Journal of the Korean
OR/MS Society
Vol. 10, No. 1, Apr. 1985

—Research Paper—

Influence Diagram Approach for Strategic Decision Structuring Process

Gi-Hyo Kim*
Soung-Hie Kim*

Abstract

The influence diagram is a new conceptual tool that can be used for structuring a strategic decision problem in decision analysis. It has a graphical representation of probabilistic dependence among variables in the decision problem. In this paper formal procedures for constructuring the influence diagram and for translating it into the corresponding decision tree are studied. An example that shows the power of the infuence diagram is shown.

In many strategic decision problems, two major aspects are uncertainty and complexity. Because of uncertainty strategic decision makers almost never know the exact consequences of choosing an alternative at the time that they make a decision. Also, for the complexity which means the existence of large number of variables, it is difficult for decision makers to understand precise situation of the problem.

To communicate with these two characteristics, the first step we can take is to identify the variables in the problem and to investigate their relationships. That is, the primary function of decision maker or decision analyst is to capture the relationships among many variables in a decision problem, a process called structuring.

There are several structuring tools associated with solving decision problems. Sage (1977), Hill and Ollila (1978, 193) introduced the interaction matrix. This method indicates the existence or non-existence of interactions in a matrix form rather than diagramatically. Miller, et al (1981) presented a structuring tool called function graph which is composed of entities and operators. The function graph is used to graphically represent the decision model's deterministic structure. Diffenbach (1982) developed the concept of influence diagram which is practical desk-top tool for mapping complex strategic issues so as to make the issues more comprehensive than otherwise. It represents causal and feedback relationships of variables. It is a good qualitative methodology useful for mapping strategic issues made complex by interaction among a variety of economic, technological, social and political factors. Though it

* Department of Industrial Engineering,
Korea Advanced Institute of Science and Technology

is useful to understand the situation of the complex problem, it requires some modifications to be applied to decision analysis. For instance, it cannot be applied directly to decision tree analysis since it has loops. As an auxiliary aid of structuring, Merkhofer and Leaf (1981) developed a manual simulation device, called simulation board. The simulation board is a manual war game for simulating force movement and engagement outcomes resulting from a given course of action. It does not provide a way of describing the independencies among random variables. The necessity to search the tree to discover dependencies makes it difficult to visualize or alter the independence assumptions in the decision tree formed.

As an efficient concept, influence diagram has been developed in decision analysis (Howard, et al, 1980). Influence diagram is a graphical device specifically designed to summarize the probabilistic dependencies that exist among the variables in a decision problem. Structuring strategic decision problem with influence diagram provides several advantages as follows: First, influence diagrams have considerable intuitive appeal apparently because their graphical representations correspond closely to the way many decision makers conceptualize their problems. Second, since influence diagrams are relatively simple decision makers can easily understand the diagrams. Third, influence diagrams facilitate communication between the individuals who are involved in the decision and provide a mechanism for identifying differences of opinion and resolving them.

Influence diagrams provide a good tool for structuring decision problem under uncertainty. From the practical viewpoint, how can we construct influence diagram precisely and how can we translate influence diagram into corresponding decision tree to solve the decision problem? In this work formal procedures for constructing influence diagrams and translating them into corresponding decision trees are developed.

1. Introduction of Influence Diagrams

The reason for using influence diagram is that it can serve at the three levels of specification of relation, function, and number, and in both deterministic and probabilistic cases. In the deterministic case, relation means that one variable can depend in a general way on several others: for example, profit is a function of revenue and cost. At the level of function we specify the relationship: namely, that profit equals revenue minus cost. Finally, at the level of number, we can specify the numerical values of revenue and cost and hence determine the numerical value of profit.

In the probabilistic case, at the level of relation we mean that given the information available, one variable is probabilistically dependent on certain other variables and probabilistically independent of still other variables. At the level of function, the probability distribution of each variable is assigned conditioned on values of the variables on which it depends. Finally, at the level of number, unconditional distributions are assigned on all variables that do not depend on any other variable and hence determine all joint and marginal probability distributions (Kim, 1982).

In this work, we shall focus on the probabilistic use of influence diagrams since the deterministic use is a special case of the probabilistic.

Influence diagrams represent the dependencies among state and decision variables. State variable is represented by a circle containing its name or number and decision variable by a square. An arrow pointing from variable A to variable B means that the outcome of variable A can influence the probabilities associated with variable B. The direction of the arrows in an influence diagram is significant and in general they cannot be reversed without it changing the independence assumptions implied by the diagram, even though, dependence is inherently non-directional. But, without changing the independence assumptions, we can have many alternative representations of an influence diagram. The two rules about manipulations are:

(1) An arrow can always be added between two nodes as long as no loops are created.

(2) An arrow joining two nodes in an influence diagram can be reversed provided that all probability assignments are based on the same set of information. For instance, if variables A and B are based on the same variable C, then the arrow between A and B can be reversed by the probability expansion rule as follows:

$$Pr(A, B|C, s) = Pr(A|C, s) \quad Pr(B|A, C, s)$$
$$= Pr(B|C, s) \quad Pr(A|B, C, s)$$

where $Pr(X|Y, s)$ means conditional probability of event X, given event Y and state of information s.

Though the alternative forms of influence diagram are logically equivalent, they again differ in their suitability for assessment purposes. In large decision problems, the influence diagram can display the needed assessments in a very useful way. And influence diagram is a directed graph having no loops since it could not represent any possible expansion order. Naturally, influence diagram contains two types of nodes: decision nodes represented by boxes (□) and chance nodes represented by circles (○). Influences on a decision node represent a basic cause/effect ordering whereas influences into a chance node represent a somewhat arbitrary order of conditioning that may not correspond to any cause/effect notion and that may be changed by application of Bayes' rule. Furthermore, influence diagram asserts that the only information available when any decision is made is that represented by the direct predecessors of the decision. For a given chance node x, if we let Nx be the set of all non-successors of x and Dx be the set of direct predecessors of x, then the influence diagram asserts that $Pr(x|Nx, s) = Pr(x|Dx, s)$ in the decision tree.

In influence diagrams, two concepts related to the decision tree are decision network and decision tree network:

A decision network is an influence diagram:

(1) that implies a total ordering among decision nodes,

(2) where each decision node and its direct predecessors directly influence all successor decision nodes.

A decision tree network is a decision network:

(3) where all predecssors of each decision node are direct predecessors.

For an influence diagram to have a corresponding decision tree, it must satisfy the conditions (1) and (2). Especially, for the decision tree network, the condition (3) assures that no probabilistic processing is needed so that a decision tree can be constructed in direct correspondence with the influence diagram.

Based on this introduction of influence diagrams, formal procedures for constructing influence diagrams and translating them into corresponding decision trees are studied respectively.

2. Procedure for Constructing Influence Diagrams

The mostefundamental and important thing is how we practically elicit an influence diagram which precisely represents the decision maker's perception about the situation of a decision problem. An influence diagram may be constructed by the decisiòn maker himself or a decision analyst. For complex problem, the decision analyst can construct the influence diagram through an interactive interview with the decision maker as the subject. Once the subject is satisfied that the influence diagram accurately represents the influences among the variables in his problem, the decision analyst will attempt to translate the influence diagram into a decision tree.

The construction of the influence diagram consists of identification of system variables and their relationships. The system variables are composed of outcome variables, decision variables, and aleatory state variables. Though the fixated variable is also a system variable, it need not be included in the influence diagram. At first, the outcome variable which is associated with the objective of the decision should be specified. Then, the primary and downstream decision variables are listed, and then, the aleatory variables that represent the uncertainties in the problem are listed. The number of outcome variables may be either one or multiple. Usually, most problem have a single outcome variable which represents the value of a specific outcome. When there are several outcome variables, we need an appropriate value model to convert the influence diagram into corresponding decision tree. This value model indicates the relative importance of the outcome variable. The measures of outcome variable should be quantifiable, that is, they must be quantities that can be expressed by numbers.

After these system variables are listed their probabilistic dependencies should be specified. The assertions of probabilistic independencies among variables must also be carefully investigated because these independence assertions are important to reduce the number of conditioning variables.

The formal procedure for constructing an influence diagram and the checklist are given below.

step 1. Specify the outcome variable: The outcome represents the subsequent event that will determine the ultimate desirability of the whole issue. The number of outcome variables may be several.

step 2. List the decision variables and the aleatory variables: List all the variables relevant to the problem without concern for redundancy and relative order of importance.

step 3. Refine the list of variables: Eliminate the redundant variables and relatively trivial variables after careful considerations. As the number of variables increases the number of assessments increases rapidly and the problem becomes more complex. Therefore, this step is important for the appropriate size of the problem.

step 4. Specify the influences among the variables: To specify the probabilistic relationships, the decision maker can work backwards from the outcome variable to state variables. That is, direct predecessors of the outcome variable are specified, and then, direct predecessors of them are specified, etc. The direct predecessors of a variable are those which the decision maker would most like to know to reduce the uncertainty in that variable.

step 5. Review the diagram: Wrong influence arrows are corrected, the missing important variables and influences may be added.

For an influence diagram to be converted to the corresponding decision tree, it must satisfy the conditions to be a decision network. We list the corresponding checklist.

Checklist

1. Check that the influence diagram represent total ordering among variables. If not, it cannot be converted into the corresponding decision tree.
2. Check that loops exist. A loop represents no possible expansion order of the joint distribution and this situation cannot be represented in a decision tree.
3. Check that the decision node and its direct predecessors are also the direct predecessors of a subsequent decision node, which is the second condision to be a decision network. In addition to this condition, check that a decision node has some non-direct predecessors having no successors except that decision variable and its predecessors. Such non-direct predecessors may be used to assess the probabilities of the direct predecessors but they represent unobservable uncertainties when the decision is made. They must be removed in order to have a correct decision tree.

4. Procedure for Translating Influences into Decision Trees

In decision analysis, usually the final choice of the alternative is made through the decision tree analysis. So, the influence diagram needs to be translated into the decision tree. If an influence diagram is a decision network then it can be changed into a decision tree network after some probabilistic processing, and then it can be directly translated into a decision tree.

In Figure 1, the needed probabilistic processing from decision network to decision tree network is to assess the probability distributions $Pr(B|s)$ and $Pr(A|B,s)$ (in Figure 1(b)) from the probability distribution $Pr(A|s)$ and $Pr(B|A,s)$ (in Figure 1(a)) which are known initially to the decision maker. That is, the probability distribution of B is obtained as follows:

$$Pr(B|s)=\int_A Pr(A|s)\ Pr(B|A,s) \qquad \text{(using Expansion rule)}$$

and the conditional probability distribution of A, given B is derived as follows:

$$Pr(A|B,s)=\frac{Pr(A|s)\ Pr(B|A,s)}{Pr(B|s)} \qquad \text{(using Bayes' rule).}$$

From these rusults, we can obtain the probabilities in decision tree network, shown in Figure 1 (b). Then, the corresponding decision tree in Figure 1 (c) can be directly drawn from the decision tree network(Figure 1(b)).

The formal procedure for translating the influence diagram which is a decision tree network into the corresponding decision tree is presented as follows:

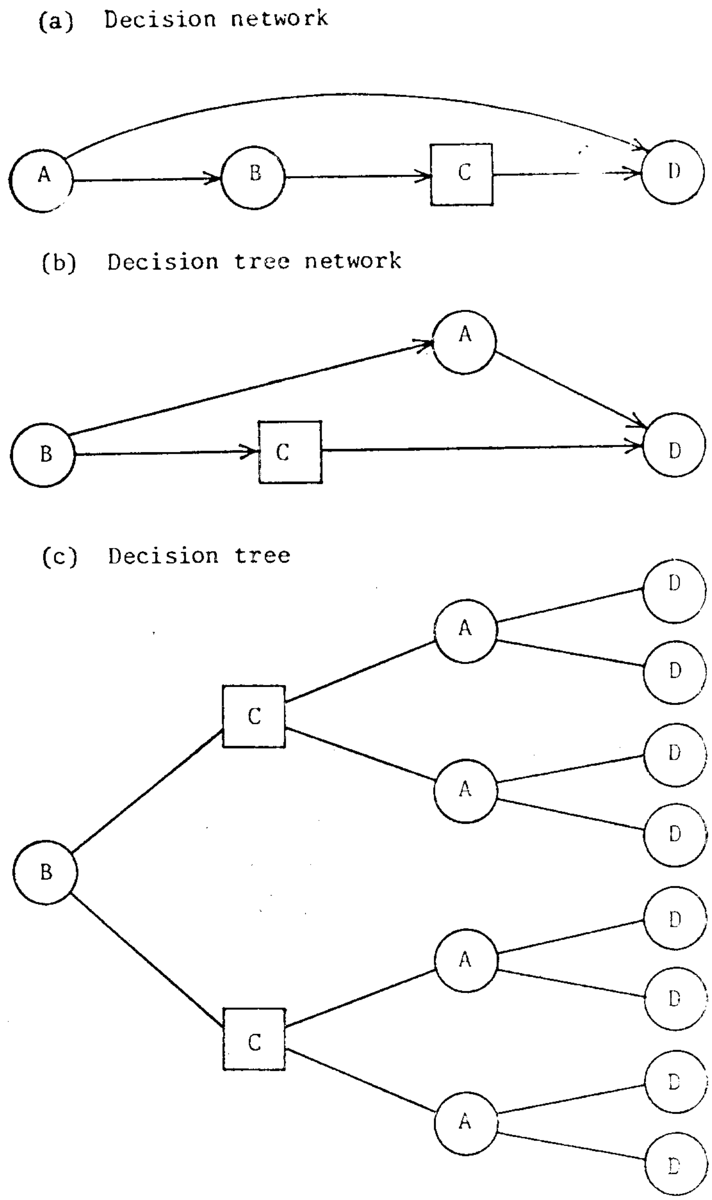


Figure 1. Change of an Influence Diagram into a Decision Tree Network and corresponding Decision Tree

step 1. Identify a node with no predecessors: Because the diagram is assumed to be a decision tree network, there will be no loops, and therefore at least one node will have no predecessors. One of the following three cases occurs.

(1) If there is only one such node, the aleatory variable or decision variable corresponding to this node is placed at the beginning of the decision tree.

(2) If there is a choice between decision variable and aleatory variable, decision variable must be selected.

(3) If all the nodes which have no predecessors are aleatory variables, then place at the beginning of the decision tree anyone of the variables that satisfies the condition (2) given above.

step 2. Once you have determined the first node in the decision tree, remove the correspo-nding node from the influence diagram along with all the arrows that leave this node. The reduced influence diagram contains at least one node which has no predecessors. Repeat step 1, The same procedure is repeated until all of the variables have been removed from the influence diagram.

5. Illustration

To see how the influence diagram is used in complex, uncertain decision problem, we shall apply it to the strategic decision problem of the oil drilling company. Though this example is a simplified form, it clearly shows the power of the influence diagram.

The company's primary decision is whether or not to drill at the given site. In addition, it wishes to determine whether or not to undertake any information gathering regarding the existence of the dome since the existence of the dome assures high probability of the oil existence. The outcome variable is the net profit of the company and the uncertainties are the total cost, the dome existence, the oil existence, and the quality of the oil. The primary decision problem can be formulated by drawing the influence diagram as in Figure 2.

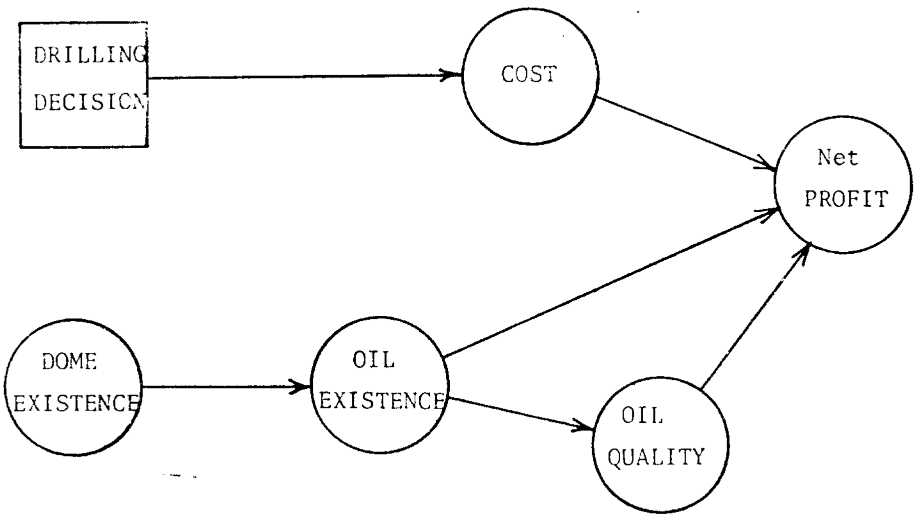


Figure 2. Influence Diagram for Primary Decision

In Figure 2 the drilling cost is dependent on the drilling decision. The oil existence is dependent on the the dome existence. The net profit is directly dependent on the drilling cost, the oil existence, and the oil quality. The oil quality is dependent on the oil existence in the sense that the probabilities of the oil quality can be assigned only when the oil exists

In addition, the dome existence, the oil existence and the oil quality is independent on the. drilling decision.

The next step is to obtain probability and value assignments corresponding to the influence diagram. The drilling cost will be one of low (\$90,000), moderate(\$120,000), andhigh (\$150,000) and so, the expected value of the cost is \$123,000. The dome existence and the oil existence have two outcomes, to exist or not and the outcomes of the oil quality are good, medium, and bad. When the oil exists, the net profit corresponding to each combinations of these outcomes are given in Table 1.

Table 1. Net Profits (\$ thousands)

Oil Quality	Cost		
	Low	Moderate	High
Bad	300	270	240
Medium	450	420	390
Good	600	570	540

The probability assignments are given in Figure 3.

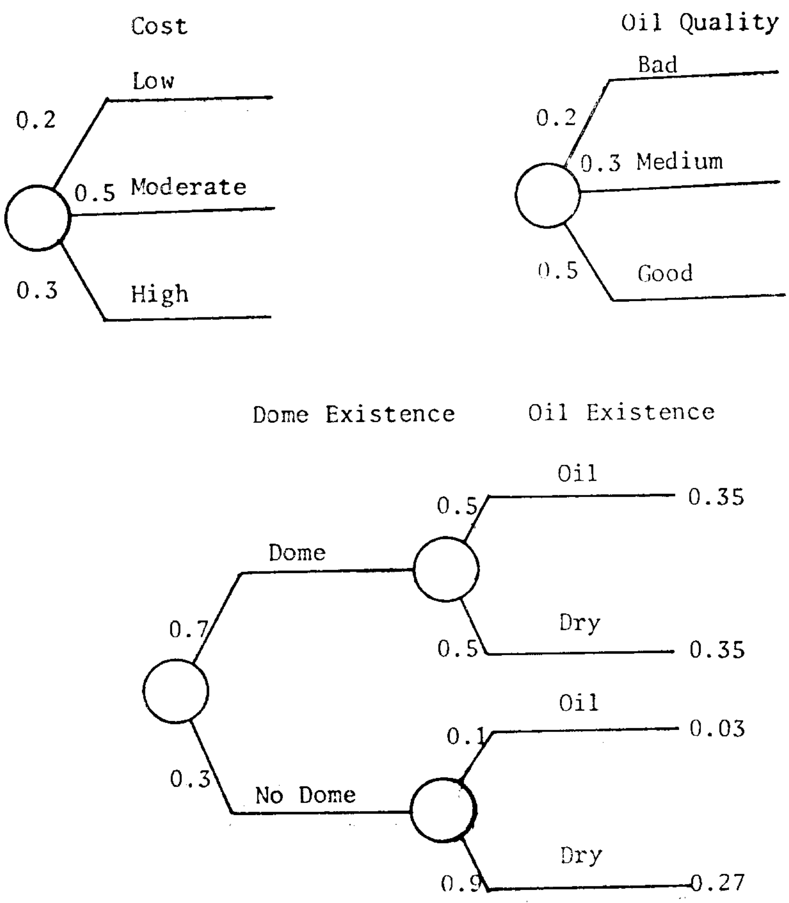


Figure 3. Initial Probability Assessment

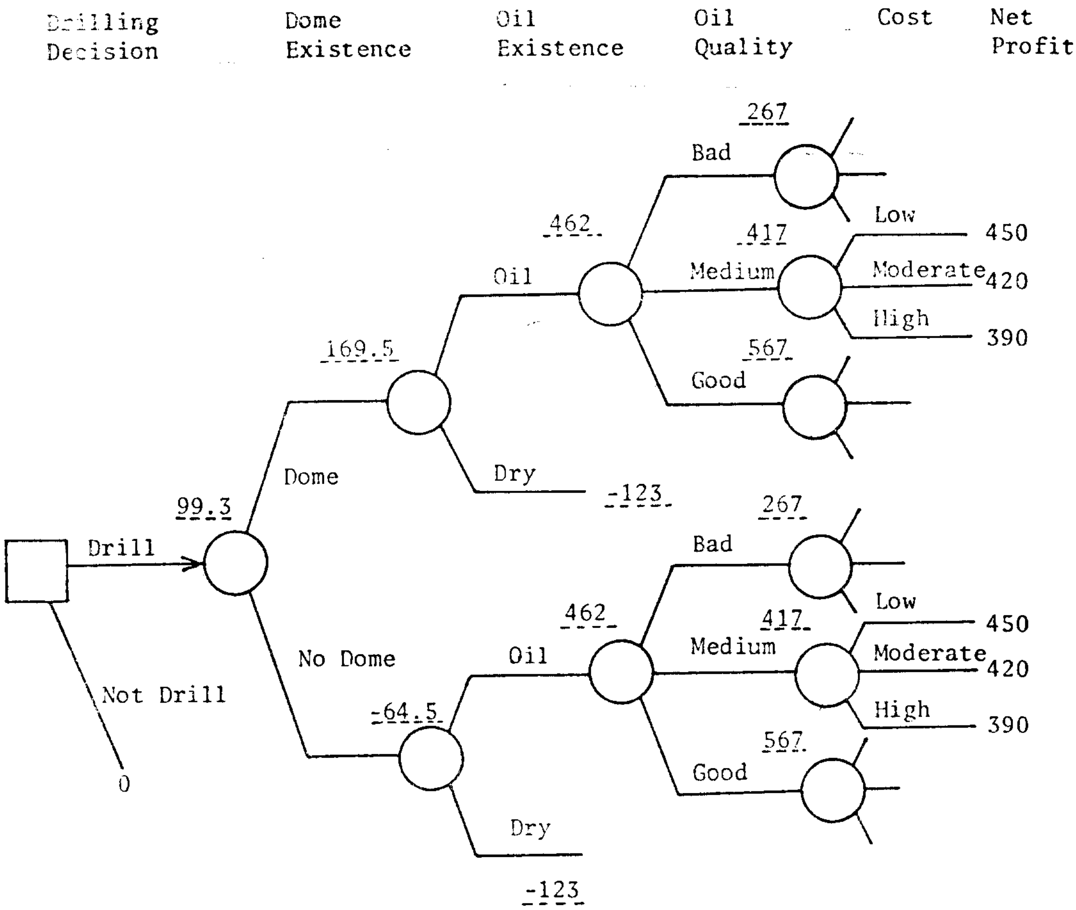
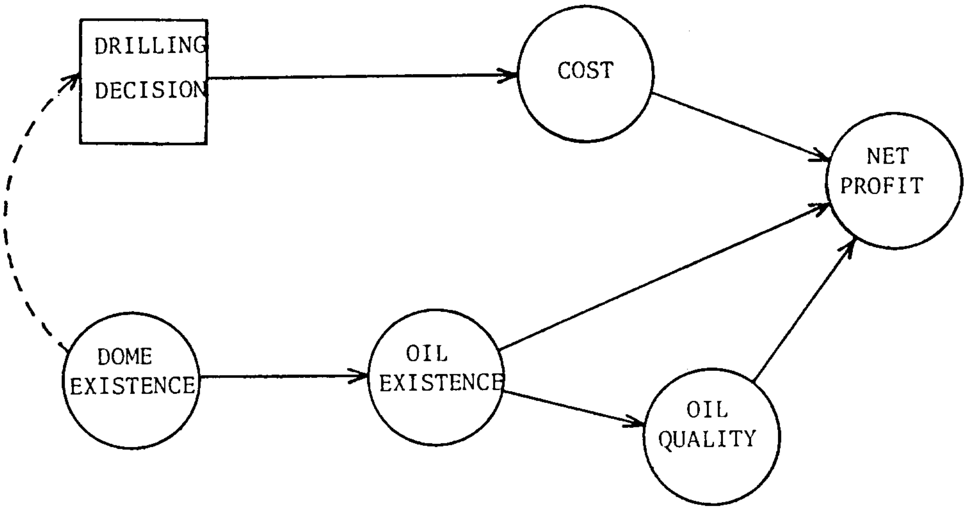


Figure 4. Decision Tree for Primary Decision

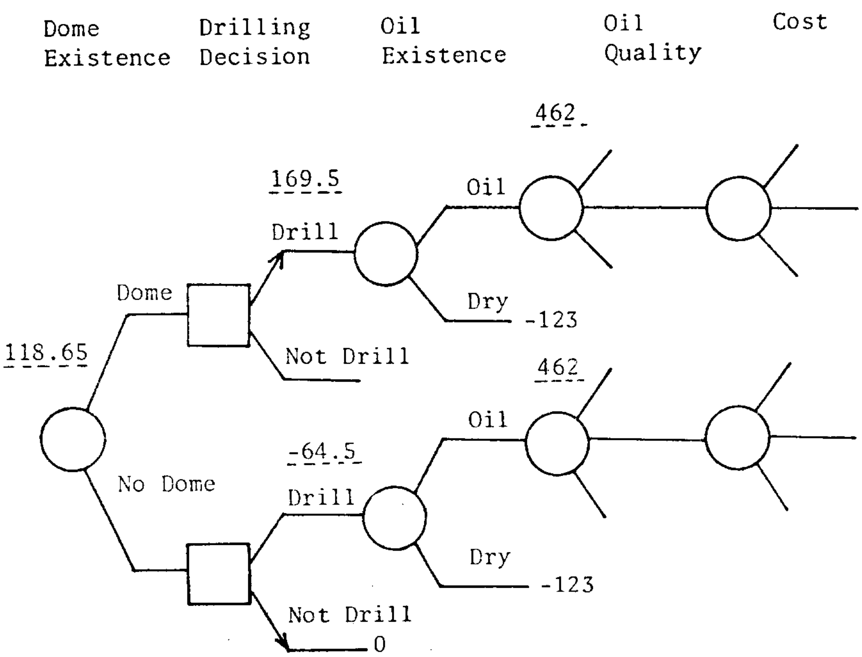
Then, the decision tree for the primary decision is given in Figure 4.
The best primary decision is to drill, and the expected value given this decision is $99,300.

Before investigating actual information gathering alternatives, the value of the perfect information about the dome existence can be calculated. In influence diagram, the influence arrow indicated by the broken line in Figure 5(a) should be added.
This modification states that the decision maker knows about the dome existence before he makes the decision. Figure 5(b) shows the corresponding decision tree. The decision rule is to drill if the dome exists, and not to drill if the dome does not exist. The expected value with the perfect information is $ 118,650 in Figure 5(b). And so, the expected value of the perfect information is $118,650 minus $99,300 which is $19,350.

Now, we consider the actual information gathering activity. We can take the seismic test



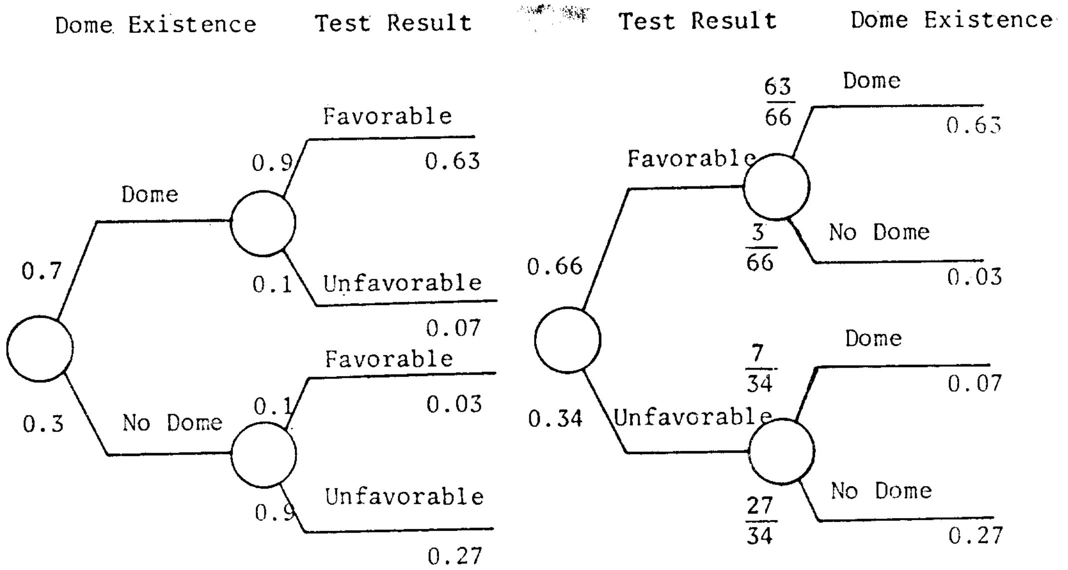
(a)



(b)

Figure 5. Modified Influence Diagram and corresponding Decision Tree

to investigate whether or not the dome exists in that site. The test result is not precise and is probabilistically dependent on the dome existence. That is, the test gives an imperfectinformation. The probabilities of the test result is given in Figure 6(a).



(a) Probabilities of test result given dome existence

(b) Probabilities of dome existence obtained by Bayes's rule

Figure 6. Probabilities

Figure 7 shows the new influence diagram involving the testing acsivity.

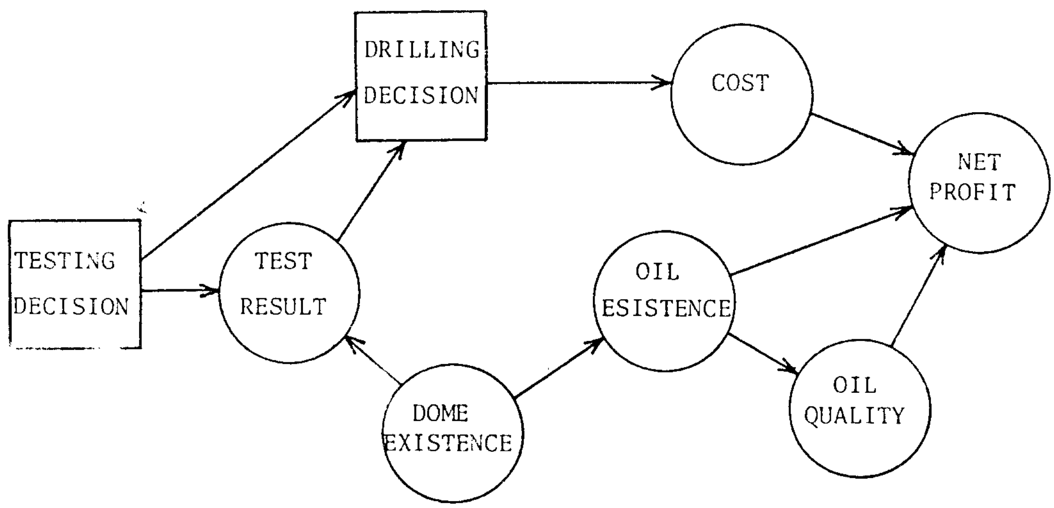


Figure 7. Influence Diagram to Determine the Value of Imperfect Information on Dome Existence

Note that the test result is directly dependent on the dome existence. Since this influence diagram is a decision network, but not a decision tree network, it cannot be directly translated into a decision tree. Using the Bayes' rule we can change the direction of the arrow between the test result and the dome existence, and then the resulting influence diagram will be a decision tree network. The result of calculations is summarized in Figure 6(b). The corresponding decision tree is given in Figure 8.

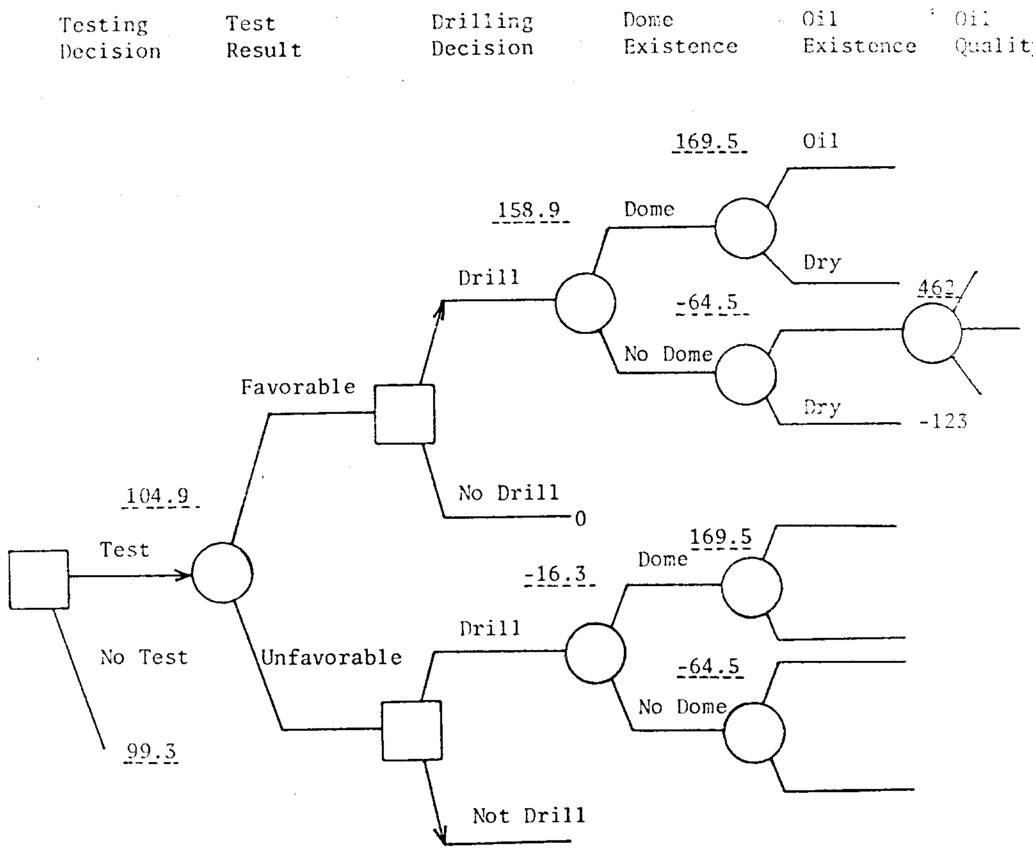


Figure 8. Decision Tree for Imperfect Information

The decision rule is to drill if the test is favorable and not to drill if the test is not favorable. The expected value is $104,900. Therefore, the value of the imperfection information of the seismic test is $104,900 minus $99,300 which is $5,600.

6. Conclusions

The influence diagram captures the logic of the decision problem in a fundamental way, and so, it makes the probabilistic mode ing and decision making process simple. The suggested procedures for constructing influence diagrams and corresponding decision trees will be a practical aid to solve a strategic decision problem with complexity and uncertainty.

Because of the simplicity of usefulness of the influence diagram, it can be easily understood by people in all degrees of technical proficiency. Especially, strategic decision makers can use it effectively as a tool for solving the problem directly or as a communication tool with the decision analysts.

References

1. Diffenbach, J., "Influence Diagrams for Complex Strategic Issues", *Strategic Management Journal*, Vol. 3, pp. 133-146, 1982.
2. Hill, J. D., and Ollila, R. G., "Analysis of Complex Decision Making processes", *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. SMC-8, No. 3, pp. 193-204, 1978.
3. Howard, R. A., et. al., *Influence Diagrams*, SRI International, Menlo Park, California 1980.
4. Kim, Soung H., *Markovian methodology for encoding and updating the prior probability assessment on dynamic processes*, Ph. D. Dissertation, Stanford University, 1982.
5. Merkhofer, M. W., and Leaf, E. B., *A Computer Aided Decision Structuring Process-Final Report*, SRI International, Menlo Park, California, 1981.
6. Miller, A. C., et. al., *Development of Automated Aids for Decision Analysis*, SRI International, Menlo Park, California, 1981.
7. Sage, A. P., *Methodology for Large scale Systems*, McGraw-Hill, Inc., 1977.