슬라이드 모우드를 이용한 모델추종 적응 제어에 관한 연구

論 文
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A Study on the Adaptive Model-Following Control Systems by Slide Mode

千 熙 英*·朴 貴 泰**·權 誠 河***·李 昌 勳***
(Hee-Young Chun · Gwi-Tae Park · Sung-Ha Kwon · Chang-Hoon Lee)

요 약

본 논문은 슬라이드 모우드 이론을 이용하여 모델추종 적응제어계의 새로운 설계법을 연구한 것이다. 파라미터 변동과 외란이 존재함에도 불구하고 시변 다변수입력계통이 모델을 잘 추종하고 편차동특성이 원하는 형태가 되며 전체계통의 안정도가 확보되는 설계방법을 고찰하고 단일입력계통, 다변수입력계통, C-131B항공기 예에 적용하여 이론의 타당성과 유효성을 검토하였다. 연구결과 다변수입력계통에 쉽게 적용할 수 있고 제어칙이 가변구조 모델추종 제어계보다 심하게 변동하지 않으며 제어이득이 구하기 쉽고 구조가 간단하며 계산시간이 매우 적다는 결론을 얻었다.

Abstract

This paper describes a new method for the design of adaptive model-following control systems. This design concept is developed using the theory of slide mode. Authors present new results on the sliding control methodology to achieve accurate tracking for a class of multi-input multi-output, time-varying systems in the presence of parameter variations and disturbances.

This algorithm can be easily applied to the multivariable control systems and obtained a smoother control signal in comparison to variable structure model following control systems. The design technique is easy and the control structure is simple. The design requires little computational effort. The control system is less sensitive to plant parameter variations and noise disturbances.

1. Introduction

The application of optimal control theory to multivariable control systems has the difficulty of specifying a performance index. Linear model-following control (LMFC) is an efficient control method that avoids this difficulty by using a model, which specifies the design objectives. However, LMFC systems are inadequate when there are large parameter variations or disturbances. Adaptive model-following control (AMFC) systems are capable of retaining high performance in the presence of parameter variations.[1),2)] Presently, there is a variety of basic and applied research of AMFC systems.

The theory of variable structure systems (VSS) has been developed in the U.S.S.R. in the last fifteen years,[3),4)] and nowdays the study of the single-input single-output (SISO) systems is enlivened,[5),6),7)] but the study of the multi-input multi-output (MIMO) systems is unexhausted. Variable structure model-following control (VSMFC) for MIMO systems by K.K.D Young in 1978 is not only difficult to choose the variable structure control gain but also

*正 會 員：高麗大 工大 電氣工學科 敎授·工博
**正 會 員：高麗大 工大 電氣工學科 副敎授·工博
***正 會 員：昌原大 電氣工學科 專任講師
****正 會 員：高麗大 大學院 電氣工學科 博士課程
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afraid of stability and convergence.[8], [9] And it may be impractical to apply the discontinuous chattering input directly to the plant.

In this paper, a new method to AMFC for MIMO systems is considered. This paper discusses the slide mode with regard to the design of AMFC systems. A smooth control signal is obtained with a little computer effort.

In section 2, the adaptive model-following control systems is introduced. In section 3 and 4, the sliding motion and the control function is described, respectively. In section 5, the perfect model-following condition and the stability is discussed. Finally, the theory is applied to the design of adaptive controllers for a scalar system, a 4th-order multivariable system and an aircraft control system.

2. Adaptive Model-following Control Systems

The state equations of linear time varying multivariable systems are represented by the following equations

$$\dot{\mathbf{X}}p\ (t)=Ap\ (t)\mathbf{X}p\ (t)+Bp\ (t)\mathbf{U}p\ (t)+\mathbf{h}\ (t) \quad (1)$$

$$\dot{\mathbf{X}}m\ (t)=Am\ \mathbf{X}m\ (t)+Bm\ \mathbf{U}m\ (t) \quad (2)$$

where $\mathbf{X}p \in R^n$, $\mathbf{X}m \in R^n$, $\mathbf{U}p \in R^m$, $\mathbf{U}m \in R^l$ and $\mathbf{h} \in R^n$. **U**m is the input vector of reference model and **U**p is the input vector of controlled plant. The vector **h** represents disturbances and parameter variations. We will assume that the pair (Ap, Bp) and (Am, Bm) are stabilizable and matrix Am is stable. The plant matrices Ap and Bp may be uncertain and time-varying, but the nominal value of the elements of these matrices are assumed to be known to the designer.

In model-following systems, the plant is controlled in such a way that the dynamic behavior of plant approximates that of a specified model. The adaptive controller should force the error between model and plant to zero as time tends to infinity. The state error vectors are represented by the following equations.

$$\mathbf{e}\ (t)=\mathbf{X}m\ (t)-\mathbf{X}p\ (t) \quad (3)$$

$$\begin{aligned}\dot{\mathbf{e}}\ (t)&=\dot{\mathbf{X}}m\ (t)-\dot{\mathbf{X}}p\ (t)\\&=Am\ \mathbf{e}\ (t)+[Am-Ap\ (t)]\mathbf{X}p\ (t)+Bm\ \mathbf{U}m\ (t)\\&\quad -Bp\ (t)\ \mathbf{U}p\ (t)-\mathbf{h}\ (t)\end{aligned} \quad (4)$$

3. Sliding Motion

VSS differs from other control systems mainly in the changes can occur in the system structure during the transient process. The basic concept of VSS can be illustrated with a simple second-order scalar system described by the error state equations.

$$\dot{e}_1=e_2$$
$$\dot{e}_2=a_1e_1-a_2e_2+bu$$

The control law is given by

$$u=-\Psi e_1$$

where Ψ is a piecewise constant feedback gain whose values are given by

$$\Psi=\begin{cases}\Psi^+ & \text{if } e_1s>0\\ \Psi^- & \text{if } e_1s<0\end{cases}$$

and $s=c_1e_1+c_2e_2=0$.

The scalar s is the switching line in the phase plane. Therefore, the value of Ψ can be adjusted according to the location of the system-representative point in the phase plane as shown in Fig. 1.

In multivariable systems, let us denote the vectors is the switching hypersurfaces. s_i (**e**)=0 is the i-th component of the m (same as control input $\mathbf{U}_p$) switching hyperplanes in the error state space.

$$\mathbf{s}\ (\mathbf{e})=G\ \mathbf{e}=\mathbf{0} \quad (5)$$

$$\dot{\mathbf{S}}\ (\mathbf{e})=G\ \dot{\mathbf{e}}=\mathbf{0} \quad (6)$$

where G is the switching surface matrix (m×n).

In the VSS theory,[3] the system-representative point could be brought from any initial position to this switching hyperplanes. Once the operating point reaches the switching hyperplanes, the control will switch between the gains to force the representative point to move along the switching hyperplanes. Every time the representative point leaves the switching hyperplanes the controller changes the feedback structure to force the point to return to the switching hyperplanes. This special motion is called the

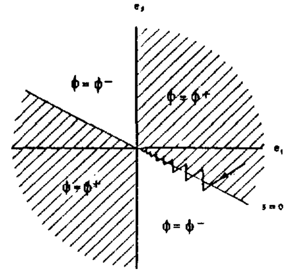

Fig. 1. Phase plane.

sliding motion. Sliding motion occurs if, at a point on a switching surface s_i (**e**) = 0, the direction of motion along the error state trajectories on either side of the surface are not away from the switching surface. The state then slides and remains for some finite time on the surface s_i (**e**) = 0. Then the error between model and plant goes to zero and the model-following is obtained. In order to satisfy the Eq. (5), the sign of $\dot{s}_i$ different from that of s_i. Therefore, the condition for sliding motion to occur on the i-th hyperplane may be stated Eq. (7)

$$s_i\dot{s}_i<0 \tag{7}$$

in the neighbourhood of s_i (**e**) = 0. Eq. (7) is the condition to occur sliding motion.

As before, the control function of the general VSS theory will switch between the gains Ψ^+ and Ψ^-, but we consider a continuous control function.

4. Control Function

In the slide mode the system satisfies the Eq. (5)-(6). Substituting Eq. (4) into Eq. (6), we obtain Eq. (8).

$$\begin{aligned}\dot{\mathbf{s}}(\mathbf{e}) &= G\,\dot{\mathbf{e}}\\ &= G\,[Am\,\mathbf{e}+(Am-Ap)\mathbf{X}p+BmUm-BpUp-\mathbf{h}]\\ &= G\,[Am\,\mathbf{e}+(Am-Ap)\mathbf{X}p+BmUm-\mathbf{h}]-GBpUp\end{aligned} \tag{8}$$

Because s is scalar in SISO systems, G is vector (1 × n), therefore we can choose the element of G so that the transient state error response is desirable. However, in MIMO systems, **s** and **Up** are vectors (m × 1), thus G is matrix (m × n). In order to correspond one by one between **s** and **Up**, the matrix GBp in Eq. (8) should be unit matrix (m × m).

$$GBp=\mathbf{I} \tag{9}$$

So we have Eq. (10)

$$G=(QBp)^{-1}Q \tag{10}$$

The selection of matrix Q is very important. We will discuss it in section 5.

Substituting Eq. (10) into Eq. (8), we obtain Eq. (11). Rearranging about **Up**, we have Eq. (12).

$$\dot{\mathbf{s}}=GAm\mathbf{e}+G\,(Am-Ap)\mathbf{X}p+GBmUm-G\mathbf{h}-\mathbf{U}p \tag{11}$$

$$\mathbf{U}p=Ke\mathbf{e}+Kp\mathbf{X}p+KmUm-G\mathbf{h}-\dot{\mathbf{s}} \tag{12}$$

The sign of **s** and $\dot{\mathbf{s}}$ are alternative from Eq. (7), the condition to occur sliding motion, we obtain Eq. (13), taking $\alpha\mathbf{s}$ instead of $-\dot{\mathbf{s}}$. As the matrices G and Q are selected by Eq. (10) and Eq. (25) and **h** satisfies the invariance condition Eq. (21.b), the term G **h** is same form of G**e**. And Eq. (7) is an inequality, thus the term $\alpha\mathbf{s}$ (= αG**e**) can supply disregard of disturbance term G**h**.

$$Up=Kee+Kp\mathbf{X}p+KmUm+\alpha\mathbf{s} \tag{13}$$

where α is a weighting value. The matrices Ke, Kp and Km are control gain matrices.

$$Ke=GAm,\quad Kp=G\,(Am-Ap^{o}),\quad Km=GBm \tag{14}$$

where, Ap^{o} is nominal value of time varying matrix Ap (t).

We take the weighting value α as a reciprocal of sampling time T so that the error between model and plant goes to zero in every one sampling time.

$$\alpha=1/T \tag{15}$$

Eq. (15) means that α is inversely proportional to the sampling time and it can be increased 10 times over that reciprocal in our experience.

In Eq. (13), control function can be partitioned into a time invariant part described by Kee + KpXp + KmUm and a time varying part described by $\alpha\mathbf{s}$. The time invariant part is LMFC mechanism and the time varying part is a adaptive mechanism. Fig. 2 shows that the total block diagram of adaptive model-following systems by slide mode.

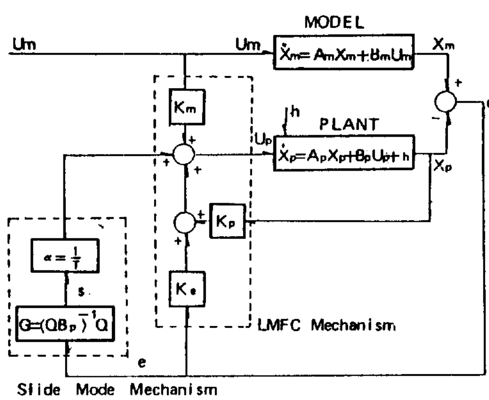

Fig. 2. Adaptive model-following control systems by slide mode.

5. Stability

The behaviour of the error dynamics during sliding needs to be considered. Substituting Eq. (12) into Eq. (4), Eq. (16) is obtained.

$$\dot{\mathbf{e}} = \mathrm{Am}\mathbf{e} + (\mathrm{Am}-\mathrm{Ap})\mathbf{X}\mathrm{p} + \mathrm{Bm}\mathrm{Um} - \mathrm{Bp}\,(\mathrm{K}\mathbf{e}\mathbf{e} + \mathrm{Kp}\mathbf{X}\mathrm{p} + \mathrm{KmUm} - \mathbf{G}\mathbf{h} - \dot{\mathbf{s}}) - \mathbf{h} \qquad (16)$$

During sliding $\dot{\mathbf{s}} = \mathbf{0}$, thus

$$\mathbf{e} = (\mathrm{Am}-\mathrm{BpKe})\mathbf{e} + (\mathrm{Am}-\mathrm{Ap}-\mathrm{BpKp})\mathbf{X}\mathrm{p} + (\mathrm{Bm}-\mathrm{BpKm})\mathrm{Um} - (\mathbf{h}\text{-}\mathrm{Bp}\mathbf{G}\mathbf{h}) \qquad (17)$$

To have perfect model-following, one must assure that for any Um, piecewise continuous, and $\mathbf{e}(o) = \mathbf{0}$, we shall have $\mathbf{e}(t) = \mathbf{0}$, $\dot{\mathbf{e}}(t) = \mathbf{0}$. One concludes immediately that this can be achieved if [10]

$$(\mathrm{Am}-\mathrm{Ap}-\mathrm{BpKp})\mathbf{X}\mathrm{p} + (\mathrm{Bm}-\mathrm{BpKm})\mathrm{Um} = 0 \qquad (18)$$

So that Eq. (18) holds for any **Xp** and **Um**, one must have

$$\mathrm{Am}-\mathrm{Ap}-\mathrm{BpKp} = 0 \quad \mathrm{Bm}-\mathrm{BpKm} = 0 \qquad (19)$$

or

$$\mathrm{BpKp} = \mathrm{Am}-\mathrm{Ap} \qquad \mathrm{BpKm} = \mathrm{Bm} \qquad (20)$$

So that the linear systems, Eq.(20), have a solution[11] about Kp and Km

$$\begin{array}{l} \mathrm{rank}\,[\mathrm{Bp}] = \mathrm{rank}\,[\mathrm{Bp};\mathrm{Am}-\mathrm{Ap}] \\ \mathrm{rang}\,[\mathrm{Bp}] = \mathrm{rank}\,[\mathrm{Bp};\mathrm{Bm}] \end{array} \qquad (21.a)$$

Eq. (21.a) is perfect model-following conditions which is general necessary conditions to the adaptive model-following control systems. In Eq. (17), for total disturbance rejection, $\mathbf{h}-\mathrm{Bp}\mathbf{G}\mathbf{h} = (\mathbf{I}-\mathrm{Bp}\mathbf{G})\mathbf{h} = \mathbf{0}$ requiring

$$\mathrm{rank}\,[\mathrm{Bp}] = \mathrm{rank}\,[\mathrm{Bp};\mathbf{h}] \qquad (21.b)$$

Eq. (21.b) is invariance condition. We will assume through out that these conditions hold.

Eq. (21.a) means that matrices Am, Ap and Bp can be transformed to Eq. (22)

$$\mathrm{Bp} = \begin{bmatrix} O \\ \hline B_2 \end{bmatrix} \qquad \mathrm{Am},\ \mathrm{Ap} = \begin{bmatrix} O & I_{n-m} & O \\ \hline & a_{ij} & \end{bmatrix} \qquad (22)$$

where, B_2 = nonsingular matrix (m×m)

$i = n-m+1,\ n$

$j = 1,\ n$

For the perfect model-following case, Eq. (17) is reduced to Eq. (23)

$$\begin{aligned} \dot{\mathbf{e}} &= (\mathrm{Am}-\mathrm{BpKe})\,\mathbf{e} \\ &= (\mathrm{Am}-\mathrm{BpGAm})\,\mathbf{e} \\ &= [\mathbf{I}-\mathrm{Bp}\,(\mathrm{QBp})^{-1}\mathrm{Q}]\,\mathrm{Am}\mathbf{e} \end{aligned} \qquad (23)$$

During the sliding mode, n error state variables can be expressed in terms of the remaining (n−m) error state variables using Eq. (5). Order reduction is due to the motion of the error state which is constrained to lie on the intersection of the m switching hyperplanes.[12] Since $[\mathrm{Bp}\,(\mathrm{QBp})^{-1}\mathrm{Q}]^2 = \mathrm{Bp}\,(\mathrm{QBp})^{-1}\,\mathrm{QBp}\,(\mathrm{QBp})^{-1}\,\mathrm{Q} = \mathrm{Bp}\,(\mathrm{QBp})^{-1}\mathrm{Q}$, rank $[\mathrm{Bp}\,(\mathrm{QBp})^{-1}\,\mathrm{Q}]$ = rank [Bp] = m. Thus the most rank of $[\mathbf{I}-\mathrm{Bp}\,(\mathrm{QBp})^{-1}\,\mathrm{Q}]$ is n−m. Therefore any Am matrix (n×n) pre-multiplied by $[\mathbf{I}-\mathrm{Bp}\,(\mathrm{QBp})^{-1}\,\mathrm{Q}]$ will have at most rank, n−m. The remaining unforced system Eq. (23) must be asymptotically stable, which implies that all n−m eigenvalues of the matrix $[\mathbf{I}-\mathrm{Bp}\,(\mathrm{QBp})^{-1}\,\mathrm{Q}]$ Am have negative real parts. And the eigenvalues of Eq. (23) can be placed arbitrarily in the complex plane by suitable choice of matrix Q (m×n). The design objective is to choose Q so that the error tends to zero with suitable transient motion. The polynomial from the desired eigenvalues is reduced to

$$P(\lambda) = C_1 + C_2\lambda + C_3\lambda^2 + \cdots\cdots + C_{n-m}\lambda^{n-m-1} + \lambda_{n-m} \qquad (24)$$

In this paper, we set up the matrix Q (m×n) as Eq. (25).

$$Q = \begin{bmatrix} C_1 \cdots\cdots C_{n-m} & \\ \hline O & \mathbf{I}_m \end{bmatrix} \qquad (25)$$

Thus, $\mathrm{QBp} = B_2$

$$\mathrm{Bp}(\mathrm{QBp})^{-1}\mathrm{Q} = \begin{bmatrix} O \\ \hline B_2 \end{bmatrix} B_2^{-1} \begin{bmatrix} C_1 \cdots\cdots C_{n-m} & \\ \hline O & \mathbf{I}_m \end{bmatrix} = \begin{bmatrix} O & O \\ \hline C_1 \cdots C_{n-m} & \\ \hline O & \mathbf{I}_m \end{bmatrix}$$

$$[\mathbf{I}-\mathrm{Bp}(\mathrm{QBp})^{-1}\mathrm{Q}]\mathrm{Am} = \begin{bmatrix} \mathbf{I}_{n-m} & O \\ \hline -C_1 \cdots -C_{n-m} & \\ \hline O & O_m \end{bmatrix} \begin{pmatrix} O & \mathbf{I}_{n-m} & O \\ \hline a_{n-m+1\,1} & \cdots\cdots & a_{n-m+1\,n} \\ \vdots & & \vdots \\ a_{n1} & \cdots\cdots & a_{nn} \end{pmatrix}$$

$$= \begin{bmatrix} O & \mathbf{I}_{n-m} & O \\ \hline \vdots & -C_1 \cdots -C_{n-m} & O \\ \hline O & O & O_{m-1} \end{bmatrix} \qquad (26)$$

Therefore, the rank of Eq. (26) is n−m and the eigenvalues of Eq. (26) are equal to those of Eq. (24).

6. Examples

6.1 Scalar System[9]

We shall illustrate the 3rd-order scalar system.

$$\dot{\mathbf{X}}m(t)=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -3.0 & -2.5 & -3.5\end{bmatrix}\mathbf{X}m(t)+\begin{bmatrix}0\\0\\3\end{bmatrix}\mathbf{U}m(t) \qquad (27)$$

$$\dot{\mathbf{X}}p(t)=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\mathbf{X}p(t)+\begin{bmatrix}0\\0\\b(t)\end{bmatrix}\mathbf{U}p(t)+\begin{bmatrix}h1(t)\\h2(t)\\h3(t)\end{bmatrix} \qquad (28)$$

where $h_1(t)$, $h_2(t)$, $h_3(t)$ represent noise signals, $b(t)$ is an uncertain time-varying parameter, $Um(t)=1$ and all initial value are zero except $Xp_1(0)=-1$.

The model has eigenvalues $\lambda_m=-3,-0.25\pm j0.968$ with an underdamped unit step response and the unstable plant has eigenvalues $\lambda_p=0,\pm j1$. The perfect model-following conditions, Eq. (21.a)

$$\text{rank}(Bp)=\text{rank}(Bp;Bm)=\text{rank}(Bp;Am-Ap)=1 \qquad (29)$$

is satisfied. In this paper, the degree of polynomial, Eq. (24), is 2 ($n-m=3-1=2$). For an underdamped response with a damping ratio 0.5 and a natural frequency 3, then the desired eigenvalues are $\lambda=-1.5\pm j2.6$. Thus, the polynomial, Eq. (24), is

$$P(\lambda)=9+3\lambda+\lambda^2 \qquad (30)$$

and the matrix Q, Eq. (25), is

$$Q=[C_1\ C_2 \vdots\ 1]=[9\ \ 3\ \ 1] \qquad (31)$$

The Eq. (26) is

$$[\mathbf{I}-Bp(QBp)^{-1}Q]Am=\left[\begin{array}{c:c}0 & I_2\\ \hdashline 0 & -C_1-C_2\end{array}\right]=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -9 & -3\end{bmatrix} \qquad (32)$$

with $n-m$ eigenvalues $\lambda=-1.5\pm j2.6$. The remaining unforced system is stable. The control gain matrices, Eq. (10), Eq. (14), are

$$\begin{aligned} G &=[\ 0.9\quad 0.3\quad 0.1\ ]\\ Ke &=[-0.3\quad 0.65\quad -0.05]\\ Kp &=[-0.3\quad -0.15\quad -0.35]\\ Km &=[\ 0.3] \end{aligned} \qquad (33)$$

The sampling interval in the digital simulation is 0.01 (sec). For the case with $b(t)=10$ (nominal value) and $h_1(t)=h_2(t)=h_3(t)=0$, the outputs $Xm_1(t)$ and $Xp_1(t)$, the

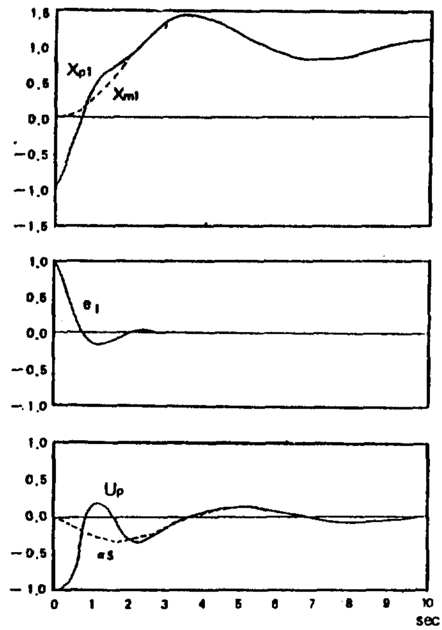

Fig. 3. Scalar control system: $h_1(t)=h_2(t)=h_3(t)=0, b(t)=10$

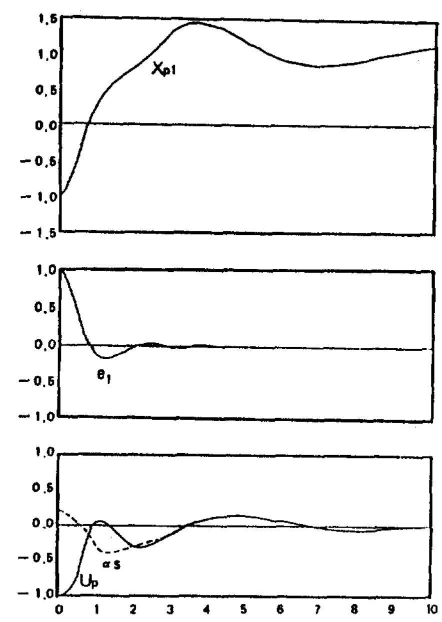

Fig. 4. Scalar control system: $h_1(t)=h_2(t)=0$, $|h_3(t)|\leq 0.05$, $b(t)=10+4\sin t$.

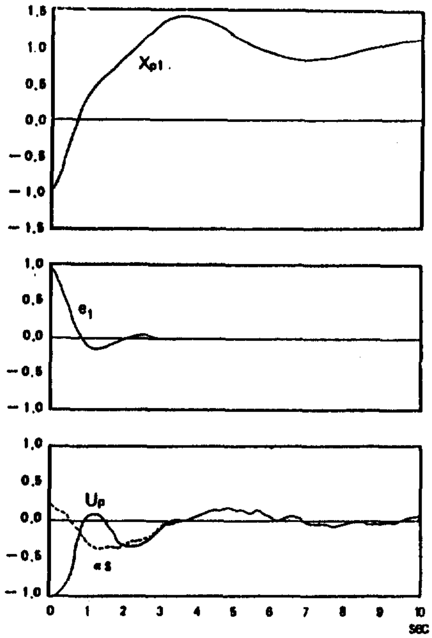

Fig. 5. Scalar control system:
$|h_1|<0.05$ $|h_2|<0.05$, $b(t)=10+4\sin t$.

ideal error transient $e_1(t)$, the control function $Up_1(t)$ and the slide mode σs are shown in Fig. 3.

For the case with time varying parameter $b(t)=10+4\sin t$, additive noise (ramdom signal $E\{h_3(t)\}=0$, $|h_3|<0.05$) and $h_1(t)=h_2(t)=0$, the response in Fig. 4 results good. The resulting response is very close to that of Fig. 3.

For the case that the noise disturbance does not satisfy the invariance condition, Eq. (21.b), satisfactory response can still be obtained in Fig. 5.

In comparison to Ref. 9), the sampling interval in the digital simulation can be increased and the control algorithm is simple, then the computational effort is little. Fig. 3-Fig. 5 show a smoother control signal to Ref. 9).

6.2 Multivariable System[9)]

We shall illustrate the 4th-order system with two controls

$$\dot{X}m(t)=\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -3 & -12 & -19 & -8\end{pmatrix}Xm(t)$$

$$+\begin{pmatrix}0\\0\\0\\3\end{pmatrix}Um(t) \qquad (34)$$

$$\dot{X}p(t)=\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -6 & -6 & -11 & a(t)\end{pmatrix}Xp(t)$$

$$+\begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 1\\ 1 & b(t)\end{pmatrix}Up(t)+\mathbf{h}(t) \qquad (35)$$

where, **h** (t) represent noise signals, a (t) and b (t) are an uncertain time varying gains, $Um(t)=4$ $(t<15)$ or 0 $(t>15)$ and all initial values are zero except $Xp_1(0)=-1$. .

The model has eigenvalues $\lambda_m=-3.6\pm j0.27, -0.4\pm j0.27$ and the plant has eigenvalues $\lambda_p=-1.16, -8.82, -0.008\pm j0.76$. The perfect modelfollowing conditions, Eq. (21.a)

$$\text{rank}(Bp)=\text{rank}(Bp;Bm)=\text{rank}(Bp;Am-Ap)=2 \qquad (36)$$

is satisfied.

The degree of polynomial, Eq. (24), is 2 $(n-m=4-2=2)$. In this paper, we take the desired eigenvalues $\lambda=-1,-1$. Thus the polynomial, Eq. (24), is

$$P(\lambda)=1+2\lambda+\lambda^2 \qquad (37)$$

and the matrix Q, Eq. (25), is

$$Q=\left[\begin{array}{cc|cc}C_1 & C_2 & 1 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]=\begin{bmatrix}1 & 2 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad (38)$$

The Eq. (26) is

$$[I-Bp(QBp)^{-1}Q]Am=\left(\begin{array}{c|cc|c}0 & 1 & 0 & 0\\ \hline & 0 & 1 & \\ & -C_1 & -C_2 & 0\\ \hline 0 & 0 & 0 & 0\end{array}\right)$$

$$=\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & -1 & -2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \qquad (39)$$

with $n-m$ eigenvalues $\lambda=-1,-1$. Thus remaining unforced system is stable.

The control gain matrices, Eq. (10), Eq. (14), are

$$G=\begin{bmatrix}-1 & -2 & -1 & 1\\ 1 & 2 & 1 & 0\end{bmatrix}$$

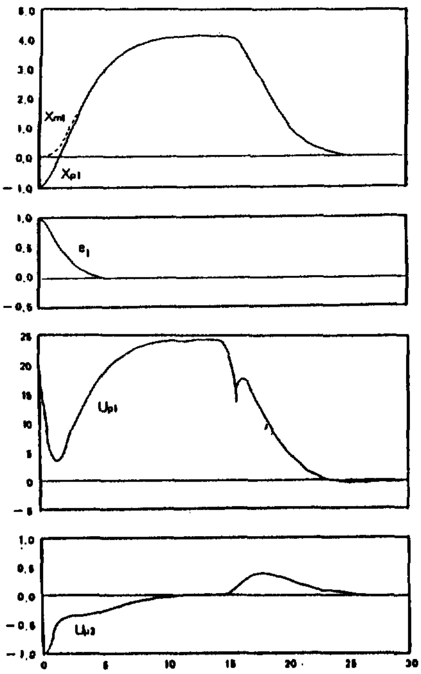

Fig. 6. Multivariable control system:
$h_1(t)=h_2(t)=h_3(t)=h_4(t)=0$, $a(t)=-10$, $b(t)=1$

$$
\begin{aligned}
Ke &= \begin{bmatrix} -3 & -13 & -21 & -9 \\ 0 & 1 & 2 & 1 \end{bmatrix} \\
Kp &= \begin{bmatrix} 3 & -6 & -8 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
Km &= \begin{bmatrix} 3 \\ 0 \end{bmatrix}
\end{aligned}
\tag{40}
$$

The sampling interval in the digital simulation is 0.01 (sec). For the case with $a(t)=-10$ (nominal value), $b(t)=1$ (nominal value) and $h_1(t)=h_2(t)=h_3(t)=h_4(t)=0$, the outputs $Xm_1(t)$ and $Xp_1(t)$, the ideal error transient $e_1(t)$, the control function $Up_1(t)$ and $Up_2(t)$ are shown in Fig. 6. For the case with time varying parameter $a(t)=-10(1+0.4\sin t)$, $b(t)=1+0.4\cos 2t$, without additive noise, response in Fig. 7 results.

For the case with the time varying parameter, additive noise (random signal $E\{h_4(t)\}=0$, $|h_4|\leq 0.05$) and $h_1(t)=h_2(t)=h_3(t)=0$, the response in Fig. 8 results.

For the case that the noise disturbance does not satisfactory the invariance condition, Eq. (21.b), satisfactory response can still be obtained in Fig. 9.

The resulting response of Fig. 7 and Fig. 8 is very close to that of Fig. 6, likewise the previous example. The reason

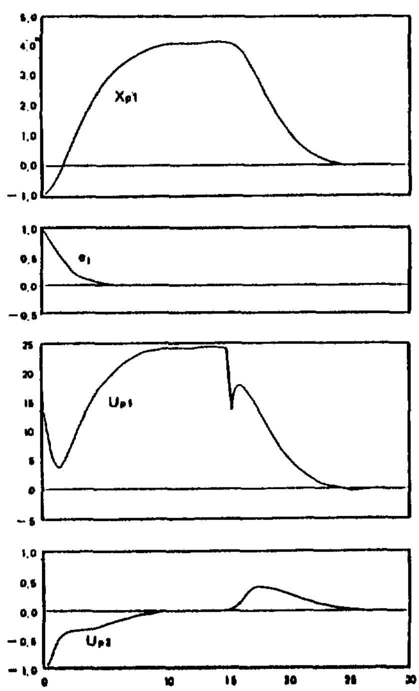

Fig. 7. Multivariable control system:
$h_1(t)=h_2(t)=h_3(t)=h_4(t)=0$
$a(t)=-10(1+0.4\sin t)$, $b(t)=1+0.4\cos 2t$

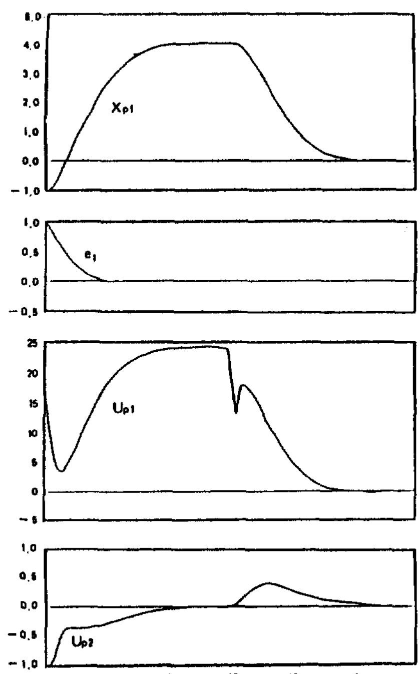

Fig. 8. Multivariable control system:
$h_1(t)=h_2(t)=h_3(t)=0$, $|h_4(t)|\leq 0.05$
$a(t)=-10(1+0.4\sin t)$, $b(t)=1+0.4\cos 2t$

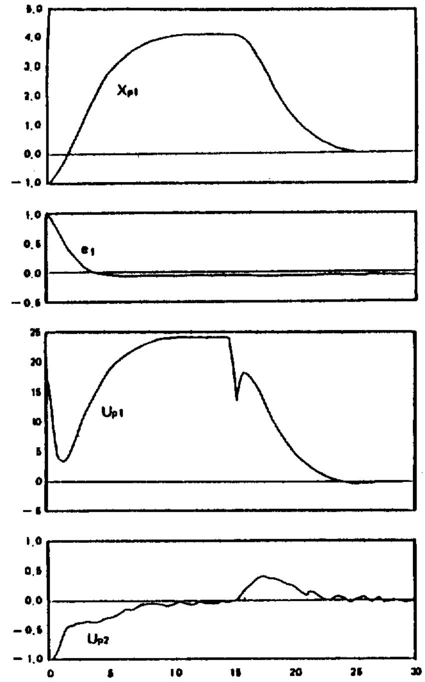

Fig. 9. Multivariable control system:
$|h_1(t)|\leq 0.05$, $|h_2(t)|\leq 0.05$, $|h_3(t)|\leq 0.05$,
$|h_4(t)|\leq 0.05$
$a(t)=-10(1+0.4\sin t)$, $b(t)=1+0.4\cos 2t$

is considered that both cases satisfy the perfect model-following conditions and the invariance condition. That is, the system response is insensitive to plant parameter variations and noise disturbance when it is operated in the slide mode.

6.3 An aircraft Control System[8]

We shall illustrate the three degree-of-freedom linearized longitudinal state equations of a conventional subsonic aircraft, a convair C-131B considered by Young[8].

$$\dot{X}m(t)=\begin{bmatrix} 0 & 1.0 & 0 & 0 \\ 5.318E\text{-}7 & -0.4179 & -0.1202 & 2.319E\text{-}3 \\ -4.619E\text{-}9 & 1.0 & -0.7523 & -2.387E\text{-}2 \\ -0.5614 & 0 & 0.3002 & -1.743E\text{-}2 \end{bmatrix} Xm(t)+\begin{bmatrix} 0 & 0 \\ -0.1717 & 7.451E\text{-}6 \\ -0.0238 & -7.783E\text{-}5 \\ 0 & 3.685E\text{-}3 \end{bmatrix} Um(t) \quad (41)$$

$$\dot{X}p(t)=\begin{bmatrix} 0 & 1.0 & 0 & 0 \\ 1.401E\text{-}4 & Ap(2,2) & -1.9513 & 0.0133 \\ -2.505E\text{-}4 & 1.0 & -1.3239 & -0.0238 \\ -0.561 & 0 & 0.358 & -0.0279 \end{bmatrix} Xp(t)+\begin{bmatrix} 0 & 0 & 0 \\ -5.3307 & 6.447E\text{-}3 & -0.2669 \\ -0.16 & -1.155E\text{-}2 & -0.2511 \\ 0 & 0.106 & 0.0862 \end{bmatrix} Up(t) \quad (42)$$

where, the state variables X_1, X_2, X_3 and X_4 are the pitch angle, pitch rate, angle of attack and air speed. The model control variables Um_1 and Um_2 are the elevator and throttle command inputs, respectively. The plant control variables Up_1, Up_2 and Up_3 are the elevator command deflection, throttle control and flap command deflection, respectively. The norminal value for Ap (2,2) is -2.038 and it is assumed to vary $-2.038+1.48\sin t$, that is, 75% variation around the nominal value. The reference model inputs are taken to be

$$Um^i(t)=\begin{cases} 1 & 0\leqq t\leqq 20 \text{ and } 40\leqq t\leqq 60 \text{ sec} \\ 0 & 20<t<40 \text{ sec} \end{cases} \quad (43)$$

All the state initial conditions are zero except $Xp_1(0)=1$.

The model has eigenvalues $\lambda_m=-0.59\pm j0.31$, $-7.12\pm j6.0$ and the plant has $\lambda_p=-1.68\pm j1.35$, $-1.32\pm j0.09$. The perfect model-following conditions, Eq. (21.a)

$$\text{rank}(Bp)=\text{rank}(Bp;Bm)=\text{rank}(Bp;Am-Ap)=3 \quad (44)$$

is satisfied.

The degree of polynomial, Eq. (24), is 1 ($n-m=4-3=1$). In this paper we take the desired eigenvalue $\lambda=-10$. Thus the polynomial, Eq. (24), is

$$P(\lambda)=10+\lambda \quad (45)$$

and the matrix Q, Eq. (25), is

$$Q=\begin{bmatrix} C_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} 10 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (46)$$

The Eq. (26) is

$$[I-Bp(QBp)^{-1}Q]Am=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -C_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (47)$$

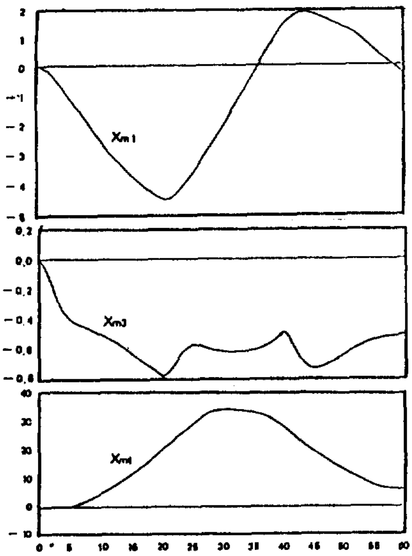

Fig. 10. Model response of the aircraft control system.

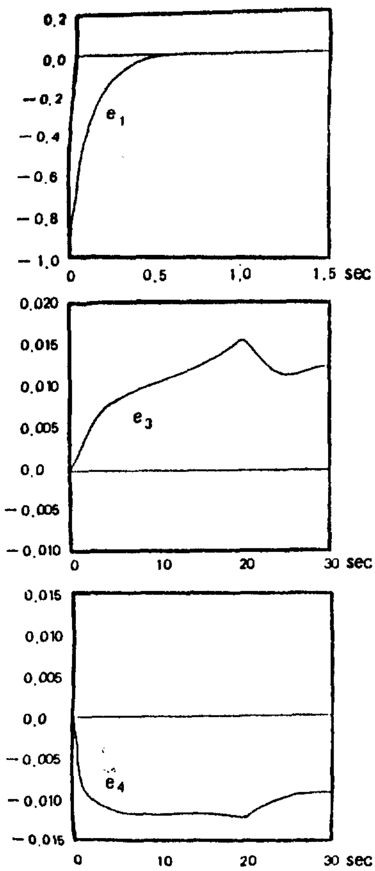

Fig. 11. Error transient of the aircraft control system.

with n−m eigenvalues $\lambda = -10$. Thus the remaining unforced system is stable.

The control gain matrices, Eq. (10), Eq. (14), are

$$
\begin{aligned}
G &= \begin{bmatrix} -1.942 & -0.194 & 0.219 & 0.036 \\ -1.045 & -0.105 & 3.482 & 9.820 \\ 1.285 & 0.129 & -4.282 & -0.474 \end{bmatrix} \\
Ke &= \begin{bmatrix} -0.020 & -1.642 & -0.130 & -0.006 \\ -5.513 & 2.481 & 0.341 & -0.255 \\ 0.266 & -3.050 & 3.063 & 0.111 \end{bmatrix} \\
Kp &= \begin{bmatrix} 0.00056 & -0.315 & 0.513 & -0.00002 \\ 0.00030 & -0.169 & 8.165 & -0.0057 \\ -0.00037 & 0.208 & -10.041 & 0.00028 \end{bmatrix} \\
Km &= \begin{bmatrix} 0.028 & 0.00011 \\ -0.065 & 0.036 \\ 0.080 & -0.0014 \end{bmatrix}
\end{aligned}
\qquad (48)
$$

The sampling interval in the digital simulation is 0.01 (sec). The model trajectories which are to be tracked by the plant trajectories are illustrated in Fig. 10.

Fi. 11 shows the error transients, and Fig. 12 shows the control functions.

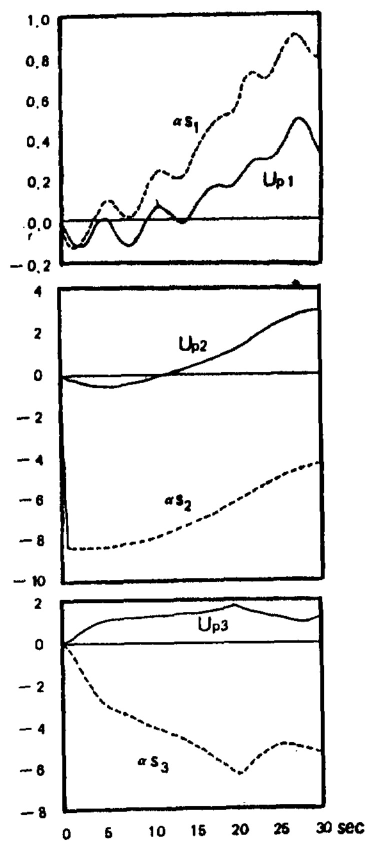

Fig. 12. Control input of the aircraft control system.

6.4 Discussions

The simulation results show that the algorithm in this paper offer some direct quantitative control over the error transient. But, most of the AMFC systems based on Lyapunov functions or the hyperstability concept do not offer any direct quantitative control[1), 2), 10)].

Though Young's approach[8)] for AMFC using the VSS theory offer a direct quantitative control, the control is discontinuous on a number of switching hyperplanes. It may be impractical to apply the discontinuous chattering input directly to the plant. In this paper, the simulation results show a smoother control signal in Fig. 3-Fig. 12.

For the case that perfect model-following conditions and the invariance condition are satisfied, a good model-following is obtained. Fig. 3-Fig. 4 or Fig. 6-Fig. 8 show that the control system exhibits insensitivity to parameter variations and noise disturbances. For the case that the noise disturbance does not satisfy the invariance condition, satisfactory response can still be obtained such as in Fig. 5 and Fig. 9.

In this paper, moreover, a suitable error transient motion is obtained arbitrarily. The reason is that the eigenvalues of the total control system can be placed arbitrarily in the complex plane by suitable choice of matrix Q. In the first example, the desired eigenvalues $-1.5\pm j2.6$ is selected. Thus the damping ratio is 0.5 and the natural frequency is 3. Then the error transient shows an underdamped response. It is in accord with Fig. 3-Fig. 5.

7. Conclusions

This paper describes a new method to the design of adaptive model-following control systems by slide mode. A summary of the results is shown below.

(1) Taking the switching surface matrix G such as Eq. (10), this algorithm can be easily applied to the multivariable control systems.

(2) Such as Eq. (13), the author takes αs instead of $-\delta$, therefore the algorithm is very simple. Satisfactory results are obtained in comparison to Young's Algorithm for computational time.

(3) The error transient can be prescribed by the design of matrix Q. The eigenvalues of the total control system can be placed arbitrarily in the complex plane by suitable choice of matrix Q such as Eq. (25). The choice of the model and matrix Q yields a suitable model transient and error transient motion.

(4) In examples, the figures show a smoother control signal. In other designs such as variable structure model-following control systems[6), 8), 9)], it may be impractical to apply the discontinuous chattering Up (t) directly to the plant. Therefore the insertion of a low pass filter ahead of the plant is not necessary.

(5) The gain matrices are obtained simply. And the design technique is easy. Therefore the designs can be carried out with a little computer effort. The implementation of the controller is easily carried out with the aid of microprocessors.

(6) The control system exhibits insensitivity to parameter variations and noise disturbances.

(7) Control function can be partitioned into a time invariant part and a time varying part, the former is LMFĈ mechanism and the latter is slide mode. Significant improvements are made on the transient behavior using this slide mode.

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