論 文
34~10~2

# 橫磁場內 螢光燈 플라즈마의 電流 및 光束에 대한 變化式 誘導

# The Derivation of Variation Equations for the Discharge Current and the Luminous Flux of Flourescent Lamp Plasma in a Transverse Magnetic Field

池 哲 根*・呂 寅 善**・朴 旺 烈***
(Chol-Kon Chee・In-Seon Yeo・Wang-Ryeol Park)

## 要 約

磁場內 低壓 放電管의 物理的 現象에 대해서는 많은 硏究가 이루어져 오고 있다. 본 論文은 螢光燈에 橫磁場을 印加하여, 이 磁場의 變化에 따른 放電電流와 光束의 變化式을 誘導하였는데, 磁場의 增加와 더불어, 電界, 電子溫度 및 光束은 增加 하지만 電流는 감소한다.

이들 誘導式을 실험값과 定量的으로 비교한 결과 여러가지 誤差要因을 고려하면, 잘 부합됨을 알 수 있다.

## Abstract

In this paper, an attempt is made to derive the variation equations of the discharge current and the luminous flux for flourescent lamp subjected to a transverse magnetic field. As the field increases, so do the electric field, the electron temperature, and the luminous flux, while the current decreases.

The theory is in a reasonable agreement with the experimental results, taking account of many approximations and restrictions.

## 1. Introduction

In the glow or arc discharge, when the positive column is subjected to a magnetic field, plasma parameters such as electron temperature and electron density undergo a change. It has been shown that when the magnetic field is transverse to the direction of the discharge current, the electron temperature increases, and the azimuthal electron density decreases, whereas if the magnetic field is longitudinal the reverse effect takes place[1].

In this paper, when the plasma of fluorescent lamp (ab.: F/L) is acted upon by a transverse magnetic field, the equations for variations of discharge current and luminous flux are to be derived, and then compared with the experimental results.

In F/L, the positive column of a low pressure discharge in a rare gas (Ar: 3 torr) plus mercury vapour ($5\times10^{-3}$ torr) produces ultra-violet radiation (253.7 or 185nm) which is transformed into visible radiation by the thin phosphor coating on the inside of the tube wall. The rare gas filling is necessary to ignite the lamp at a reasonable voltage and to arrive at a low mean free path for the electrons during the operation. By this addition, the electrons now follow a zigzag path which enhances enormously the chance of a collision with mercury atoms[2, 3].

*正 會 員: 서울大 工大 電氣工學科 敎授・工博
**正 會 員: 서울大 大學院 電氣工學科 博士課程修了
***正 會 員: 서울大 大學院 電氣工學科 博士課程
接受日字: 1985年 4月 12日

## 2. The Current Variation in a Transverse Magnetic Field

When a homogeneous magnetic field is applied at a right angle to the F/L axis, it deflects the column toward the wall with the result that the total loss of electrons and ions is increased, and then in order to compensate this loss the axial electric field increases, thus increasing the ionization and the electron temperatue[4].

From the Langevin equation, the conduction current density only due to the electric field $\overline{E}$ is:[5]

$$\overline{J} = \frac{Ne\, e^2}{m(\nu + jw)}\overline{E} \qquad (1)$$

Where $N_e$ is the electron density, e the electron charge, m the electronic mass, $\nu$ the collision frequency, and $w$ the source frequency.

In a low frequency discharge, we can neglect the source frequency term, so the discharge current relation as follows can be deduced.

$$J = C_1 \frac{N_e L_e}{\sqrt{T_e}} E \qquad (2)$$

Where $C_1$ is constant, $L_e$ is the electronic mean free path and $T_e$ the electron temperature.

The variation of electric field due to a transverse magnetic field was approximated by Sen et al. (1973)[6,7] as follows:

$$E_f/E = (\alpha + \frac{\beta^2}{\alpha})^{\frac{1}{2}} = (1 + C_2 B^2)^{\frac{1}{2}} \qquad (3)$$

$$\text{Where } \alpha \cong 1,\ \beta \cong \frac{e L_e}{m v_r} B,\ C_2 = (\frac{e L_e}{m v_r})^2, \qquad (4)$$

$V_r$ is the electron thermal velocity, and B the magnetic field strength.

Further it has been shown by Beckman (1948)[4] that due to a transverse magnetic field the electron density at a distance r from the axis is given by

$$N_{ef} = N_0 \exp(-\frac{C_3 r\cos\phi}{2D_a}) J_0 \{ r(\frac{z'}{D_a} - \frac{C_3^2}{4D_a^2})^{\frac{1}{2}} \} \qquad (5)$$

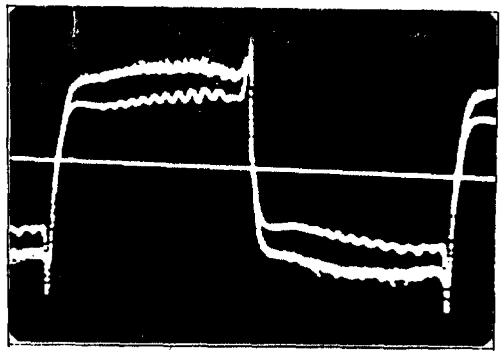

**Fig. 1.** The voltage increase in a transverse magnetic field of 300 (gauss).

Where $C_3 \cong \beta/\alpha\ \mu_i E$, $N_0$ is the electron density at the axis, $J_0$ the zero-order Bessel function, $D_a$ the ambipolar diffusion coefficient, z' the ionization frequency, $\mu_i$ the ionic mobility, and $\phi$ the azimuth in Fig. 2.

By the boundary condition at the tube wall with radius of R, $N_{ef}(R) = 0$,

$$R(\frac{z'}{D_a} - \frac{C_3^2}{4D_a^2})^{\frac{1}{2}} = 2.405 \qquad (6)$$

$$\text{and, } D_a = \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \cong \mu_i \frac{KT_e}{e} \quad \text{for } T_e \gg T_i \qquad (7)$$

Where $\mu_e$ is the electron mobility.

Substituting $\alpha$, $\beta$, $C_2$ and $C_3$ in Eq. (5), and considering the electron density profile of $N = N_0 J_0 (r\sqrt{z/D_a})$ in the absence of a magnetic field, it varies:

$$N_{ef}/N_e = \exp(-C_4 B) \qquad (8)$$

$$\text{Where } C_4 = \frac{\mu_i E r C_2^{\frac{1}{2}}}{2D_a}\cos\phi = \frac{e E r C_2^{\frac{1}{2}}}{2KT_e}\cos\phi \qquad (9)$$

It is well known that when a magnetic field acts upon an ionized gas, the equivalent pressure concept as developed by Blevin & Haydon (1958)[8] provides that the electronic mean free path and gas pressure F change as follows:

$$L_{ef}/L_e = (1 + C_2 B^2)^{-\frac{1}{2}} \qquad (10)$$

$$P_f / P = (1 + C_2 B^2)^{\frac{1}{2}} \quad (11)$$

Further, from the theory of positive column and assuming the Maxwell-Boltzmann distribution law, Von Engel (1965)[9] deduced that

$$z = C_5 P \left(\frac{eV_i}{KT_e}\right)^{-\frac{1}{2}} \exp\left(-\frac{eV_i}{KT_e}\right) \quad (12)$$

$$\text{thus, } \exp\left(\frac{eV_i}{KT_e}\right)\left(\frac{KT_e}{eV_i}\right)^{\frac{1}{2}} = C_6 P \quad (13)$$

Where $C_5$ and $C_6$ are the constants dependent upon a gas, k being Boltzmann constant and $V_i$ the ionization potential.

From Eqs. (4), (5), (6), (11), (12), and (13)

$$D_a\left\{\left(\frac{2.405}{R}\right)^2 + \frac{C_2 \mu_i^2 E^2}{4D_a^2} B^2\right\} = C_6 P_f \exp\left(\frac{-eV_i}{KT_{ef}}\right)^{-\frac{1}{2}} \quad (14)$$

Where $T_{ef}$ is the electron temperature in the case of applying a transverse magnetic field. Hence, Eqs. (11), (13), and (14) yield the following relation:

$$\exp\left\{\frac{eV_i}{K}\left(\frac{1}{T_{ef}} - \frac{1}{T_e}\right)\right\}\left(\frac{T_e}{T_{ef}}\right)^{\frac{1}{2}} \times (C_7 + C_8 B^2) = (1 + C_2 B^2)^{\frac{1}{2}} \quad (15)$$

Where $C_7 = D_a\left(\frac{2.405}{R}\right)^2$, and $C_8 = \frac{C_2 \mu_i^2 E^2}{4D_a}$

The variation of the electron temperature with the magnetic field will be represented by Eq. (15) as follows, for values of $T_{ef}$ not much different from $T_e$, $T_{ef}/T_e =$

$$\frac{\frac{2eV_i}{KT_e} + 1}{\frac{2eV_i}{KT_e} + Ln(1 + C_2 B^2) - 2Ln(C_7 + C_8 B^2) + 1} \quad (16)$$

Therefore, the discharge current in a transverse magnetic field can be deduced by Eqs. (2), (3), (8), (10), and (14), $J_f/J = \exp(-C_4 B) \times$

$$\left\{\frac{\frac{2eV_i}{KT_e} + 1}{\frac{2eV_i}{KT_e} + Ln(1 + C_2 B^2) - 2Ln(C_7 + C_8 B^2) + 1}\right\}^{-\frac{1}{2}} \quad (17)$$

## 3. The Variation of Luminous Flux in a Transverse Magnetic Field

The radiation energy of a spectral line arising from the transition between upper level u and lower level d is:[9]

$$W_r = N_U A_{U,d} h \nu_{U,d} \quad (18)$$

Where $N_u$ is the excited atom density (u-state), $A_{u,d}$ the transition probability, h Planck's constant, and $\nu_{u,d}$ the radiation frequency.

Suppose that the thermodynamic equilibrium is established, then according to Boltzmann's law the atomic density $N_u$ excited to a upper level $V_u$ volts from the ground state is:[10]

$$N_u = N_d \left(\frac{g_u}{g_d}\right) \exp\left(-\frac{eV_u}{KT_e}\right) \quad (19)$$

Where $g_u$, $g_d$ is statistical weight of the excited or ground state, respectively.

The densities of all the particles in the discharge tube, which consist of electrons, ions, ground state and excited state atoms, vary with the magnetic field.

If $N_k$ is k-level excited atom density, the total atom density $N_a$ is:

$$N_a = N_d + N_i + \sum_K N_K \quad (20)$$

As the ion density $N_i$ is equal to $N_e$, the following relation can be established from Eqs. (8) and (20)

$$N_{df}/N_d = 1 + C_9\{1 - \exp(-C_4 B)\} \quad (21)$$

$$\text{Where } C_9 = \frac{N_i}{N_a - N_i - \sum_K N_K} \quad (22)$$

Hence, the variation of luminous flux L, being proportional to the intensities of spectral lines, can be derivied from Eqs. (16), (18), (19), and (21) as follows:

$$L_f/L = [1 + C_9\{1 - \exp(-C_4 B)\}] \times \exp\left[\frac{eV_u}{KT_e} \times \left\{1 - \frac{\frac{2eV_i}{KT_e} + Ln(1 + C_2 B^2) - 2Ln(C_7 + C_8 B^2) + 1}{\frac{2eV_i}{KT_e} + 1}\right\}\right] \quad (23)$$

## 4. Experiments and Discussions

The discharge tube for F/L is cylindrical with the plasma length of about 330 (mm), and the diameter of 26 (mm) consuming power 10 Watts. This is shown in Fig. 2.

The magnetic field is provided by the electromagnet poles 140×70 (mm) and it is applied perpendicularly to

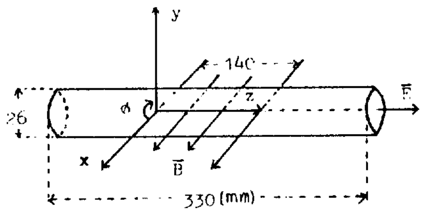


**Fig. 2.** The discharge tube in a transverse magnetic field: the magnetic field and the electric field are applied to it in the direction of x and z, respectively.

the axis of the positive column. That field will be varied from 0 to about 600 (gauss) with the current being supplied from d.c. source voltage.

Though four terms are known as the elements varying the discharge current, in this case only two terms, electron density and electron temperature are valid.

Therefore, the current must decrease because the electron density decreases and the electron temperature increases with a transverse magnetic field. This is represented quantitatively in Fig. 3, being compared with the experimental results.

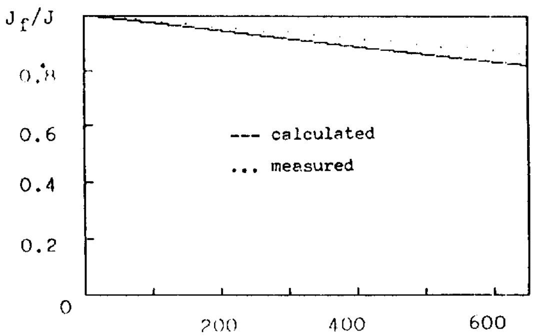


**Fig. 3.** The variation of discharge current in a transverse magnetic field (10W F/L).

The phenomena of voltage increase and current decrease in a transverse magnetic field can be regarded as an increase of the equivalent resistance in F/L circuit, which agrees with a little increase of power factor

$$(\text{p.f.} \quad \cos\rho = \frac{R}{|R + jwL_1|})$$

Further, the luminous flux, which increases according as the spectral line-intensities increase with a magnetic field, has been observed as shown in Fig. 4.

This shows bigger discrepancy between theory and ex-

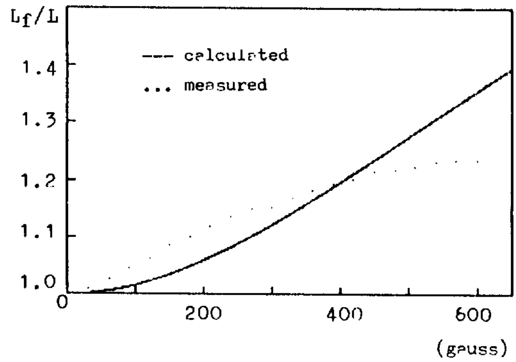


**Fig. 4.** The variation of luminous flux in a transverse magnetic field (10W F/L).

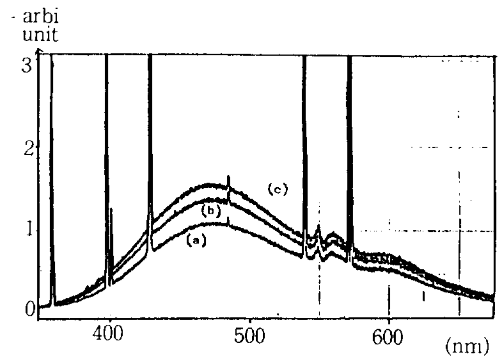


**Fig. 5.** The variation of spectrum distribution for 10W F/L in a transverse magnetic field:
(a) zero (b) 300 gauss (c) 600 gauss

**Table 1.** Measured results & the difference between theory and experiment for 10W F/L.

| ** | zero field | (%) increase | | (%) difference | |
|---|---|---|---|---|---|
| | | 300 (gauss) | 600 (gauss) | 300 (gauss) | 600 (gauss) |
| current | 0.217 (A) | -5.5 | -12.0 | 2.5 | 4.9 |
| luminous flux | 24.3 (Lux) | 15.2 | 23.5 | 2.7 | -8.9 |
| voltage | 43.9 (V) | 10.9 | 21.7 | 12.1 | 70.3 |

perimental result than that of the current (cf. Table 1), which can be explained as follows:

(1) Application of Boltzmann's law assuming a thermodynamic equilibrium.

(2) Experimental restrictions–the partiality of applying magnetic field to F/L, etc.

(3) Not considering the variation of all the densities of excited atoms.

The spectrum distribution shows that the intensities of spectral lines increase with a magnetic field without any shift as shown in Fig. 5.

## 5. Conclusion

Taking advantage of Beckman's expression for the electron density distribution, Sen et al.'s approximation for the electric field strength, and Blevin's equivalent pressure concept, three equations of electron temperature, discharge current, and luminous flux have been derived for the F/L plasma.

In a transverse magnetic field, as the field increases, so do the electric field, the electron temperature, the luminous flux, and the power factor, while the discharge current decreases.

To test the derived equations, some measurements have been made on 10W F/L. Theory and experiment show fair agreement, taking account of the limits of accuracy and of uncertainty in the constants and many approximations in the calculations.

However, the following problems need further attention:

(1) Regarding the ionizing process as only a one-step ionization,

(2) The application of Boltzmann's law to obtain the density of excited states,

(3) Neglecting the variation of electron density along the axis.

**References**

1) S.K. Sadhya & S.N. Sen, "Mercury Arc Plasma in an Axial Magnetic Field", Int. J. Electronics, Vol. 49, No. 3, pp. 235-247, 1980.

2) W. Elenbaas, "Light Sources", Macmillan, 1972, pp. 103-110, 10-13.

3) Chee Chol Kon, "Illuminating Engineering", Moon-un-dang, 1982, pp. 69-72.

4) Lars Beckman, "The Influence of a Transverse Magnetic Field on a Cylindrical Plasma", Proc. Phys. Soc., 61, pp. 515-520, 1948.

5) Blake E. Cherrington, "Gaseous Electronics and Gas Lasers", Pergamon Press, 1979, pp. 11-12.

6) S.N. Sen & R.P. Das, "Voltage-Current Characteristic and Power Relation in Arc Plasma in a Transverse Magnetic Field", Int. J. Electronics, Vol. 34, No. 4, pp. 527-536, 1973.

7) S.N. Sen & R.N. Gupta, "Variation of Discharge Current in a Transverse Magnetic Field in a Glow Discharge", J. Phys. D (Appl. Phys.) pp. 510-517, 1971.

8) H.A. Blevin & Haydon, "The Townsend Ionization Coefficients in Crossed Electric and Magnetic Fields", Aust. J. Phys., 11, pp. 18-34, 1958.

9) A. Von Engel, "Ionized Gases (2nd ed)", Oxford at the Clarendon Press, pp. 242-243, 1965.

10) S.N. Sen, R.P. as & R.N. Gupta, "Spectral Line Intensity in a Glow Discharge in a Transverse Magnetic Field", J. Phys. D (Appl. Phys.), Vol. 5, pp. 1260-1266, 1972.