

# Shape Measurement and Representation of 3-D Curved Objects using Simple Back-Projection Algorithm

## (단순역투영법을 이용한 3 차원 곡면물체의 형상계측 및 표현)

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### 要 約

C. T. (computed tomography)에서 쓰이는 단순 역투영법(simple back-projection)이라는 기초적인 재구성방법을 이용한 컴퓨터 시각장치를 개발하였다. 운동입체시(motion stereo vision)의 중요한 요소인 시방향의 연속적인 변화에 착목하여, 곡면체의 수평면에 평행한 윤곽선을 구하고, 일정한 높이로 측정된 이들 각각의 윤곽선을 집적시킴에 의해 결과적으로는 3 차원 물체를 정량적으로 계측할 수 있다.

또한, 어떠한 곡선이라도 곡률에 의해서 특징지워 질 수 있음에 착안하였다. 본 논문에서는 미분기하학의 곡률론을 도입하여, 이미 측정된 3 차원 곡면물체를 표시하였다. 이로써 목하 연구중인 3차원 곡면물체의 인식에 이 곡률론을 이용하려고 한다.

### Abstract

The new computer vision system which can reconstruct contours of parallel fault planes with horizon of 3-D curved objects has been developed. With the system, the shape of 3-D objects was measured by Simple Back-Projection algorithm which is a fundamental one in C.T. (Computed Tomography).

And, the curvature in differential geometry characterizes any curve. Devising it, the method to represent each contour of 3-D curved objects with the system is described in this paper.

### I. Introduction

In recent times a great deal of interest has been shown in the recognition of three dimensional objects since the feasibility of 'Auto-Man' which attached eyes, ears and hands on

the computer suggested by C.E. Shannon and M. Minsky, members of Artificial Intelligence Group in America (1958).<sup>[1]</sup>

The pioneer research for the recognition with the computer had been conducted by L. G. Roberts in MIT (1965).<sup>[2]</sup> He had obtained input pictures using the TV camera and characteristic points of polyhedral solids by Spatial Differential Method, and experimented the recognition with line drawing from them. The paper proposed fundamental problems for the recognition of polyhedral objects, which

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has been based on the recognition of solids.

In order to recognize the object, machine, that is robot, must be able to obtain informations about its position, feature and configuration. For it, the vision system must be attached to the robot to enable to do measuring spatial coordinate parameters of objects.

The vision system can be distinguished broadly in two ways. One is Gray Level Method which deals with informations of 2-D images, and another is Range Finding Method of 3-D ones.[3][4][5]

It is the fundamental concept of the latter that humans use a great variety of vision-based depth cues.[6] A vision system which can extract depth informations from images obtained by various view points was developed by Y. Sato in Japan (1978).[7]

He aimed at the motion stereo vision that the effect of stereo vision could be taken by the movement of observation points. His algorithm has a merit in the accuracy, but when spatial coordinate parameters are decided, takes a long time for solving simultaneous equations for the use of the statistical inference called least square method.

In this paper, the sequential variation of vision direction which is the important requisite of the motion stereo vision is conceived. Also, the new computer vision system which was theoretically introduced in conference on KIEE [1983] has been developed.[13] With the system, we obtain contours of parallel fault planes with horizon by which introduced the Simple Back-Projection (SBP) algorithm in C.T. (Computed Tomography),[8] and then, pile them up. Consequently, the shape of 3-D curved objects is measured.

In addition, devising the curvature in differential geometry characterizes any curve,[9] the representation method of 3-D curved objects with the system is described in this paper.

Finally, the measured and represented results with the system are shown.

## II. Shape Measurement

### 1. Principle of Measurement (Simple Back-Projection)

Principle which measures any horizontal section of 3-D curved objects is explained as follows.

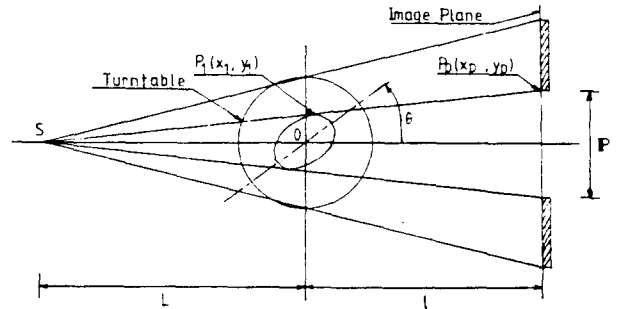


Fig. 1. The coordinate system for the measurement.

Where S is a source point,

O, an origin of the system

P, a part which the light isn't appeared on the image plane.

L, a distance between S and O,

l, between O and the plane, and  $\theta$  is any rotated angle.

As shown in Fig. 1, if any object is rotated in angle  $\theta$ , projection data can be obtained at one point on a screen. If  $P_1(x_1, y_1)$  is a point of the object, projection data are given by

$$x_p = l$$

$$y_p(\theta, x_1, y_1) = \frac{(L+l)(y_1 \cos \theta - x_1 \sin \theta)}{x_1 \cos \theta + y_1 \sin \theta + L} \quad (1)$$

from eq. (1), component x is constant with l, but on the other hand y varies in depending on the  $\theta$ . If  $\theta$  is varied from  $0^\circ$  to  $360^\circ$  with the equiangle interval, projection data as many as the number of sampling are obtained.

The method which constructs the horizontal section of 3-D objects using each projection data produced in such a principle is SBP which is a fundamental algorithm in C.T.

A back projection equation is derived from data, eq. (1). This is given by

$$f(\theta, x_b, y_b) = \frac{-x_p \sin \theta + y_p \cos \theta - L \sin \theta}{x_p \cos \theta + y_p \sin \theta + L \cos \theta} \cdot x_b$$

$$+ \frac{L \cdot y_p}{x_p \cos \theta + y_p \sin \theta + L \cos \theta} - y_b. \quad (2)$$

Where,  $x_p$  and  $y_p$  are projection data. Therefore, the function  $f$  is determined by  $\theta$ ,  $x_b$  and  $y_b$ . And then, the following equation may be obtained

$$P(x,y) = \begin{cases} 1 & \text{for } f(\theta, x_b, y_b) \in P \\ 0 & \text{for } f(\theta, x_b, y_b) \notin P \end{cases} \quad (3)$$

where, as shown in Fig. 1, the region  $P$  is the part where the light isn't appeared on the screen.

We will reconstruct a section of the object if  $P(x,y)$  is composed with rotating  $\theta$  from  $0^\circ$  to  $360^\circ$  with the equiangle interval.

The principle is shown in Fig. 2 (a) and its computer simulation in Fig.3 (b). Fig.3 (a) is a supposed original image.

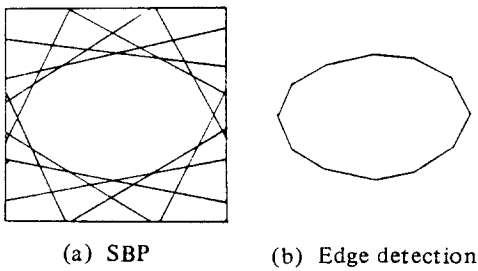
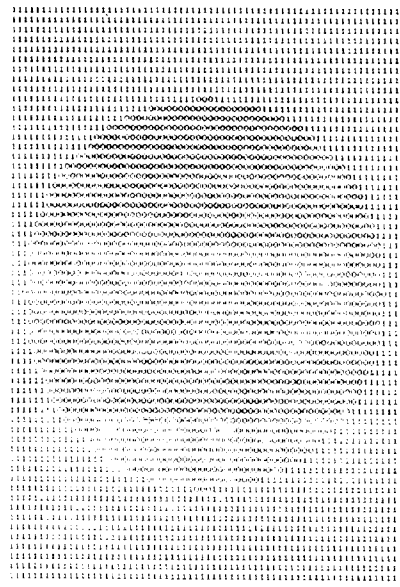


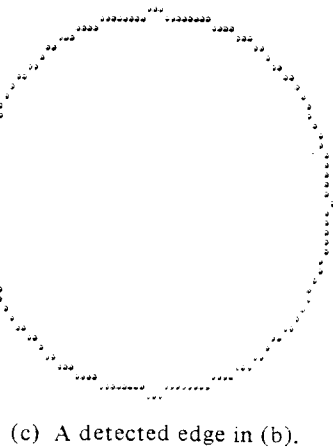
Fig. 2. The principle of SBP.



(a) A supposed original image.



(b) A result by SBP.



(c) A detected edge in (b).

Fig. 3. Computer simulation of the principle.

### 2. Edge Detection

The contour is obtained from a 2-D image which is taken by SBP. The methods that detect the edge are Laplacian Method, Gradient Method, Sobel Technique, and etc.

In this paper, 3 X 3 masking method was employed.[10][11] The center of the mask forms in 3 X 3 small pixels range with 8 or 4 connection. It is the method that puts always the center on the edge, and then, that moves it step by step.

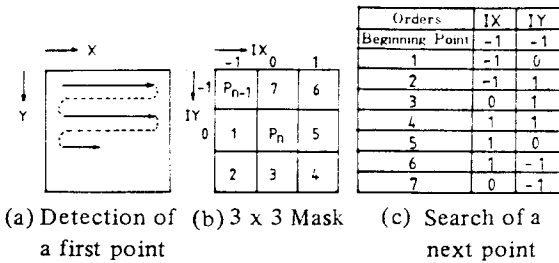


Fig. 4. Edge detection (3 x 3 masking method).

Using the 8 connection, first, the image scanned as shown in Fig. 4 (a). And then, a first edge point P<sub>1</sub> is searched. Next, there is a description about the (n + 1)'th edge point P<sub>n+1</sub> after P<sub>n</sub> has done.

Suppose P<sub>n-1</sub> to [I<sub>(n-1)</sub>, J<sub>(n-1)</sub>] and P<sub>n</sub> to [I<sub>(n)</sub>, J<sub>(n)</sub>]. Then, the direction of P<sub>n-1</sub> to P<sub>n</sub> can be known from the following equation.

$$\begin{aligned} IX &= I_{(n-1)} - I_{(n)} \\ IY &= J_{(n-1)} - J_{(n)} \end{aligned} \tag{4}$$

And whether the proceeded pixel is white or not is detected as shown in Fig. 4 (b).

After P<sub>n-1</sub> is detected as this, relating equations with IX and IY are

$$\begin{aligned} JX &= IX + IY \\ JY &= IY - IX \end{aligned} \tag{5}$$

Then, if JX and JY are zero or -1 or 1, put as they stand, and if -2 or 2, replace with -1 or 1, respectively. Doing so, the position of the next detected edge point P<sub>n+1</sub> will be calculated in order as in Fig. 4 (c). The principle of the edge detection is shown in Fig. 2 (b) and its computer simulation in Fig. 3 (c).

### III. Computer Vision System

#### 1. Block Diagram of the System

The block diagram of the vision system is shown in Fig. 5.

Illustration of the system is as follows.

Eye of the observer in the system is CCTV

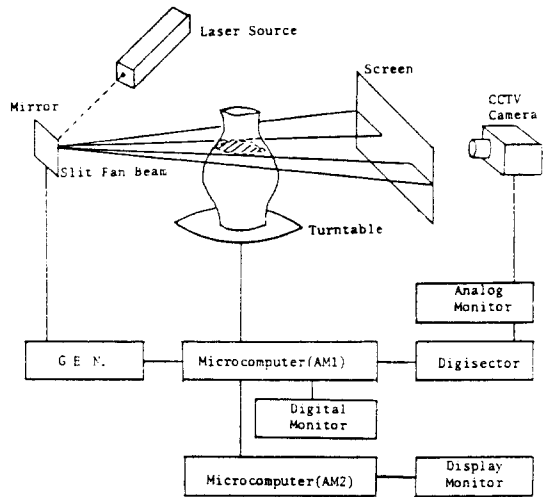


Fig. 5. Block diagram of the measurement system.

camera used commonly. By using Digisector the camera can be easily interfaced with Apple Microcomputer (AM1) which manages input pictures and projection data. Main microcomputer (AM2) calculates SBP from data and plots results, and represents objects.

AM1 sends operating signals to the camera and displays digital images to a digital monitor translated input pictures that show in an analog one through Digisector. And then, it detects projection data from digital images and sends them to AM2.

After the end of these processes, AM1 sends operating signals to a turntable and lets it turn with equiangle intervals. And then, AM1 sends operating signals to the camera and repeats the former processes.

Light source is a He-Ne gas laser. Let scan laser beam to a mirror with a generator, then, a slit fan beam is made by the reflected beam. This slit source irradiates to an object which lies on the turntable. A projection data image shows on a screen.

#### 2. The Computer Vision System

The computer vision system based on the block diagram is setted up as shown in Fig. 5. The total system is shown in Fig. 6.

Eventually, we measure the 3-D object quantitatively as pile up the reconstructed

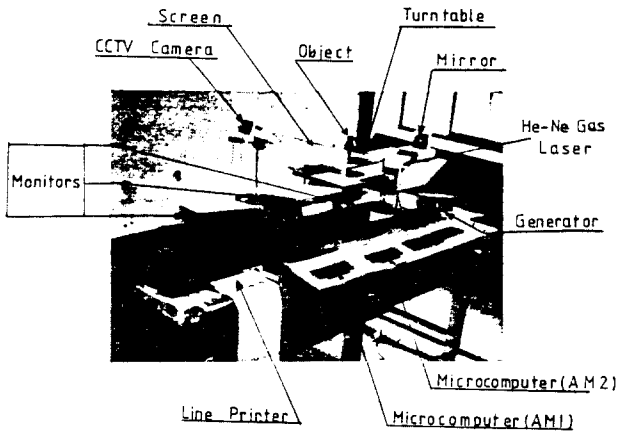


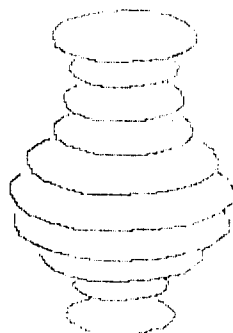
Fig. 6. A computer vision system.

sections measured from the object with constant spaces.

The experimental result is shown in Fig. 7. Fig. 7 (a) shows an object, and Fig. 7 (b) its plot of the result, and components x, y and z of the measured object are in Table 1. In this experiment,  $\theta$  is  $15^\circ$  in an equiangle interval. And the distance L between the source and the center of turntable is 735 in millimeters, l between the center and the screen 245.5. The interval of component z is 13.



(a) An object



(b) Its result

Fig. 7. A measurement result of a 3-D curved object.

IV. Curvature and Representation

The enclosing surface, or boundary, of a

Table 1. Sampling components x, y and z of a measured object in Fig. 7 (b).

component			component		
x	y	z	x	y	z
-2.12794926	20.5924487	5	-2.79711212	31.7346162	70
16.7496091	12.1390625	5	26.0844278	18.2896782	70
18.3975583	-8.45336634	5	18.8815399	-13.4449381	70
2.12794927	-20.5924487	5	2.79711221	-31.7346162	70
-16.7595091	-12.1390825	5	-26.0844279	-18.2896781	70
-18.8975584	8.45336623	5	-28.8815399	13.444938	70
-1.37670086	15.2536328	18	-2.73583864	30.4234454	85
12.4482856	9.14620243	18	24.9795573	17.5810283	85
4.1449864	-6.29743032	18	27.7153959	-12.842417	85
1.69670091	-15.2526327	18	27.73583867	-30.4234454	85
-12.4482856	-9.14620224	18	-24.9795574	-17.5810282	85
-4.1449864	6.29742037	18	-27.7153959	12.8424171	85
-1.6871425	17.3176767	31	-12.31565242	25.2132724	96
14.9629767	10.2758312	31	18.9454574	13.61205	96
15.9211192	-7.04184541	31	-11.2611098	-9.46122245	96
1.26714251	-17.2176767	31	2.21565245	-23.2132724	96
-14.0639767	-10.275831	31	-18.9454574	-13.6120499	96
-15.9211191	7.04184556	31	-21.2611098	9.46122255	96
-2.07811468	19.9373722	44	-1.51579525	13.3903556	109
16.2272125	11.74893862	44	10.8384471	8.00787092	109
18.3053281	-8.14898609	44	12.2542424	-5.38243557	109
2.07811473	-19.9373722	44	1.5157953	-13.3903555	109
-16.2272126	-11.7487886	44	-10.8384471	-8.00786992	109
-18.3053282	8.148986	44	-12.2542424	5.38243567	109
-2.56229796	27.1457201	57	-1.69670086	15.3536328	122
22.2277245	15.791875	57	12.4482856	9.14620243	122
24.7900321	-11.3538451	57	14.1449864	-6.29743032	122
2.56229803	-27.1457201	57	1.69670091	-15.2526327	122
-22.2277242	-15.791875	57	-12.4482856	-9.14620234	122
-24.7900321	11.352845	57	-14.1449864	6.29742037	122

well-measured 3-D object should unambiguously specify the object. Since surfaces are what is seen, these representation are important for computer vision.

A general technique for approximating surfaces with four-sided surface patches is that of Coons (1974).<sup>14</sup> Coons specifies the four-sides of the patch with polynomials. Although this is appropriate for synthesis, it is not so easy to use for analysis. This is because of the difficulty of registering the patch edges with image data.

In the other hand, the volume of many biological and manufactured objects is naturally described as the "swept volume" of a 2-D set moved along some 3-space curve. General sweeps are quite a popular representation in computer vision, where G.J. Agin and T.O. Binford go by name generalized cylinders (sometimes "generalized cones").<sup>[3]</sup>

And, there is a method for the description and identification of curved objects. Y. Sato considered the shape of a curved object as a surface which is an accumulation of horizontal section boundaries. He described the coefficients of a Fourier series expansion as the representative function.<sup>15</sup>

In this paper, we devise that the curvature

characterizes any curve. Because surface element coordinate points of an object were measured by the computer vision system, curvatures be obtained from them.

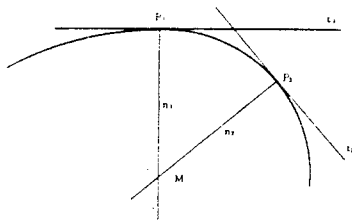


Fig. 8. Curvature.

Consider a curve as shown in Fig. 8. Let \$t\_1\$ and \$t\_2\$ be the tangents, \$n\_1\$ and \$n\_2\$ the normals, at two neighboring points \$P\_1\$ and \$P\_2\$ on the curve. Let the point of intersection of the two normals be at \$M\$. Clearly, the angle between the tangents is equal to an one between the normals. That is,

$$\angle(t_1 t_2) = \angle(n_1 n_2). \tag{6}$$

Let \$P\_2\$ approach \$P\_1\$ along the curve, and calculated the ratio between the angle \$n\_1 n\_2\$ and the distance between the two points of the curve. This ratio approaches a limit,

$$\lim_{P_1 P_2 \rightarrow 0} \frac{\angle(n_1 n_2)}{P_1 P_2} = K_{12}. \tag{7}$$

This limiting value \$K\_{12}\$ is called the curvature of the curve at the point \$P\_1\$. From eq. (7), it is derived as follows,

$$\begin{aligned} K_{12} &\equiv \lim_{P_1 P_2 \rightarrow 0} \frac{\angle(n_1 n_2)}{P_1 P_2} = \lim_{P_1 P_2} \frac{\sin(n_1 n_2)}{P_1 P_2} \\ &= \lim_{P_1 P_2 \rightarrow 0} \frac{P_1 P_2}{MP_1 \cdot P_1 P_2} \\ &= \lim_{P_1 P_2 \rightarrow 0} \frac{1}{MP_1} = \frac{1}{r_{12}} \end{aligned} \tag{8}$$

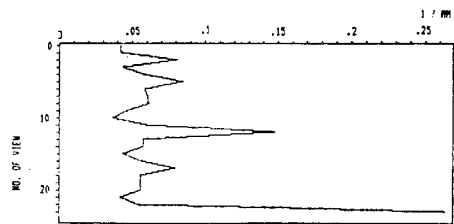
As shown in eq.(8), \$K\$ is the reciprocal of the length \$r\$ of the line-segment that is the com-

mon limit of the two segments \$MP\_1\$ and \$MP\_2\$ of the normals.

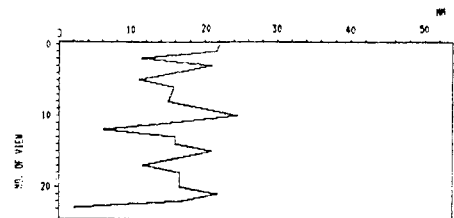
The quantity \$r\$ is also obtained in another way, as follows. We draw a circle through \$P\_1\$ and two neighboring points of the curve. If the two neighboring points approach \$P\_1\$, the circle should expect that the center of the limiting circle is at the limiting position of the point of intersection \$M\$ of the normals, and that its radius is therefore equal to \$r\$.

Table 2. Centers of curvatures, radiuses of curvatures and curvatures of a bottom contour of the object as shown in Fig. 7.

centers of curvature		radiuses of curvature	curvatures
x	y		
-5.32047355	-1.44541713	22.545929	.0442539055
-4.83634659	-1.43464518	22.0802255	.0452897925
4.84046503	1.460156e	11.9996099	.0823360425
-3.5150221	-3.87531551	21.5444427	.0464156815
3.66492104E-07	5.74022624E-07	16.4523377	.06067421E
2.45855016	4.48597256	11.4401269	.0874116177
1.1204427	-1.173839261	16.2279957	.061621904E
1.31182681	.230175396	15.8529225	.063159374
1.23650522	.509691657	15.5421901	.0643409964
3.6698347	-5.52311425	20.1732243	.0495706579
7.30701523	-7.51104072	25.1902187	.0396977907
-1.09895997	-2.4354042	15.9555043	.0626742961
-16.0649945	.223947586	6.64311787	.150331726
2.70520775E-07	2.70718691E-06	16.4922386	.0606342146
-1.94891507E-07	-1.68019155E-07	16.4922382	.0606342164
4.54181854	2.63724604	21.5290823	.0464272118
1.49799211	-.851148541	16.8551897	.0597299117
-1.19755552	-4.80451818	12.1658851	.0820305175
3.49286521E-08	-1.81794314E-07	17.1791577	.0582100729
4.24258426E-08	-2.27525066E-07	17.1791577	.0582100734
-2.52461157E-07	6.89697615E-07	4.7391562	.1552190699
-2.64142145	4.7391279	22.2288872	.0449975797
.80511394	1.04924512	17.5389629	.0570181057
13.8261494	-6.10495544	2.75553084	.361594232



(a) Curvatures, \$K\$.



(b) Radiuses of curvature, \$r = 1/K\$.

Fig. 9. Plotting the curvature.

This circle is called the circle of curvature of the curve at  $P_1$ , its center is called the center of curvature, and its radius  $r$ , the radius of curvature. The shape of a 3-D curved object is represented in following figures and a table.

The centers of curvatures, radiuses of curvatures and the curvatures which calculate by the algorithm are shown in Table 2. In addition, results of curvatures are plotted in Fig. 9. Fig. 9 (a) is plotting the curvatures, and Fig. 9 (b) plotting the radiuses of the curvatures, which a contour is bottom of Fig. 7 (b).

## V. Conclusion

The method for the shape measurement of 3-D curved objects was based on the SBP algorithm. Contours of the parallel section with horizon was obtained by this algorithm, and then, the objects could be measured by piling them up. Fully, this method might be reconstructed the ones constructed with the convex section, but the ones with the concave one could not be done.

And, the shape of a 3-D curved object was experimentally represented by the computer vision system. A study on the recognition is proceeded.

The procedure of the recognition may be as follows. Model bases of objects are memorized in computer or diskettes with curvatures. An object which should be recognized is begun the measurement of the curvature, and then, the calculated information is compared with models by graph-searching procedure.<sup>12</sup> This procedure is to use a popular searching method in Artificial Intelligence. By using the above procedure, the position, feature and configuration of the objects may be recognized.

The shape measurement of 3-D curved objects using SBP can be applicated in storing objects quantitatively or designing of dress or studying of antiquities, etc.

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