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Network Function Characterizing the General n-Line 2n-Port Coupled Transmission System

(일반화한 n선로 결합 전송구조의 회로망 함수)

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要 約

비균질성 매질(Inhomogeneous medium) 내에서 n-선로가 서로 결합한 전송선 구조의 이미탄스(Immittance)를 normal mode정수를 사용하여 구하는 방법을 제시했다.

이미탄스를 구하는 절차가 체계적이고 단순하여 공식화할 수 있어 수치계산에 적합하다. 4-선로 8-포트에 관한 이미탄스식을 예로 들었다.

Abstract

A general procedure for finding the immittances of a general, uniformly coupled, n-line structure in an inhomogeneous medium is presented. The expressions derived in terms of the normal modes of the system are in a convenient matrix form and can be used to compute or to derive the explicit expressions for the elements of the 2n-port immittance matrix. As an example, the closed form expressions for the elements of the admittance matrix of a symmetrical four-line eight-port structure are given.

I. Introduction

A coupled transmission line system is a set of transmission lines in which the voltages and currents of each line can influence the voltages and currents of all the others. The systems to be considered are lossless and uniform n-line structures in an inhomogeneous medium. The n-line 2n-port structure can be characterized in terms of an immittance matrix, a chain matrix, or scattering parameters. The immittance matrix is suitable for studying two-port circuits, while the scattering parameters are more suitable for studying multiport circuits, including couplers, since the immittance matrix of a coupler does not conveniently describe the coupler performance. The general forms of the immittance and chain matrices of an n-line structure were derived by Chang[1], Marx[2], and Paul [3,4]. The explicit closed expressions for non-symmetrical two line and symmetrical three-line structures were derived by Tripathi [5,6] in terms of the normal modes of the coupled system.

The expressions derived in terms of the normal mode parameters of an n-line coupled system are in a convenient matrix form and can be used to compute or to derive the explicit

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expressions for the elements of the 2n-port immittance matrix. These normal mode parameters are the phase constants and the characteristic impedance of the individual lines for all the modes of the system and can be readily computed for various cases of multiple coupled lines(7).

II. Immittance Functions

The transmission line equations for the n-line system shown in Fig. 1 are given by:

$$\frac{d[V]}{dx} = -[Z][I] \qquad (1)$$

$$\frac{d[I]}{dx} = -[Y][V] \qquad (2)$$

where [V] and [I] are the n-dimensional column vectors representing the voltages and currents on the lines, and [Z] and [Y] are n x n impedance and admittance matrices as given by

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix}$$

and

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix}$$

where Z_{ii} and Y_{ii} ($i = 1,2, \ldots, n$) are the equivalent self-impedance and admittance per unit length of the ith line, and Z_{ij} and Y_{ij} ($i \neq j$) are the mutual impedance and admittance per unit length between the ith line and the jth line.

The voltages and currents for the case of uniformly coupled lines considered here are then the solutions of the following characteristic equations:

$$\frac{d^2[V]}{dx^2} + [Z][Y][V] = 0 \qquad (3)$$

$$\frac{d^2[I]}{dx^2} + [Y][Z][I] = 0 \qquad (4)$$

where, for the n-line 2n-port case

$$[Y][Z] = \left\{ |[Z][Y]| \right\}^T$$

The general solutions for the port voltages on n-line 2n-port structures are given by

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_n \\ V_{n+1} \\ \cdot \\ \cdot \\ \cdot \\ V_{2n} \end{bmatrix} = \begin{bmatrix} [M_V] & [M_V] \\ & \\ [M_V][e^{-j\theta}i]_{diag} & [M_V][e^{j\theta}i]_{diag} \end{bmatrix} \begin{bmatrix} A_1 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_{2n-1} \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_{2n} \end{bmatrix} \qquad (5)$$

The corresponding currents for n-line 2n-port are determined by substituting the expressions for voltages (5) into (1). These currents are given by

$$\begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_n \\ I_{n+1} \\ \cdot \\ \cdot \\ \cdot \\ I_{2n} \end{bmatrix} = \begin{bmatrix} [M_I] & -[M_I] \\ & \\ -[M_I][e^{-j\theta}i]_{diag} & [M_I][e^{j\theta}i]_{diag} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_{2n-1} \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_{2n} \end{bmatrix}$$

where, $i = 1,2, \ldots, n$.

In the above equation, the eigenvalues,

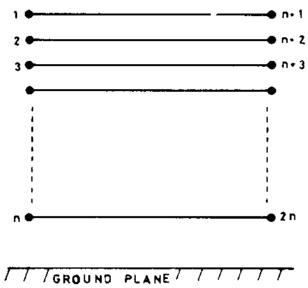

Fig. 1. Schematic of the coupled line 2n-port.

β_i's, representing the solutions of $\det\{[Z][Y] + \beta^2 [U]\} = 0$, are the propagation constants for the normal modes of the system.

A_j ($j = 1, 2, \ldots, 2n$) is an arbitrary amplitude coefficient and $\theta_i = \beta_i \ell$ is the electric length of lines for the n normal modes.

$[M_V]$ is the voltage eigenvector matrix (n by n) corresponding to eigenvalues:

$$[M_V] = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \alpha_2 & \beta_2 & \cdots & \zeta_2 \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \alpha_n & \beta_n & \cdots & \zeta_n \end{bmatrix}$$

$[M_I]$ is the current eigenvector matrix (n by n) defined as $[M_I] \triangleq [Y]_c^T * [M_V]$ with the element $M_{Iij} = Y_{ji} \cdot M_{Vij}$:

$$[M_I] = \begin{bmatrix} Y_{11} & Y_{21} & \cdots & Y_{n1} \\ \alpha_2 Y_{12} & \beta_2 Y_{22} & \cdots & \zeta_2 Y_{n2} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \alpha_n Y_{1n} & \beta_n Y_{2n} & \cdots & \zeta_n Y_{nn} \end{bmatrix}$$

where, Y_{jk} is the characteristic admittance of line k for mode j.

Eliminating A_1, A_2, ... , and A_{2n} leads to 2n equations for the 2n-port currents where coefficients represent the immittance parameters. These admittance parameters of the 2n-port are found to be:

$$[Y] = \begin{bmatrix} [M_I] & -[M_I] \\ -[M_I][e^{-j\theta_i}]_{diag} & [M_I][e^{j\theta_i}]_{diag} \end{bmatrix} \begin{bmatrix} [M_V] & [M_V] \\ [M_V][e^{-j\theta_i}]_{diag} & [M_V][e^{j\theta_i}]_{diag} \end{bmatrix}^{-1} \quad (7)$$

where $[\ ]_{diag}$ indicates a diagonal matrix.

The second matrix can be inverted as follows:

$$\begin{bmatrix} [U] + [\frac{e^{-j\theta_i}}{2j \sin \theta_i}]_{diag} & -[\frac{1}{2j \sin \theta_i}]_{diag} \\ -[\frac{e^{-j\theta_i}}{2j \sin \theta_i}]_{diag} & [\frac{1}{2j \sin \theta_i}]_{diag} \end{bmatrix} \begin{bmatrix} [M_V]^{-1} & [o] \\ [0] & [M_V]^{-1} \end{bmatrix}$$

Hence

$$[Y] = \begin{bmatrix} [M_I][0] \\ [0][M_I] \end{bmatrix} \begin{bmatrix} -j[\cot \theta_i]_{diag} & j[\csc \theta_i]_{diag} \\ j[\csc \theta_i]_{diag} & -j[\cot \theta_i]_{diag} \end{bmatrix} \begin{bmatrix} [M_V]^{-1}\ [0] \\ [0]\ [M_V]^{-1} \end{bmatrix}$$

Manipulating the above matrices leads to

$$[Y] = -\begin{bmatrix} [M_I][\coth \gamma_i \ell]_{diag} [M_V]^{-1} & -[M_I][\text{csch}\ \gamma_i \ell]_{diag} [M_V]^{-1} \\ [M_I][\text{csch}\ \gamma_i \ell]_{diag} [M_V]^{-1} & [M_I][\coth \gamma_i \ell]_{diag} [M_V]^{-1} \end{bmatrix} \quad (8)$$

where $\theta_i = j\gamma_i \ell$.

The above matrix elements for the admittance matrix can be readily evaluated for a given 2n-port structure as illustrated for the case of a four-line microstrip structure in the following example.

Example: For a four-line symmetrical microstrip structure, shown in Fig. 2, the quasi-TEM normal mode parameters can be computed by the technique given by Lee(7). The capacitance matrix $[C]_d$ of the structure with the dielectric present is given by:

$$[C]_d = \begin{bmatrix} C_{11} & -C_{12} & -C_{13} & -C_{14} \\ -C_{12} & C_{22} & -C_{23} & -C_{13} \\ -C_{13} & -C_{23} & C_{22} & -C_{12} \\ -C_{14} & -C_{13} & -C_{12} & C_{11} \end{bmatrix}$$

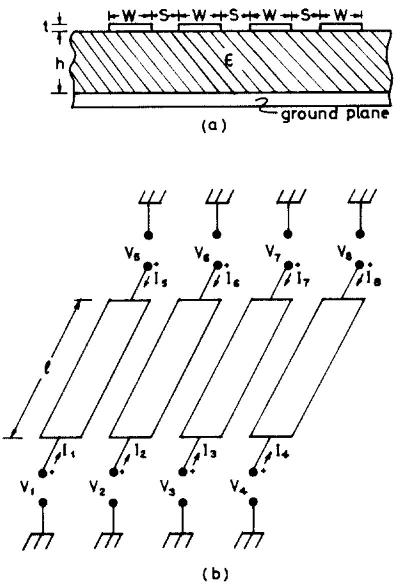

Fig. 2. (a) Cross sectional view of the symmetrical four-line microstrip structure. (b) Schematic of the coupled line eight-port.

For the relative dielectric constant $\epsilon_r = 10$ and the geometry given by w/h = 0.11 and s/h = 0.08, these parameters are found to be:

$C_{11} = 110.38$ pf, $C_{12} = 50.61$ pf, $C_{13} = 9.93$ pf, $C_{14} = 5.16$ pf, $C_{22} = 134.30$ pf, $C_{23} = 46.52$ pf.

These parameters, together with the same parameters with dielectric removed, lead to the four phase constants, the eigenvector matrix, and the characteristic admittance matrix of the structure, as given by:

$\beta_1 = 8.502 \times 10^{-9}$ sec/m, $\beta_2 = 7.849 \times 10^{-9}$ sec/m

$\beta_3 = 7.824 \times 10^{-9}$ sec/m, $\beta_4 = 7.823 \times 10^{-9}$ sec/m

where subscript i (=1,2,3,4) indicates a mode.

$$[M_V] = \begin{bmatrix} 1. & 1. & 1. & 1. \\ 1.0105 & 0.3436 & -1.5643 & -4.7330 \\ 1.0105 & -0.3436 & -1.5643 & 4.7330 \\ 1. & -1. & 1. & 1. \end{bmatrix}$$

and

$$[Y]_c = \begin{bmatrix} 1/192.998 & 1/\ 77.272 & 1/39.132 & 1/25.393 \\ 1/305.077 & 1/125.673 & 1/61.856 & 1/41.299 \\ 1/305.077 & 1/125.673 & 1/61.856 & 1/41.299 \\ 1/192.998 & 1/\ 77.272 & 1/39.132 & 1/25.393 \end{bmatrix} \text{(mho)}$$

The eight-port admittance matrix elements, at the center frequency as defined by $\bar{\theta} = \frac{\Sigma \theta_i}{n} = \frac{\pi}{2}$ are calculated by using equation (8) and found to be:

$Y_{11} = -.2144 \times 10^{-3}$mho, $Y_{12} = .3538 \times 10^{-3}$mho

$Y_{13} = .1719 \times 10^{-3}$mho, $Y_{14} = .2053 \times 10^{-3}$mho

$Y_{15} = .1395 \times 10^{-1}$mho, $Y_{16} = -.6552 \times 10^{-2}$mho

$Y_{17} = -.1350 \times 10^{-2}$mho, $Y_{18} = -.7717 \times 10^{-3}$mho

$Y_{22} = -.4697 \times 10^{-3}$mho, $Y_{23} = .2796 \times 10^{-3}$mho

$Y_{26} = .1710 \times 10^{-1}$mho, $Y_{27} = -.5995 \times 10^{-2}$mho

III. The Symmetrical Four-Line Eight-Port Structure

The immittance matrix parameters of the coupled line eight-port shown in Figure 2 are found in terms of the normal mode characteristic immittance of the four lines in a straight forward manner by using the approach described in the previous section.

Let the voltage and current eigenvector matrices for the symmetrical four-line structure be presented by

$$[M_V] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ R_a & R_b & R_c & R_d \\ R_a & -R_b & R_c & -R_d \\ 1 & -1 & 1 & -1 \end{bmatrix} \qquad (9)$$

$$[M_I] = \begin{bmatrix} Y_{a1} & Y_{b1} & Y_{c1} & Y_{d1} \\ R_aY_{a2} & R_bY_{b2} & R_cY_{c2} & R_dY_{d2} \\ R_aY_{a2} & -R_bY_{b2} & R_cY_{c2} & -R_dY_{d2} \\ Y_{a1} & -Y_{b1} & Y_{c1} & -Y_{d1} \end{bmatrix} \qquad (10)$$

where Y_{xi} represents the mode admittance of line i (1,2,3,4) for mode χ (a,b,c,d).

The inverse matrix of (9) is then given by:

$$[M_V]^{-1} = \begin{bmatrix} -R_c/R_1 & 1/R_1 & 1/R_1 & -R_c/R_1 \\ -R_d/R_2 & 1/R_2 & -1/R_2 & R_d/R_2 \\ R_a/R_1 & -1/R_1 & -1/R_1 & R_a/R_1 \\ R_b/R_2 & -1/R_2 & 1/R_2 & -R_b/R_2 \end{bmatrix} \qquad (11)$$

where

$$R_1 = 2\,(R_a - R_c)$$
$$R_2 = 2\,(R_b - R_d)$$

The orthogonal condition $[M_I]^T[M_V] = [U]$ leads to the following relations;

$$\frac{Y_{a1}}{Y_{a2}} = \frac{Y_{c1}}{Y_{c2}} = \frac{Z_{a2}}{Z_{a1}} = \frac{Z_{c2}}{Z_c} = -R_a R_c$$

$$\frac{Y_{b1}}{Y_{b2}} = \frac{Y_{d1}}{Y_{d2}} = \frac{Z_{b2}}{Z_{b1}} = \frac{Z_{d2}}{Z_{d1}} = -R_b R_d$$

According to equation (8), the admittance matrix for the symmetrical eight-port structures are given by

$$[Y] = \begin{bmatrix} [Y]_{11} & [Y]_{12} \\ [Y]_{12} & [Y]_{11} \end{bmatrix} \qquad (12)$$

where

$$[Y]_{11} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{12} & Y_{22} & Y_{23} & Y_{24} \\ Y_{13} & Y_{32} & Y_{33} & Y_{34} \\ Y_{14} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$$= \begin{bmatrix} Y_{a1} & Y_{b1} & Y_{c1} & Y_{d1} \\ R_aY_{a2} & R_bY_{b2} & R_cY_{c2} & R_dY_{d2} \\ R_aY_{a2} & -R_bY_{b2} & R_cY_{c2} & -R_dY_{d2} \\ Y_{a1} & -Y_{b1} & Y_{c1} & -Y_{d1} \end{bmatrix}$$

$$\begin{bmatrix} \coth \gamma_a \ell & 0 & 0 & 0 \\ 0 & \mathrm{Coth}\ \gamma_b \ell & 0 & 0 \\ 0 & 0 & \mathrm{Coth}\ \gamma_c \ell & 0 \\ 0 & 0 & 0 & \mathrm{Coth}\ \gamma_d \ell \end{bmatrix} \cdot$$

$$\begin{bmatrix} -R_c/R_1 & 1/R_1 & 1/R_1 & -R_c/R_1 \\ -R_d/R_2 & 1/R_2 & -1/R_2 & R_d/R_2 \\ R_a/R_1 & -1/R_1 & -1/R_1 & R_a/R_1 \\ R_b/R_2 & -1/R_2 & 1/R_2 & -R_b/R_2 \end{bmatrix}$$

and

$$[Y_{12}] = \begin{bmatrix} Y_{15} & Y_{16} & Y_{17} & Y_{18} \\ Y_{25} & Y_{26} & Y_{27} & Y_{28} \\ Y_{35} & Y_{36} & Y_{37} & Y_{38} \\ Y_{45} & Y_{46} & Y_{47} & Y_{48} \end{bmatrix}$$

$$= - \begin{bmatrix} Y_{a1} & Y_{b1} & Y_{c1} & Y_{d1} \\ R_aY_{a2} & R_bY_{b2} & R_cY_{c2} & R_dY_{d2} \\ R_aY_{a2} & -R_bY_{b2} & R_cY_{c2} & -R_dY_{d2} \\ Y_{a1} & -Y_{b1} & Y_{c1} & -Y_{d1} \end{bmatrix}$$

$$\begin{bmatrix} \operatorname{csch}\gamma_a\ell & 0 & 0 & 0 \\ 0 & \operatorname{csch}\gamma_b\ell & 0 & 0 \\ 0 & 0 & \operatorname{csch}\gamma_c\ell & 0 \\ 0 & 0 & 0 & \operatorname{csch}\gamma_d\ell \end{bmatrix} \cdot$$

$$\begin{bmatrix} -R_c/R_a & 1/R_1 & 1/R_1 & -R_c/R_1 \\ -R_d/R_2 & 1/R_2 & -1/R_2 & R_d/R_2 \\ R_a/R_1 & -1/R_1 & -1/R_2 & R_d/R_2 \\ R_b/R_2 & -1/R_2 & 1/R_2 & -R_b/R_2 \end{bmatrix}$$

The expressions for the admiitance parameters are obtained as follows by manipulating the partitioned matrix above.

$$Y_{11} = Y_{44} = Y_{55} = Y_{88} = -(A+B)$$

$$Y_{14} = Y_{41} = Y_{58} = Y_{85} = -(A-B)$$

$$Y_{12} = Y_{21} = Y_{34} = Y_{43} = Y_{56} = Y_{65} = Y_{78} = Y_{89} = C+D$$

$$Y_{13} = Y_{31} = Y_{24} = Y_{42} = Y_{57} = Y_{75} = Y_{68} = Y_{86} = C-D$$

$$Y_{15} = Y_{51} = Y_{48} = Y_{84} = E+F$$

$$Y_{18} = Y_{81} = Y_{45} = Y_{54} = E-F$$

$$Y_{16} = Y_{61} = Y_{25} = Y_{52} = Y_{38} = Y_{83} = Y_{47} = Y_{74} = -(G+H)$$

$$Y_{17} = Y_{71} = Y_{28} = Y_{82} = Y_{35} = Y_{53} = Y_{46} = Y_{64} = -(G-H)$$

$$Y_{22} = Y_{33} = Y_{66} = Y_{77} = I+J$$

$$Y_{23} = Y_{32} = Y_{67} = Y_{76} = I-J$$

$$Y_{26} = Y_{62} = Y_{37} = Y_{73} = -(K+L)$$

$$Y_{27} = Y_{72} = Y_{36} = Y_{63} = -(K-L)$$

where

$$A = \frac{1}{R_1}\left(R_c Y_{a1} \coth\gamma_a\ell - R_a Y_{c1} \coth\gamma_c\ell\right)$$

$$B = \frac{1}{R_1}\left(R_d Y_{b1} \coth\gamma_b\ell - R_b Y_{d1} \coth\gamma_d\ell\right)$$

$$C = \frac{1}{R_1}\left(Y_{a1} \coth\gamma_a\ell - Y_{c1} \coth\gamma_c\ell\right)$$

$$D = \frac{1}{R_2}\left(Y_{b1} \coth\gamma_b\ell - Y_{d1} \coth\gamma_d\ell\right)$$

$$E = \frac{1}{R_1}\left(R_c Y_{a1} \operatorname{csch}\gamma_a\ell - R_a Y_{c1} \operatorname{csch}\gamma_c\ell\right)$$

$$F = \frac{1}{R_2}\left(R_d Y_{b1} \operatorname{csch}\gamma_b\ell - R_b Y_{d1} \operatorname{csch}\gamma_d\ell\right)$$

$$G = \frac{1}{R_1}\left(Y_{a1} \operatorname{csch}\gamma_a\ell - Y_{c1} \operatorname{csch}\gamma_c\ell\right)$$

$$H = \frac{1}{R_2}\left(Y_{b1} \operatorname{csch}\gamma_b\ell - Y_{d1} \operatorname{csch}\gamma_d\ell\right)$$

$$I = \frac{1}{R_1}\left(R_a Y_{a2} \coth\gamma_a\ell - R_c Y_{c2} \coth\gamma_c\ell\right)$$

$$J = \frac{1}{R_2}\left(R_b Y_{b2} \coth\gamma_b\ell - R_d Y_{d2} \coth\gamma_d\ell\right)$$

$$K = \frac{1}{R_1}\left(R_a Y_{a2} \csc\gamma_a\ell - R_c Y_{c2} \csc\gamma_c\ell\right)$$

$$L = \frac{1}{R_2}\left(R_b Y_{b2} \operatorname{csch}\gamma_b\ell - R_d Y_{d2} \operatorname{csch}\gamma_d\ell\right).$$

The validity and correctness of the above expressions has been tested and the results have been compared with the values computed from general matrix expressions given in the previous section.

IV. Conclusions

The procedure for finding the immittance of a general n-line coupled structure in an inhomogeneous medium has been presented in terms of the normal mode parameters of the n-line coupled system. The expressions are in a convenient form for both computational purposes and for deriving the explicit closed form expressions for the elements of the 2n-port immittance matrix. As an example, the closed form expressions for the elements of the admittance matrix of a symmetrical eight-port structure are given.

The scattering parameters of a general coupled line four-port with arbitrary terminations are derived directly from the normal mode parameters of two-line structure by using the definition of the scattering parameters.

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