of a galaxy can be calculated using equation (4) (Brandt and Belton. 1962).

$$M_T = (\frac{A^2}{B}) \frac{1}{BG} = (\frac{3}{2})^{3/n} \frac{V_m^2 R_m}{G} \quad \cdots\cdots (4)$$

For the practical purpose, Gordon *et al.* (1968) used the modified form of equation (4) as

$$M_T/M_\odot = 7.4 \times 10^4 (1.5)^{3/n} R_m D V_{m\cdot rad}^2 \operatorname{cosec}^2 i \quad \cdots\cdots (5)$$

where R_m is in arc min. and $V_{m\cdot rad}$ is in kms^{-1} and D is in megaparsecs.

The mass equations (4) and (5) give a total mass of 3.1×10^{11} $M_\odot$ for the case 1, and 2.8×10^{11}

$M_\odot$ for the case 3.3. These are comparable with the results from the previous studies (Gorden *et al.* 1968, Rogstadt and Shostak 1973). The ratio of the total mass to luminosity of the optical image $M/L_{opt.}$ is about 17 for the case 1, and 16 for the case 3.3, since the luminosity of the optical image is 1.75×10^{10} $L_\odot$ (Kim and Chun 1984) in the blue light. From these values we can calculate the total mass to luminosity ratio M_T/L_T as 14 for the case 1, and 13 for the case 3.3 (total luminosity L_T will be discussed in section III). These are slightly larger than the average value of 10 for the spiral galaxy (Brosche and Reinhardt 1977), and of 9.9 for 16 spiral galaxies with radially increasing the integrated M/L (Blackman and van Moorsel 1984). From these facts, it seems like to be the conventional value of about 4 for the average spiral (Voigt 1982) is too low.

2. Integrated Mass and Mass to Luminosity Ratio

The integrated mass M(R) can be calculated through the accumulation of the fractional mass, and the numerical results were tabulated by Brandt and Scheer (1965). Using the fractional mass table we can derive the distribution of integrated mass in NGC 6946. Figure 2 shows the radial variation of the integrated mass up to 19$'$.7 (31.52 kpc) for the case 1, and 34$'$.0 (54.4 kpc) for the cases 3.3.

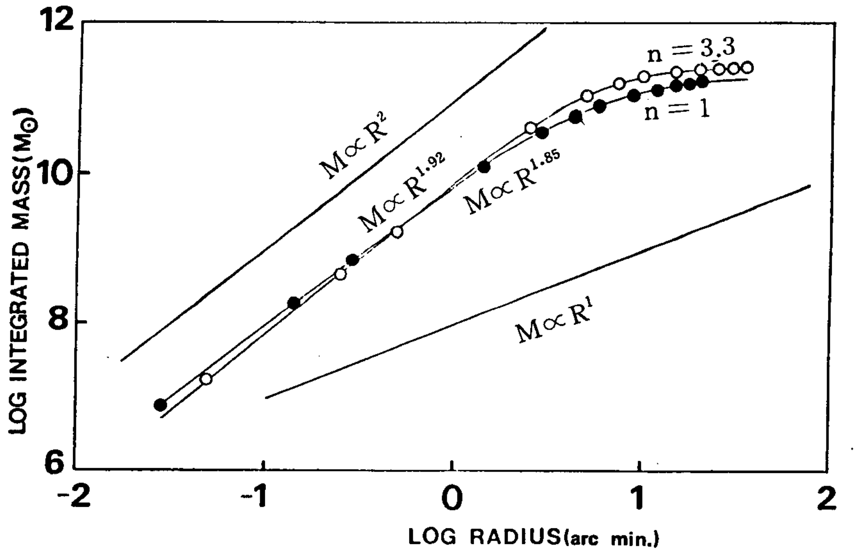

Figure 2. Radial variation of the integrated mass.

The integrated mass M(R) increased with radius roughly by

$M(R) \propto R^{1.85}$, for $R < 4'$, case 1 .... (6)

$M(R) \propto R^{1.92}$, for $R < 4'$, case 3.3 .... (7)

However it is not possible to fit with any power laws beyond $4'$ for both cases. The integrated mass distribution of NGC 6946 does not show on exponential distribution (Freeman 1970, Boroson 1981) as well as the distribution produced from an identically flat rotation curve, $M(R) \propto R$. Our results are consistent with Burstein *et al.* (1982)'s one. However for larger R the curves in Figure 2 has steeper curvature than the integrated mass distribution suggested by Burstein *et al.* (1982). From Figure 2 the optical image extended to $R = 6'$ has a mass of 8.0×10^{10} $M_\odot$ which is about 26% of the total mass for the case 1, and for the case 3.3 it is 1.4×10^{11} M which is about 49% of the total mass.

Using the accumulated light distribution in NGC 6946 (Kim and Chun 1984), we derived the integrated M/L curves up to the optical image and presented them in Figure 3, in which the integrated M/L increases with the positive gradient similar to that of NGC 224 (de Vaucouleurs 1958) which is a luminosity class I-II spiral galaxy (RC2). In spite of using Nordsiek's (1973) technique, Blackman and van Moorsel (1984) showed the integrated M/L increases significantly with radius in 16 out of 26 spiral galaxies (13 pairs), while the remains show constant. Even both cases show difference for the integrated M/L curves, it is important to notice that they have similar increasing pattern with radius. This increasing pattern may indicate the existence of the massive unluminous halo. Figure 3 also give us the integrated M/L values at the limiting of the optical image ($R \leqslant 6'$) as 5 for the case 1 and 8 for the case 3.3, which are quite similar to the conventional value of M/L for the spiral galaxy.

3. Surface Density and Local Mass to Luminosity Ratio

The radial distributions of the surface density ρ_s, mass in the unit area parallel to the galactic plane, were derived within the radius of 19'.7 (31.58 kpc) for the case 1, and of 19'.5 (31.2 kpc) for the case 3.3 using the Ω function table provided by Brandt and Scheer (1965) and it was plotted in Figure 4. For the case 1, the central surface density is of 1199 $M_\odot pc^{-2}$ and the surface density at 31.5 kpc is about 12 M pc^{-2}. However for the case 3.3, the surface density decreases from 912 $M_\odot pc^{-2}$ at the centre to 1 $M_\odot pc^{-2}$ at 31.2 kpc. We may expect that these distributions will be extended to a few hundreds parsecs (see subsection 4). It is worth while to notice that the overall shape of the surface density for the case 1 is more similar to those of NGC 5055 and M31 (Brandt and Belton 1962) than that for the case 3.3.

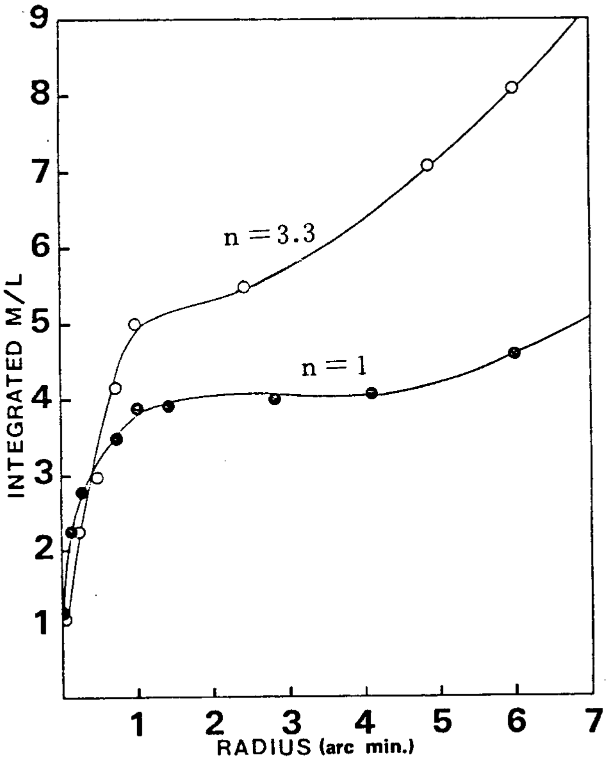

Figure 3. The integrated M/L radial variation.

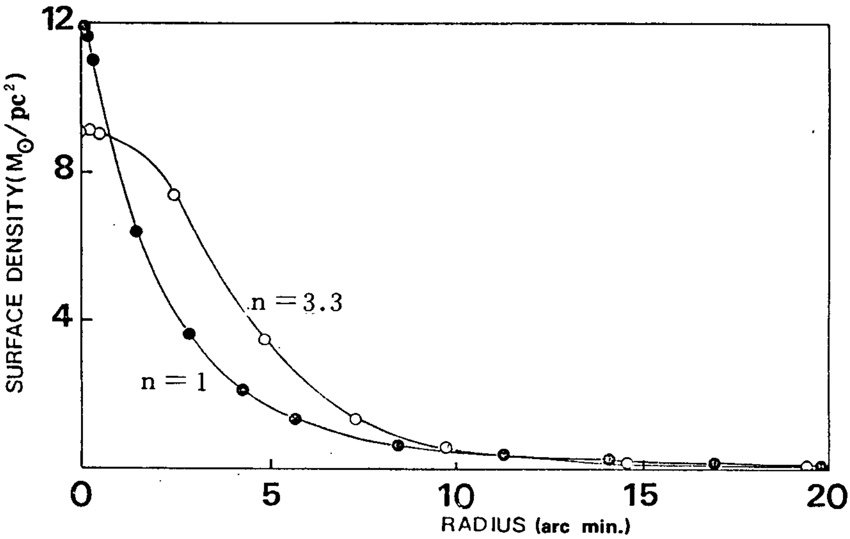

Figure 4. Local surface density distribution as a function of radius.

The local M/L distributions as a function of radius are presented in Figure 5 for both cases. To calculate the local value of M/L at $R > 6'$, we extrapolated the surface brightness distribution of the exponential disk described as the following equation given by Kim and Chun (1984).

$$\mu(R^*) = 20.58 + 0.583\, R^*, \quad R^* < 6' \qquad (8)$$

where $\mu(R^*)$ is the surface brightness in magnitude per square arc second and R^* is the equivalent radius in arc minute. From Figure 5 one can see that the local M/L increases sharply with radius quite similar to that of M31 (de Vaucouleurs 1958). It is prominent from the Figure 5 that beyond the optical image the local M/L in NGC 6946 increases very sharply with radius. For example, the local M/L reaches rapidly 370 at 16'.9 (27 kpc) and 1200 at 19'.7 (31.5 kpc) for the case 1, and 108 at 14'.6 (23.4 kpc) and 615 at 19'.5 (31.2 kpc) for the case 3.3. We compared the local values of M/L in NGC 6946 with those of NGC 224 (de Vaucouleurs 1958). From to our calculation and Figure 5, the local M/L in NGC 6946 is 43 at 11'.3 (18.1 kpc) and 121 at 14'.1 (22.6 kpc) for the case 1, and 28 at 9'.7 (15.5 kpc) and 108 at 14'.6 (23.4 kpc). In NGC 224, the local M/L is 33 at 16.5 kpc, 70 at 22 kpc, and about 130 at 27.5 kpc (de Vaucouleurs 1958). These results are in good agreement with the Burstein *et al* (1982)'s suggestion, where the local M/L increases rapidly with radius.

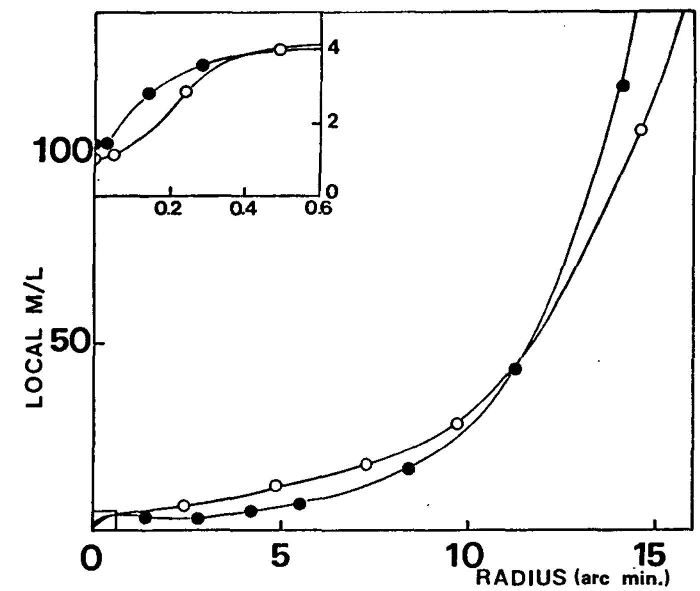

Figure 5. The radial local M/L distribution, in which the radial variation of the local M/L for $R < 0'.6$ (0.96 kpc) is shown in the left side upper panel.

Blackman (1979a, b, c) showed that there is middle dip in the distributions of the local M/L at the outer disk, and tried to explain it by using the density wave theory. As a matter of fact, there was such a middle dip in the local M/L distribution for NGC 5055 with the negative gradient (Fish 1961), and for NGC 224 with the positive gradient (de Vaucouleurs 1958). In the case of NGC 6946 there is no such a middle dip, and the local M/L increases gradually in the middle region ($1' < R < 5'$) for the case 3.3. It is not clear whether the middle dip really exist or not. The local M/L is almost constant within the known error in the middle region for the case 1. But these results are not conclusive because of the uncertainty of estimation of the surface density.

4. Space Density

The space density is defind as a mass contained within a unit volume element at radius R, and can be calculated using the $\tilde{\beta}$ function table provided by Brandt and Scheer (1965). From the space density formula given by Brandt and Belton (1963) and Brandt and Scheer (1965), we have to choose the true axial ratio c/a of NGC 6946 to calculate the space density. Wyse and Mayall (1942) suggested that the average axial ratio of several edge on spirals is $\langle c/a \rangle = 1/12$ ignoring the nuclear bulge. Blackman (1977) proposed that the axial ratio c/a = 1/16 is typical for several edge on Sc type galaxies in the Hubble atlas, and adapted c/a = 1/10 to allow for the nuclear bulge of NGC 935 which is Sc type galaxy. Blackman (1979a) also used c/a = 1/12 for NGC 157 which is SAB(rs)bc (T = 4) type and luminosity class I galaxy. Our rough estimate of c/a for NGC 6946 using D/B = 14.4 (Kim and Chun) shows that at least the true axial ratio is smaller than c/a = 1/10. So we choose c/a = 1/15 to be suitable for NGC 6946.

The resultant space density distributions in NGC 6946 is provided in Figure 6. The central space density is 5.1 $M_{\odot}pc^{-3}$ for the case 1 and 0.9 $M_{\odot}pc^{-3}$ for the case 3.3. The former is comparable with 3.8 $M_{\odot}pc^{-3}$ for NGC 935 (Blackman 1977), 2.2 $M_{\odot}pc^{-3}$ for NGC 1084 and 3.6 $M_{\odot}pc^{-3}$ for NGC 7331 (Blackman 1979b). The later value is comparable with 0.57 $M_{\odot}pc^{-3}$ for NGC 157 (Blackman 1979a), which has a surface density of 307.9 $M_{\odot}pc^{-2}$ at R $\doteq$ 2 kpc using the spiral density wave theory (Basu 1979). However NGC 6946 has a surface density of 641 $M_{\odot}pc^{-2}$ at R = $1'.41$ (2.3 kpc) for the case 1 and 739 $M_{\odot}pc^{-2}$ at R = $2'.43$ (3.9 kpc) for the case 3.3. These mean that NGC 6946 is about two times heavier and denser than NGC 157 whose total mass is 1.61 x 10^{11} $M_{\odot}$ (Blackman 1979a).

The radial distribution of the space density for the case 1 in Figure 6 agrees well with that in Our Galaxy which is a SAB(rs)bc (T = 4) type and Luminosity class I-II galaxy (Voigt 1982). For example, the space density in Our Galaxy is 1.2 $M_{\odot}pc^{-3}$ at 3 kpc, 0.1 $M_{\odot}pc^{-3}$ at 10 kpc, 0.09 $M_{\odot}pc^{-3}$ at 20 kpc, and so on. Similarly, in NGC 6946, the space density is 1.2 $M_{\odot}pc^{-3}$ at 2.3 kpc, 0.1 $M_\odot pc^{-3}$ at 9.0 kpc, 0.009 $M_\odot pc^{-3}$ at 22.5 kpc, and so on. However the space density is 0.9 $M_\odot$ pc^{-3} at 2.3 kpc, 0.2 $M_\odot pc^{-3}_3$ at 10.1 kpc, 0.01 $M_\odot pc^{-3}$ at 19.5 kpc, and so on, for the case 3.3. So the distribution for the case 3.3 does not agree well with that in our galaxy.

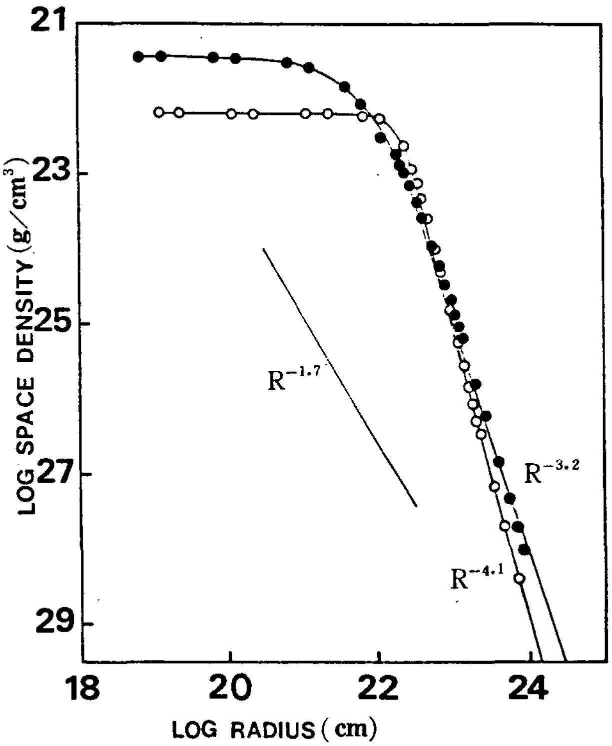

Figure 6. Mass space,density decreases rapidly after log R = 22 cm.

Our calculation shows that the space density distribution with radius can be described as

$\rho_{space} \propto R^{-3.2}$, for R > 3.′7 (5.9 kpc), n = 1 .... (9)

$\rho_{space} \propto R^{-4.1}$, for R > 4.′9 (7.8 kpc), n = 3.3 .... (10)

i.e., space density falls off more rapidly that $1/r^2$. The generally adapted mean desnity in the universe is expressed by

$\langle \rho \rangle \sim 10^{-30}\ \Omega\ gcm^{-3}$ .... (11)

where Ω is the ratio of the observed density to that in a closed universe (Peebles 1979). If the space density in NGC 6946 continues to decrease as $R^{-3.2}$ or $R^{-4.1}$, then it will reach the mean density 10^{-30} gcm^{-3} of the closed universe in a 1.3 Mpc or 0.6 Mpc for both cases respectively. This leads to the conclusion that the dynamical size of NGC 6946, which is an optically normal galaxy, is significantly larger than the characteristic size (a few tens kpc) of normal bright spiral galaxies which one have thought conventionally. This also means that Local Group dominated by Our Galaxy and Andromeda Galaxy (M31) which are optically normal bright spiral galaxies and have the similar structures of mass and M/L distribution with NGC 6946 may be filled with the nonluminous matter, and the members of Local Group may be continuously connected with each other by the nonluminous matter. The dynamical sizes and the distance of NGC 6946 do not cause any confusion to the fact that NGC 6946 is not a member of Local Group (de Vaucouleurs 1966), but support the fact above again.

However Burstein *et al.* (1982) found the fall off in space density with radius is described by $R^{-1.7}$ for several Sc galaxies, and then emphasized the wonderful agreement with the density-radius power law $\rho \sim R^{-1.7}$ (de Vaucouleurs 1971) and the power law of spatial correlation function in R for the distribution of galaxies with the exponent equal to -1.7 (Peebles 1980). Burstein *et al.* (1982) thought the dynamical radius of a few Mpc, where the space density in a galaxy reaches the mean density of the closed universe. If $R^{-1.7}$ distribution of the space density is common property in spiral galaxy, then the size of Local Group will be extened beyond NGC 6946.

5. Vertical Thickness

The vertical thickness T at certain radius R in a galaxy is defined by deviding the surface density ($M_{\odot}pc^{-2}$) to the space density ($M_{\odot}pc^{-3}$). To calculate T we assumed the space density to be constant along the perpendicular direction to the galactic plane. The vertical thickness in NGC 6946 is given in Figure 7. For the case 1 the vertical thickness of the mass distribution increases linearly with radius from 0.2 kpc at the center to 3.9 kpc at $19'.7$ (31.5 kpc). On the other side, it remains constantly at about 1.0 kpc from $0'$ (0 kpc) to $4'.9$ (7.84 kpc), and then increases linearly to 3.2 kpc at $19'.5$ (31.2 kpc). The vertical thickness rises with the increasing radius. This may be an evidence to insist that in NGC 6946 the significant fraction of the mass is placed in a vertically extended disk like shape beyond the optical image, and therefore it may be a shape of the nonluminous massive halo. The vertical thickness in the optical image of NGC 6946 is less than 1.4 kpc at $6'$ (9.6 kpc) for the case 1, and 1.2 kpc at $7'.3$ (11.7 kpc) for the case 3.3. The oblateness (ϵ) of the nuclear bulge of NGC 6946 is defined by,

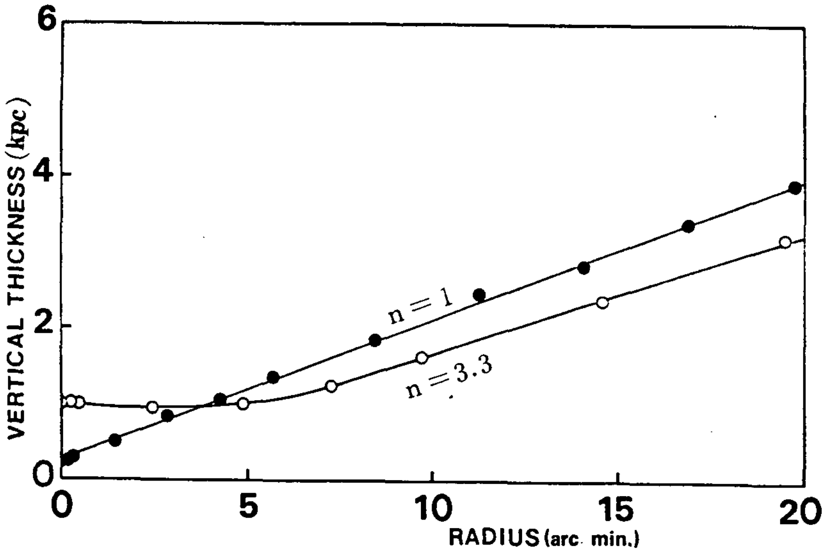

Figure 7. Vertical thickness becomes thicker for the shape parameter n = 1 than for n = 3.3 in the outer region of NGC 6946.

$$\epsilon = \frac{a - c}{a} = 1 - \frac{c}{a} \qquad (12)$$

where c/a is a true axial ratio of the nuclear bulge. If we substitute a as radius of nuclear bulge R* = 0.′74 (1.18 kpc) (Kim and Chun 1984) and c as vertical thickness at R = 0′ (0 kpc) to equation (12), then the oblateness ϵ will be 0.8 for the case 1 and 0.14 for the case 3.3. This result reveals the fact that the nuclear bulge for the case 1 rotates more rapidly than that for the case 3.3, which is clearly shown in Figure 1.

III. Discussions

We have calculated the mass and M/L values of a fair sample of Scd galaxy NGC 6946 which is optically quite normal. In Table 2 we summarize the mass, luminosity and M/L in the galaxy, which is divided into three regions: nuclear bulge (0′ < R < 0.′74), optically seen disk (0.′74 ⩽ R ⩽ 6′), and halo (R ⩾ 6′) which contains the optically undetectable disk.

Table 2. Integral Mass and Luminosity Structure in NGC 6946

	Mass ($M_\odot$)		Luminosity ($L_\odot$)	M/L ($M_\odot/L_\odot$)	
n	1	3.3		1	3.3
Total	3.16×10^{11}	2.78×10^{11}	2.12×10^{11}	14.43	13.11
Nuclear	3.97×10^{9}	4.73×10^{9}	1.14×10^{9} (a)	3.48	4.14
Disk	7.64×10^{10}	1.37×10^{11}	1.64×10^{10} (b)	4.66	8.35
Halo	2.25×10^{11}	1.36×10^{11}	3.65×10^{9} (c)	61.64	37.26

Note (a), (b) – Kim and Chun (1984)
(c) – this paper

The luminosity of 3.65×10^9 $L_\odot$ of the optically undetectable disk from 6′ to 58′.7 contained within the extended halo (at least a few hundred kpc) is calculated by numerical integration extrapolating the exponential distribution of the surface brightness described by equation (8). This enables us to give the corresponding M/L value of 62 for the case 1, and of 37 for the case 3.3. These are M/L values of the outer nonluminous halo of NGC 6946.

Since Ostriker and Peebles (1973) suggested that there is a significant fraction of the hidden mass in the outer halo of a disk galaxy from N-body experiment, the increasing evidences for the massive halo have been found for the past ten years. Among them an extensive study for the binary galaxies (Turner 1976) shows that spiral galaxies have massive halo containing about 3 times more than and about 10 times less than the disk mass, and M/L of the massive halo is about 65 ($H_\odot$ = 50 kms^{-1} Mpc^{-1}). The ratio of the nonluminous halo mass for the case 1, which is adapted to be reliable, to the disk mass in NGC 6946 is quite similar to the lowest boundary of the range of halo mass to disk mass ratio ($3 < M_H/M_D < 10$) given by Turner (1976). Moreover M/L ~ 62 for the unluminous halo of NGC 6946 agrees well with Turner's (1976) value of 65. The radius of 58′.7 (93.9 kpc), where near the end of the exponential distribution of the surface brightness may be expected to contain a mass of about 80% of total mass, bear resemblance to the characteristic radius of about 100 kpc for the dark halo (Turner 1976). Our estimate of a total mass for the case 1 also support the argument that the masses of individual galaxies (1×10^{12} $M_\odot$) based on the orbital parameters of binary galaxies is about 3~4 times greater than those based on the traditional rotation curve (Peterson 1979). Though the halo mass and M/L for the case 3.3 is less consistent with Turner's (1976) and Peterson's (1979) values than those for the case 1, it is clear that the significant fraction (~50% of a total mass) of a total mass is in the dark halo beyond the optical image of NGC 6946, and M/L of 37 for the dark halo is significantly higher than the total M/L and conventional value of M/L for spiral galaxies. M/L of the optical image, nuclear bulge and optically seeing disk is very close to the conventional value of ~ 4 (Voigt 1982) for spiral galaxies.

Consequently, our estimates of the mass, luminosity, and M/L of NGC 6946, an isolated spiral galaxy, support the massive halo hypothesis without any confusions for the mass and conventional M/L of the optical image of spiral galaxies vigorously. The fact that the calculated masses and M/L in NGC 6946 are almost independent on the shape parameter n of the rotation curves reinforces the massive halo hypothesis more strongly.

We acknowledge support from the Korea Science & Engineering Foundation for this work.

References

Ables, H. D. 1971, Publ. *U. S. Naval Obs.* Part IV.

Basu, B., Bhattacharay, T., and Sen Gupta, Anuva. 1980. Bull. *Astron. Inst. Czechosl.*, **31**, 160.

Blackman, C. P. 1977, *M.N.R.A.S.*, **178**, 15.

———. 1979a, *M.N.R.A.S.*, **186**, 701.

———. 1979b, *M.N.R.A.S.*, **186**, 717.

———. 1979c, *M.N.R.A.S.*, **188**, 93.

———. 1984, *M.N.R.A.S.*, **208**, 91.

Bosma, A. 1981, *A. J.*, **86**, 1825.

Brandt, J. C. 1960, *Ap. J.*, **131**, 293.

Brandt, J. C., and Belton, M. J. S. 1962, *Ap. J.*, **136**, 354.

Brandt J. C., and Scheer, L. S. 1965, *A. J.*, **70**, 471.

Burstein, D., Rubin, V. C., Thonnard, N., and Ford, W. K., Jr. 1982, *Ap. J.*, **253**, 70.

Buta, R. J. 1982, *A. J.*, **94**, 578.

de Vaucouleurs, G. 1958, *Ap. J.*, **128**, 465.

———. 1960, *Ap. J.*, **131**, 265.

———. 1971, Publ. *A. S. P.*, **83**, 113.

de Vaucouleurs, G., de Vaucouleurs, A., and Corwin, H. G. 1976, *Second Reference Catalogue of Bright Galaxies* (Austin: University of Texas Press).

Fish, R. A. 1961, *Ap. J.*, **134**, 880.

Freeman, K. C. 1970, *Ap. J.*, **160**, 811.

Gordon, K. J., Remage, N. H., and Roberts, M. S. 1968, *Ap. J.*, **154**, 845.

Hodge, P. W. 1966, *An Atlas and Catalogue of H II regions in Galaxies* (Seattle: University of Washington, Astron. Dept.)

Kim, S. W., and Chun, M. S. 1984, *Jour, Kor. Ast. Soc.*, **17**, 23.

Nordsieck, K. H. 1973, *Ap. J.*, **184**, 719.

Ostriker, J. P., and Peebles, P. J. E. 1973, *Ap. J.*, **186**, 467.

Peebles, P. J. E. 1979, p.213, *in Physical Cosmology: proceedings of the Les Houches Summer School 1979*, ed. R. Balian *et al.* (New York: North-Holland).

____________. 1980, *The Large-Scale Structure of the Universe* (Princeton: Princeton University Press).

Peterson, S. D. 1979, *Ap. J.*, **232**, 20.

Rogstad, D. H., and Shostak, G. S. 1973, *Astron. Astrophys.*, **22**, 111.

Sandage, A. R., and Tammann, G. A. 1974, *Ap. J.*, **194**, 559.

Schmidt, M. 1965, P. 513, *In Galactic Structure*, ed. A. Blaauw and M. Schmidt. (Chicago: University of Chicago Press).

Turner, E. L. 1976, *Ap. J.*, **208**, 304.

van den Bergh, S. 1960, *Ap. J.*, **131**, 215.

Voigt, H. H. 1982, p.252, *in Numerical Data and Functional Relationships in Science and Technology Group* IV, Vol. 2-c, ed. K. Schaifers and H. H. Voigt (Berlin: Springer-Verlag).

Wyse, A. B., and Mayall, N. U. 1942, *Ap. J.*, **95**, 24.