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Optimal Replacement of Biomass for Maximizing Gas Production

Hwa-Ki Lee*

Abstract

Biomass conversion processes have the potential for satisfying approximately 25% of the national demand for methane gas. At the current time very little analytical work has been done to optimally design and operate the production facilities associated with these processes. This study was motivated by the high cost of these proposed systems. The biomass in storage decays (exponentially) with time while the batch methane production rate decreases (exponentially) over time. The basic problem is to determine the optimal residence times for batches in the anaerobic digester to maximize total production over a fixed planning horizon. The analysis characterizes the form of the optimal policy and presents efficient algorithm for obtaining this solution.

1. Introduction

In anticipation of future world energy and food shortages, scientists have been studying methods for producing energy (in this analysis methane gas) from the residual biomass from food producing crops. For example, the grain of sorghum is used as cattle feed while the stalks can be used as a feedstock in methane producing anaerobic conversion systems. Of course, crops may be grown solely for conversion purposes.

Conservative estimates indicate that approximately one-fourth of the national annual demand could be met by biomass to methane conversion systems (Isaacson *et al.* 1984). Recent research efforts have been orientated towards anaerobic digestion methods for large scale biomass to methane production systems. The Gas Research Institute has funded several large research projects with emphasis on biomass from such sources as kelp, sugarcane, water hyacinth, napier grass and sorghum as feedstocks (Mishoe *et al.* 1983 and Sweeten *et al.* 1984). For large scale production systems, biomass storage and conversion will most likely occur at the production facility. An anaerobic conversion procedure which has received considerable attention in these large research projects relies on a batch production process (Sofer and Zaborsky 1981). With this method, the digester is filled with biomass, an inoculum such as swine manure is added, and the digester sealed and left to produce methane by microbial action. The rate of gas production from a digester declines over time as the structural and nonstructural carbohydrates in the biomass are broken down and converted to methane.

* Dept. of Industrial Engineering, Inha University

Because of the slow rate of methane production, the fixed capacity of the digesters, and the single harvest of the agricultural crop providing the biomass, |biomass must be stored during the conversion planning horizon. Since stored biomass degrades significantly over time, the batch timing replacement policy can have a major effect on the total gas produced from a fixed quantity of biomass. Storage decay is not as critical for systems that can be continuously harvestsd throughout the year such as water hyacinth. However, for agricultural crops such as sorghum which have a short term harvest period, the decay of stored biomass quality over time is an important factor affecting total gas proudction.

The major shortcoming of biomass to methane coversion systems at the current time is the cost per unit of gas produced. To become competitive with current petroleum methane sources, considerable research directed towards improving the total methane production system is needed. Thus, it is imperative that optimal production designs and operating procedures be obtained for these systems. In this paper, we derive the optimal timing of batch replacements for a system where the total biomass is available at the beginning of the production period. A recursive equation for the optimal batch timings for a fixed number of fixed sized batches is developed.

2. Problem Description

The kinetics of a digestion process are critical in determining the rate of gas production per volume of digester. A description of digestion kinetics was obtained using chemical reaction engineering theory by Chynoweth *et al.* (1981). In summary, the rate of gas production for the batch process was found to decrease with time and is proportional to $\alpha e^{-\beta t}$ where t is the time in the batch digester, and α and β are biomass-gas conversion coefficients which depend upon the biomass, innoculum and digester. The actual production rate obtained from a batch of size b is proportional to b; that is, a batch of size b produces methane at rate $b\alpha e^{-\beta t}$. The unit cumulative methane production for a fresh batch of biomass that has been in the digester for time t is given by the function

$$g(t)=(\alpha/\beta)\ (1-e^{-\beta t}),$$

and the total gas produced from a batch of size b is $bg(t)$. Hashimoto *et al.* (1981) and Hashimoto (1983) propose a rectangular hyperbola form for the cumulative methane production from anaerobic digestion processes. Their experimental results essentially support the above exponential· form.

Biomass, once it has been harvested, decays in quality as a function of the time since harvest, and this decay rate depends upon the storage process. This rate is well approximated by the exponential function

$$h(t)=e^{-rt}$$

where t is the length of time since harvest and γ is a parameter depending upon the storage prosess. The analysis that follows is based upon the additional assumption that β is greater than γ, which insures that the decay occurs at a slower rate than gas production. Otherwise, a storage-batch production system would have no merit.

The production and decay components interact by decreasing the quality of subsequent batches loaded into the digester. The first batch will have a quality level of 1 and the gas produced during the residence time t_1 is thus $bg(t_1)$. The second batch, however, will have decayed in storage to a quality level $h(t_1)$ while the first batch is being processed. Thus, the production obtained by the second batch processing time t_2 will be $bh(t_1)g(t_2)$. The third batch will produce the gas quantity $bh(t_1+t_2)g(t_3)$, etc. The basic decision variables then for the production process are the digester residence time t_1. Figure 1 illustrates the general behavior of the gas production system over the planning horizon [0, T].

This paper addresses one problem related to the residence times. In the proplem, it is assumed that a digester has already been designed to operate with a fixed capacity, c, using a given biomass supply, s. Furthermore the number of batches, $n(=s/c)$, is assumed to be an integer. The setting of the system parameters c, b, n, and s are design considerations based on the relative costs associated with the building, maintenance, and operation of the production and storage facilities and the cost of biomass. These considerations are beyond the scope of this particular study and are assumed to be fixed parameters. The problem is to determine the length of the scheduling interval for each of the n batches. The solution of this problem provides the information necessary to operate the digester under design conditions.

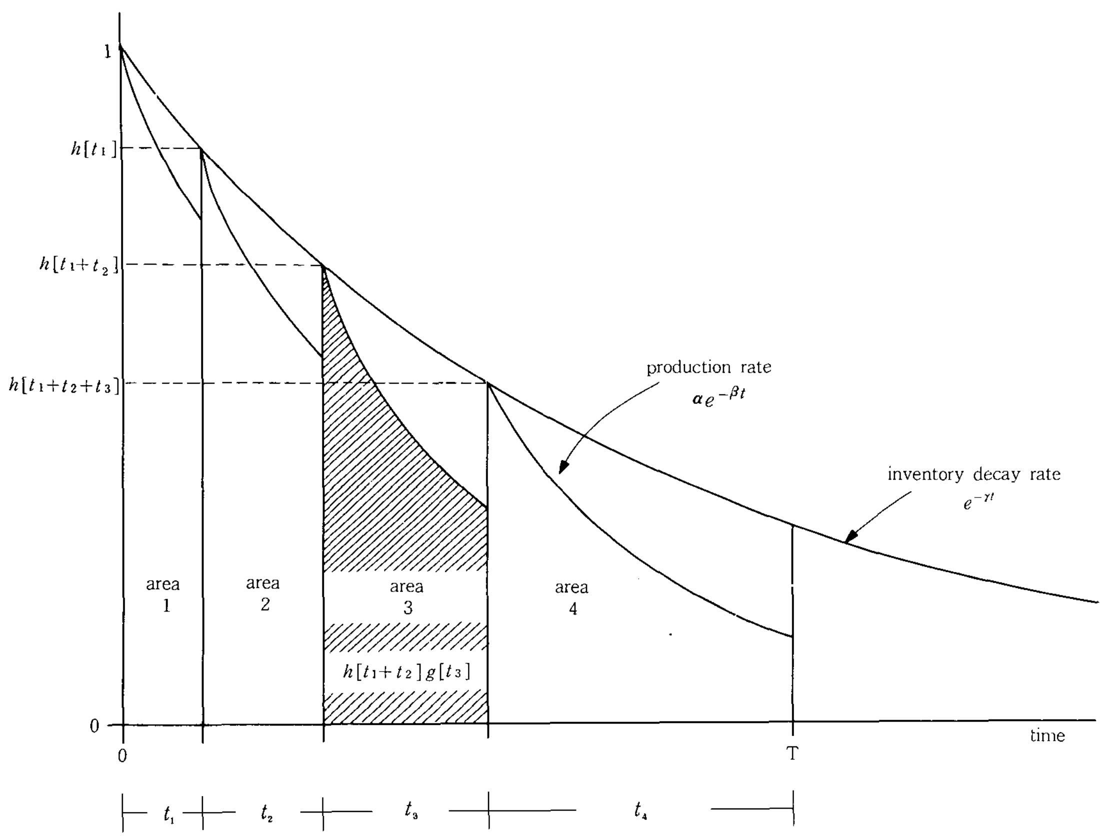

Fig. 1 : Illustration of the combined inventory decay and batch production processes. The shaded region represents the production from the third batch. The case illustrated is for a four batch system with $\alpha = 1$.

3. Analysis of the Design Problem

Assume that the digester has been designed and built with a fixed capacity, c, that a supply s (available at time $t=0$) is divided into n equal batches and that the n batches must be processed in time T (days). The problem is to determine the length of each of the n scheduling intervals so as to maximize the total gas production over T. Mathematically, the problem is to select the vector $t=(t_1, t_2, \ldots, t_n)$ so as to

$$\text{maximize } U(t, n)=g(t_1)+h(t_1)\, g(t_2)+\cdots+h(t_1+\cdots+t_{n-1})\, g(t_n)$$

$$\text{subject to } t_1+t_2+\cdots+t_n=T,$$

$$t_i>0, \quad \text{for } j=1, \cdots, n.$$

Then since total gas production is equal to the batch capacity c times $U(t, n)$, solving problem (Pl) will also maximize the total methane production over [0, T].

The solution to (P1) is given Theorem 1 below and is derived by solving the Kuhn-Tucker necessary conditions.

The statement of the optimal solution to (P1) requires a recursive procedure based upon the n−fold composition of a function with itself. Let f be a real valued function and define $f^{\circ(n)}(x)$ as follows :

$$f^{\circ(0)}(x)=x,$$

$$f^{\circ(1)}(x)=f(x),$$

$$f^{\circ(n)}(x)=f(f^{\circ(n-1)}(x)), \text{ for } n\geqq 2.$$

Of course, the above definition is valid only if the range and domain of the function f are consistent. Now, define the real·valued function $q(x)$ by

$$q(x)=1-w+wx^{1/w}, \textit{ for } 0\leqq x\leqq 1,$$

where $w=(\beta-\gamma)/\beta$. This function along with the composition operator are central to the statement of the optimal policy. The following theorem and corollaries establish the solution to (P1). The proofs are provided in the Appendix.

Theorem 1. For each positive integer n and positive T, there is a unique positive real number λ^* and positive vector t^* depending upon λ^* such that

$$\Sigma t_i^* = T,$$

$$t_i^* = [\ln q^{\circ(n-i)}(x^*)]/(\gamma-\beta), \text{ for } i=1, 2, \ldots, n, \qquad (1.1)$$

$$t_i^* > 0, \text{ for all } i, \qquad (1.2)$$

where $$x^* = (\lambda^* e^{\gamma T})/\alpha. \qquad (1.3)$$

To see how the solution works, suppose $n=3$. Then given the value of x^*, we can construct the batch residence timing solution as follows :

$$t_3^*=[\ln x^*]\ /(\gamma-\beta),$$

$$t_2^*=[\ln q(x^*)]/(\gamma-\beta),$$

$$t_1^*=[\ln q(q(x^*))]/(\gamma-\beta).$$

The solution t^* is easily determind once the value of x^* (and hence λ^*) has been specified.

Unfortunately, λ^* must be obtained using a numerical procedure because no closed form solution exists. However, Lemma 1 provides a finite interval of uncertainty known to contain λ^* so that any one of a variety of one-dimensional search procedures (*e.g.* grid search or binary search) will efficiently find λ^*. Essentially given a guess for λ, equations (1. 1)−(1. 3) are used to find the corresponding t; if $\sum t_j = T$, the procedure terminates. Otherwise, λ is increased (or decreased) and t is recomputed.

Lemma 1. For any positive integer n and positive T,

$$\alpha e^{-\beta T} \leq \lambda^* < \alpha e^{-\gamma T}.$$

Two interesting consequences of Theorem 1 are the following corollaries. These results state that the scheduling periods (batch residence times) are strictly increasing and the batch production quantities are strictly decreasing in value.

Corollary 1. For each positive integer n and positive T, the optimal scheduling vector satisfies

$$0 < t_1^* < t_2^* < \ldots < t_n^* \ ;$$

that is, successive batches remain in the digester for increasing lengths of time.

Corollary 2. For each positive integer n and positive T, the total gas produced from batch 1 is greater than the amount produced by batch 2, and in general, the amount of gas produced by batch j is greater than the amount produced by batch $j+1$, for all j.

The result in Corollary 2 is nonintuitive because it states that earlier batches produce more gas than later batches having shorter residence times. This phenomenon is a consequence of the fact that the stored biomass decays over time so that longer time intervals are required to produce comparable quantities of methane. However, it is not optimal to manage batches to obtain equivalent amounts of methane, nor to use equal residence times.

Algorithm 1. Let n be a positive integer, T be a positive real number, and assume that each of the n batches will fill the digester to capacity. The solution to (Pl) is obtained by the following procedure :

Step 0 : Set $\underline{\lambda} = \alpha e^{-\beta T}$ and set $\overline{\lambda} = \alpha e^{-\gamma T}$ (the bounds provided by Lemma 1). Select a termination tolerance, $\varepsilon < 1$.

Step 1 : Set $\lambda = (\overline{\lambda} + \underline{\lambda})/2$.

Step 2 : Set $x = \lambda e^{\gamma T}/\alpha$.

Step 3 : Compute :

$$t_j = [ln\ q^{\circ(n-j)}(x)]/(\gamma - \beta), \quad j = 1, 2, \ldots, n.$$

Step 4 : Compute SUM $= \sum t_j$. If $|T - \text{SUM}| \leq \varepsilon$ then stop ; otherwise, if SUM $> T$ then set $\overline{\lambda} = \lambda$ and return to Step 1 ; or if SUM $< T$ then set $\underline{\lambda} = \lambda$ and return to Step 1.

Example 1. Consider a system with digester capacity of 1000 tons of biomass and with production and decay paramenter values of $\alpha=10$, $\beta=0.07$ and $\gamma=0.0006$. We wish to schedule a biomass supply of 3000 tons using three batches over a 90 day time period ($T=90$). Equation (1) yields the optimal timings given that the Lagrange multiplier λ is known. Algorithm 1 determines λ at t by searching over the bounded range of λ given in Step 0. For illustration purposes, Table 1 lists the solution value for the system for various values of $\lambda \in[0.018, 9.474]$ with a grid step of 0.473. Also. included in Table 1 and denoted by bold type are entries for $\lambda^*=1.131$. This value was obtained by a binary search. The associated computations for t^* given $\lambda^*=1.131$ are :

$$q^{\circ(0)}(x)=(1.131/10)\ exp\{(0.0006)90\}=0.1194,$$
$$q^{\circ(1)}(x)=q(x)=10/0.07+(1-0.0006/0.07)(1.19)\ exp\{0.07/0.0694\}=0.125,$$
$$q^{\circ(2)}(x)=q(0.125)=0.130,$$
$$t_3^*=ln(0.1194)/(-0.694)=30.62,$$
$$t_2^*=ln(0.125)/(-0.694)=29.99, \text{ and}$$
$$t_1^*=ln(0.130)/(-0.694)=29.39.$$

Notice that $t_1^*+t_2^*+t_3^*=90$. The total gas produced is 369, 457m^3.

5. Conclusions

This paper has introduced a new type of perishable inventory and production problem, corresponding to the production of methane gas by anaerobic conversion of biomass. One optimization problem was formulated and analyzed. In this case the optimal policy was characterized and a simple numerical algorithm was provided. This analysis has demonstrated that not only can a complex applications problem be solved using analytic methods but in adition the solution can be shown to have remarkably simple structure. The results are novel, and hopefully they will stimulate more research in this area as well as contribute to the anaerobic conversion methodology.

Table 1†

Example Results of Algorithm 1 for Various Values of the Lagrange Multiplier λ for a System with $s=3000$, $n=3$, $T=90$, $c=1000$, $\alpha=10$, $\beta=0.07$ and $\gamma=0.0006$.

	Batch Times (days)				
λ	t_1	t_2	t_3	$\sum t_i$	TOTAL GAS(m^3)
0.018	57.5	65.8	90.0	213.3	409, 365.10
0.491	39.3	40.9	42.6	122.8	396, 548.30
0.964	31.4	32.1	32.9	96.5	376, 210.60
1.131	**29.39**	**29.99**	**30.63**	**90.0**	**369, 457.00**
1.437	26.3	26.7	27.2	80.2	356, 885.40

1. 910	22. 5	22. 8	23. 1	68. 4	337, 045. 10
2. 382	19. 5	19. 7	19. 9	59. 1	316, 873. 50
2. 855	17. 0	17. 1	17. 3	51. 4	296, 464. 30
3. 328	14. 9	15. 0	15. 1	44. 9	275, 873. 20
3. 801	13. 0	13. 1	13. 2	39. 3	255, 135. 80
4. 274	11. 4	11. 4	11. 5	34. 3	234, 276. 80
4. 746	9. 9	9. 9	10. 0	29. 8	213, 314. 30
5. 219	8. 5	8. 6	8. 6	25. 7	192, 262. 10
5. 692	7. 3	7. 3	7. 3	22. 0	171, 130. 70
6. 165	6. 2	6. 2	6. 2	18. 5	149, 928. 80
6. 638	5. 1	5. 1	5. 1	15. 4	128, 663. 10
7. 110	4. 1	4. 1	4. 1	12. 4	107, 339. 60
7. 583	3. 2	3. 2	3. 2	9. 6	85, 963. 06
8. 056	2. 3	2. 3	2. 3	7. 0	64, 537. 54
8. 529	1. 5	1. 5	1. 5	4. 5	43, 066. 65
9. 002	0. 7	0. 7	0. 7	2. 2	21, 553. 39
9. 474	0. 0	0. 0	0. 0	0. 0	0. 51

† The values associated with λ^* are in bold-face.

Appendix

The following notational conventions will be used thoughout this Appendix :

$$\sum_{k=u}^{v} f(k)=0, \text{ If } v<u,$$

$$\prod_{k=u}^{v} f(k)=1, \text{ If } u<v,$$

for any expression f. Also, $x^k=0$ If $x=0$ and $k<0$.

Proof of Theorem 1. For notational convenience define G by

$$G(t) = \sum_{i=1}^{n} g(t_i)\, h\Big(\sum_{k=1}^{i-1} t_k\Big).$$

It is easy to verify the following identities, for $u,\ v\geq 0$:

$$\begin{aligned} &h(u+v)=h(u)\,h(v),\\ &h'(u)=\gamma h(u),\\ &g'(u)=\alpha e^{-\beta u}, \text{ and} \\ &g''(u)=-\beta g'(u). \end{aligned} \tag{A1}$$

It is not difficult to use properties $(A1)$ to demontrate that the following relations are equivalent to the necessary conditions for (P1) :

$$h(-t_n^*)g'(t_n^*)=\lambda h(-T), \text{ and}$$

$$h(-t_j^*)g'(t_j^*)=g'(t_{j+1}^*)+\gamma g(t_{j+1}^*), \text{ for } j=1,\ldots,\ n-1.$$

Now,

$$h(-t_j^*)g'(t_j^*)=\alpha e^{(\gamma-\beta)t_j^*}$$

and

$$g'(t_{j+1}^*)+\gamma g(t_{j+1}^*)=\alpha q^{\circ(n-j)}(x^*),\ \text{for } j=1,2,\ldots,n-1,$$

Where $x^*=\lambda^* e^{\gamma T}/\alpha$. Hence, and equivalent set of relations is

$$e^{(\gamma-\beta)t_j^*}=\begin{cases} x^*, & j=n,\\ q^{\circ(n-j)}(x^*), & j=1,\ldots,n-1,\end{cases}$$

which uniquely define t^*.

At this point, a unique t^* exists that solves the Kuhn-Tucker conditions provided $\lambda^*>0$ exists and is finite, and the solution has the form stated in the hypothesis of Theorem 1. It remains to prove that this solution is a maximizer and that a finite and positive λ^* exists.

To prove that t^* is in fact optimal (*i. e.* tha Kuhn-Tucker condition are also sufficient) we show that the Hessian matrix of $G(\underline{t})$ at $\underline{t}^*$ is negative-definite. Using the properties $(A1)$ it is easy to show that the Hessian $\hat{H}$ of G at (t^*) is given by

$$\hat{H}_{ij}=\begin{cases} -\beta\lambda^*-\gamma(\beta-\gamma)\sum_{k=1}^{i-1} g(t_k^*)\,h(\sum_{u=1}^{k-1} t_u^*), & i=j;\\ -\gamma\lambda^*, & i\neq j.\end{cases}$$

Subtracting row n from every other row produces the matrix H given below, which is equivalent to the Hessian of G at (t^*) :

$$H_{ij}=\begin{cases} A_j, & i=j \text{ and } i=1,2,\ldots,n-1;\\ o, & j\neq i,\ i=1,2,\ldots,n-1,\ j=1,2,\ldots,n-1;\\ v, & i=1,2,\ldots,n-1,\ j=n;\\ -d, & i=n,\ j=1,2,\ldots,n-1,\end{cases}$$

where $v=(\beta-\gamma)>0$, $d=\gamma\lambda^*>0$ and

$$A_j=\begin{cases} -(\beta-\gamma)\,[\lambda^*+\gamma\sum_{k=1}^{j-1} g(t_k)\,h(\sum_{u=1}^{k-1} t_u^*)], & j=1,2,\ldots,n-1;\\ -\beta\lambda^*, & j=n.\end{cases}$$

Notice that $A_j<0$ for all $j=1,2,..,n$.

Now, define M_j to be the $j-$th order principal minor determinant of H, and and define D_{n-j} to be the determinant of the matrix formed from H by deleting the first j rows and columns of H. Then it is easy to show

$$M_j=(-1)^j\prod_{k=n-j+1}^{n}|A_u|,\ j=1,2,\ldots,n-1,$$

$$D_j=(-1)^j[\,|A_j D_{j-1}|+vd\prod_{u=2}^{j-1}|A_u|\,],\ j=2,3,\ldots,n, \text{ and}$$

$$M_n=(-1)^j\,[\,|A_n D_{n-1}|+vd\prod_{u=2}^{n-1}|A_u|\,].$$

Therefore, the principal minor determinants of odd order are negative and of even order are positive. Tuhs, Hessian is negative definite at $\underline{t}^*$, and (t^*) is an optimal solution to $(P1)$.

We now must establish the existence and uniqueness of the proposed solution. Since we used the fact that $\sum t_j = T$, then x must satisfy

$$Q(x) = \prod_{j=0}^{n-1} q^{\circ(j)}(x) = \prod_{j=1}^{n} e^{(\gamma-\beta)t_j^*} = e^{(\gamma-\beta)T},$$

where $x = \frac{\lambda}{\alpha} e^{\gamma t}$. If such an x exists and is unique then so is λ^* and t^*.

The following four lemmas establish critical properties of Q and $q^{\circ(j)}$, and are needed to complete the proof of Theorem 1.

Lemma A1.

1. $q(u)$ is strictly increasing for $u \geq 0$.
2. $q(u) - u$ is strictly decreasing (increasing) on $[0, 1]$ ($(1, \infty)$).
3. $q(1) = 1$ and $q(0) = \frac{\gamma}{\beta}$, so that $q(u) > u$ on $(0, 1)$.

Proof.

1. $q'(u) = u^{\frac{\gamma}{\beta-\gamma}} > 0$ for all $u > 0$.
2. $q'(u) - 1 < 0$ for $u \in (0, 1)$ and the result is immediate.
3. Obvious.

Lemma A2. Let j be any positive integer and u any nonnegative real number. Then,

1. $$\frac{d}{du} q^{\circ(j)}(u) = \begin{cases} 1, & j = 0; \\ \prod_{k=0}^{i-1} q'(q^{\circ(k)}(u)), & j \geq 1. \end{cases}$$

2. $q^{\circ(j)}(u)$ is increasing for $0 < u \leq 1$.
3. $q^{\circ(j)}(u) > u$ for $0 < u \leq 1$.

Proof. Trivial.

Lemma A3.

1. Q is continuously differentiable over $[0, 1]$.
2. Q is strictly increasing over $[0, 1]$.
3. Let $\underline{u} \in (0, 1)$, then there is a unique $\underline{u} \in [u, 1]$ such that $Q(u) = \underline{u}$.

Proof.

1. $Q'(u) = 1/2 \sum_{i=1}^{n} \sum_{j \neq i} \sum_{k \neq i, j} \frac{d}{du}(q^{\circ(i)}(u))\, q^{\circ(j)}(u)\, q^{\circ(n)}(u) > 0.$

2. $Q(u) = \prod_{j=0}^{n-1} q^{\circ(j)}(u) < q^{\circ(0)}(u) = u$, since $q^{\circ(j)}(u) < 1$ for all $u \in (0, 1)$.

Since Q is strictly increasing and continuous, there is a unique $u \in [\underline{u}, 1]$ such that $Q(u) = \underline{u}$.

Proof of Theorem 1, Continued. Now, let $\underline{u} = e^{(\gamma-\beta)T}$ in Lemma A3. Then we see there is a unique u, say $\hat{u}$ such that $Q(\hat{u}) = \underline{u}$; put differently, there is unique $x(\lambda)$ such that $Q(x(\lambda)) = e^{(\gamma-\beta)T}$ and hence a unique Lagrange multiplier λ^*

Proof of Lemma 1. Since $0 < t_n^* < T$, $0 < \frac{n(x)}{\gamma-\beta} \leq T$ and so

$$e^{(\gamma-\beta)T} \leq x < 1, \text{ and}$$
$$\alpha e^{-\beta T} < \lambda < \alpha e^{-\gamma T}.$$

Proof of Corollary 1. Let $x^* = x(\lambda^*)$,
so that
$$e^{(\gamma-\beta)} t_j^* = q^{\circ(n-j)}(x^*), \text{ and}$$

$$\begin{aligned} e^{(\gamma-\beta)t_j^*} - e^{(\gamma-\beta)t_{j+1}^*} &= q^{\circ(n-j)}(x^*) - q^{\circ(n-j-1)}(x^*) \\ &> q^{\circ(n-j-1)}(x^*) - q^{\circ(n-j-1)}(x^*) \\ &= 0, \end{aligned}$$

which implies that $t_j^* < t_{j+1}^*$ as desired.

Proof of Corollary 2. Let j be an arbitrary period index. The difference in methane produced in periods j and $j+1$ is

$$h(\sum_{k=1}^{j-1} t_k)\ [g(t_j) - h(t_j)g(t_{j+1})],$$

where t is the optimal vector of scheduling periods. All that is needed is to show the term within the brackets is positive. Toward this end, define the function

$$f(v) = 1 - q(v)^{\ell} - q(v)^{\ell-1}[1 - v^{\ell}], \text{ for } 0 \leqq v \leqq 1,$$

where $\ell = \beta/(\beta-\gamma) > 1$. A simple derivative argument show that $f'(v) < 0$ and thus f is strictly decreasing on [0, 1]. Because $f(1) = 0$, then $f(v) > 0$ for $0 \leqq v < 1$. Now, using $v = q^{\circ(n-j)}(x)$,

$$g(t_j) - h(t_j)g(t_{j+1}) = \frac{\gamma}{\beta} f(v) > 0.$$

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