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Analytical Estimation of Thermoelastic Damping

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열탄성 진동감쇠에 관한 해석적 연구

이 우 식

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초 록

다음 세대의 우주 비행선(spacecraft) 및 우주 구조물 등은 매우 정밀한 수행 능력은 물론 발사 경비의 절감을 위해 자체 무게를 최대한 줄일 수 있는 설계 방안을 요구하고 있다. 이같은 중요한 요구 조건들을 만족시킬 수 있는 진동 제어의 한 방법으로써 재료 고유진동감쇠(material damping)의 효과적인 응용이 매우 중요시 되고 있다.

따라서, 본 연구에서는 열기류(thermal currents)에 의해 발생하는 열탄성 진동감쇠(thermoelastic damping)를 연관 열탄성학(coupled thermoelasticity)에 근거하여 해석적으로 추정하고 진동감쇠율에 구조적, 기하학적 형태가 미치는 영향에 관해 고찰하였다. 단일 구조물의 형태, 경계조건 및 진동모우드 등이 진동감쇠에 미치는 영향을 새로운 계수로서 공식화 하였으며 아울러 최대 진동감쇠율을 얻을 수 있는 최적조건을 고찰하였다.

Nomenclature

a_p	: Fourier coefficients
C_k	: Amplification factors
c_v	: Constant-strain specific heat (unit mass)
D	: $Eh^3/12(1-\nu^2)$, plate flexural rigidity
e	: $\epsilon_{11}+\epsilon_{22}+\epsilon_{33}$
E	: Young's modulus
$f(\alpha_3)$	: Function defined in Eq. (4)
h	: Shell thickness
i	: $\sqrt{-1}$, imaginary unit
k	: Thermal conductivity
K	: $Eh/(1-\nu^2)$
K_{11}, K_{22}	: The changes in curvatures of middle surface
L_1, L_2	: Dimensions along α_1, α_2 coordinates
M_k	: Quantity defined in Eq. (10)
R_1, R_2	: Radii of shell curvatures in α_1, α_2

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	coordinates
t	: Time coordinate
ΔT	: $T-T_0$, temperature disturbance
T_0	: Constant reference absolute temperature
U	: Strain energy
ΔU	: Energy dissipated
w	: Deflection of shell in α_3 coordinate
W	: Normal modes (with subscripts)
$\alpha_1, \alpha_2, \alpha_3$	: Orthogonal curvilinear coordinates
α_t	: Linear thermal coefficient of expansion
Γ_k	: Quantity defined in Eq. (14)
δ	: Logarithmic decrement (with subscripts)
$\delta(\ldots)$	: Delta function
Δ	: $T_0\alpha^2E/\rho c_v$, thermal relaxation strength
ϵ_{ij}	: Mechanical strain tensor
η	: Damping loss factors (with subscripts)
η_D	: $\Delta[\omega\tau/(1+\omega^2\tau^2)]$, Debye formula
η_t	: Quantity defined in Eq. (13)
θ	: Dimensionless constant ($\cong 1$)
ν	: Poisson's ratio
Π	: Factor defined in Eq. (23)
ρ	: Material density
σ_{ij}	: Mechanical stress tensor
τ	: $\rho c_v h^2/\pi^2 k$, dimensionless time
ω	: Frequency (with subscripts)
$\Delta\omega$	: $\omega_1-\omega_2$, half-power bandwidth
∇	: Nabla operator
$\cdot$	: $\partial/\partial t$, time derivative

Subscripts

$(\ldots)_k$	: Property of k^{th} (or mn^{th}) vibration mode
$(\ldots)_{max}$	: Maximum value of $(\ldots)$
p	: Plate
s	: Shell

1. Introduction

Material damping arises from several physical sources, and it is therefore difficult to predict accurately. Nevertheless, reasonably accurate damping information is often required to design a system properly for dynamic loadings. There exists considerable literature on both analytical and experimental aspects of the subject. Lazan(1) and Nowick and Berry(2) provide useful summaries of what was known up to their dates of publication. The author has found, however, relatively few fundamental theoretical studies of internal energy dissipation (or material damping).

The inherent dissipation in monolithic solids tends to be small compared to the damping furnished artificially by dashpots, constrained viscoelastic layers or interconnections, joints and bearings. This is believed to explain why the role of material damping is frequently omitted or underplayed in the extensive literature on damping analysis and active control of Large Space Structures (LSS). Several authors (e.g., Gevarter(3), Ashley(4)) have given some consideration to the possibly important role of material damping on the stabilization of structures. Reference (4) observed that a tiny amount of structural damping is useful for meeting the control system requirements of LSS like telescopes and antennas in space.

In order to analyze the internal energy dissipation of a given structure, one should take into account all possible damping mechanisms, depending upon the specific material. In practical cases, however, one or two mechanisms generally predominate, the others being comparatively negligible.

In 1938, Zener(5) predicted that thermoelastic damping (simply thermal damping) of monolithic crystalline solids is often much greater than the damping due to all other mechanisms. Experiments of Bennewitz and Rötger(6) confirmed that his predictions are accurate. Thermal damping is almost universal. But, under certain conditions with high electromagnetic (EM) field, the electromagneto-elastic damping (simply electromagnetic damping) is even of a larger order of magnitude.

2. Thermal Damping Analysis

2.1 Background

Zener[5,7] was apparently the first to point out that the energy dissipation in vibrating metals must be sought in stress inhomogeneities, giving rise to temperature gradients and hence to local thermal currents which increase the entropy. Biot[8] discussed irreversible thermodynamics in vibrating systems and applied a generalized coordinate method to the calculation of internal energy dissipation. Tasi and Herrmann[9,10] investigated a crystal plate by means of a variational principle. Chadwick[11], in 1962, showed that his results from normal mode analysis agree with Zener's theory[5]. Later, Alblas[12] developed a general theory of energy dissipation in a three-dimensional finite body.

2.2 Basic Formulation

The two governing equations of the linearized coupled thermoelasto-dynamics are given by[13,14]

$$D\nabla_1{}^8 w+\left(\frac{E}{1-\nu}\right)\nabla_1{}^6\int_{-h/2}^{h/2}\alpha_t\Delta T\alpha_3 d\alpha_3+Eh\nabla_k{}^4 w +\rho h\nabla_1{}^4\ddot{w}=\nabla_1{}^4 F \quad (1)$$

$$\nabla^2(\alpha_t\Delta T)-\left(\frac{\rho C_v}{k}\right)\frac{\partial(\alpha_t\Delta T)}{\partial t} =\frac{T_0\alpha_t{}^2 E}{k(1-2\nu)}\frac{\partial e}{\partial t} \quad (2)$$

where

$$\nabla_k{}^2(\cdot)=\frac{1}{A_1A_2}\left\{\frac{\partial}{\partial\alpha_1}\left[\frac{1}{R_2}\frac{A_2}{A_1}\frac{\partial(\cdot)}{\partial\alpha_1}\right] +\frac{\partial}{\partial\alpha_2}\left[\frac{1}{R_1}\frac{A_1}{A_2}\frac{\partial(\cdot)}{\partial\alpha_2}\right]\right\}$$

A_1, A_2 are Lamé parameters and $\nabla_1{}^2$ and ∇^2 are two and three dimensional Laplacian operators in orthogonal curvilinear coordinates (α_1, α_2, α_3), respectively. Note that α_3 is the coordinate in the direction normal to the middle surface of a shell defined by (α_1, α_2). F is the external force and the other symbols are defined in nomenclature.

Equation (1) is the equation of transverse motion, which is believed sufficiently accurate to estimate quickly the effects of curvatures in relatively shallow shells. Equations for plates and beams are readily recovered by forcing the radii of shell curvatures, $R_1=R_2=\infty$ and aoisson's ratio, $\nu=0$. Because of this adaptability, plates and beams are easily recovered from the results obtained by solving Eqs. (1) and (2) for shallow shells. Equation (2) is the heat conduction equation, in which the influence of the curvatures of shell on thermal flux is neglected. One notes that the two governing equations include small coupling terms between elastomechanical and thermodynamic behaviors which give rise to the damping of the vibration.

The general theory of shallow shells usually assumes as above the normal stress σ_{33} is negligible along with shear strains ϵ_{13} and ϵ_{23}. Under this assumption, the reduced Hooke's law and the strain-displacement relations[13] approximately give the dilational part of the displacement field (simply dilatation) in the form

$$e=\epsilon_{11}+\epsilon_{22}+\epsilon_{33} \cong-\left[\frac{1-2\nu}{1-\nu}\right]\alpha_3\nabla_1{}^2 w+\left[\frac{1+\nu}{1-\nu}\right]\alpha_t\Delta T \quad (3)$$

From the physics of the situation and the forms of Eqs. (2) and (3), a logical approximation to the elastomechanical coupling would seem to be

$$\alpha_t\Delta T(\alpha_1,\alpha_2,\alpha_3,t)\cong f(\alpha_3)\nabla_1{}^2 w(\alpha_1,\alpha_2,t), \quad (4)$$

which reduces Eqs. (1) and (2) into the forms:

$$D(1+i\eta_t)\nabla_1{}^8 w+Eh\nabla_k{}^4 w+\rho h\nabla_1{}^4\ddot{w}=\nabla_1{}^4 F \quad (5)$$

$$\frac{d^2f}{d\alpha_3^2}+\left[\frac{\nabla_1^4 w}{\nabla_1^2 w}-\left(\frac{\rho c_v\theta}{k}\right)\frac{\nabla_1^2\dot{w}}{\nabla_1^2 w}\right]f+\left[\frac{\rho c_v\Delta}{k(1-\nu)}\frac{\nabla_1^2\dot{w}}{\nabla_1^2 w}\right]\alpha_3=0 \quad (6)$$

where

$$\eta_t\equiv\text{Imag. part of}\left\{\frac{E}{D(1-\nu)}\int_{-h/2}^{h/2}f(\alpha_3)\alpha_3 d\alpha_3\right. \quad (7)$$

$$\Delta\equiv\frac{T_0\alpha_t^2 E}{\rho c_v}$$

$$\theta\equiv 1+\Delta\left[\frac{1+\nu}{(1-\nu)(1-2\nu)}\right]\cong 1$$

One notes that only the imaginary part of the integration in Eq. (7) is taken because of its contribution to the damping of vibration. The modified equation of motion (5) now contains the complex plate flexural rigidity $D(1+\eta_t)$. θ is approximately equal to unity because always the thermal relaxation strength $\Delta\ll 1$[7].

In order to investigate free vibration, one assumes harmonic motion in the form

$$w=\sum_{mn}^{\infty}C_{mn}W_{mn}e^{i\omega t}=\sum_{k}^{\infty}C_k W_k e^{i\omega t} \quad (8)$$

(Indices mn are replaced by k for convenience.) Here C_k is the amplification factor of the k^{th} normal mode, W_k, which satisfies the following equations:

$$D\nabla_1^8 W_k+Eh\nabla_k^4 W_k-\rho h\omega_k^2\nabla_1^4 W_k=0 \quad (9)$$

$$\int_A W_k W_l dA=M_k\delta_{kl} \quad (10)$$

where $dA=d\alpha_1 d\alpha_2$ is the plate area element and δ_{kl} is the Kronecker delta. It is convenient to expand $f(\alpha_3)$ in Fourier series, following the lead of Zener[5],

$$f(\alpha_3)=\sum_{p=0}^{\infty}a_p\sin(2p+1)\frac{\pi\alpha_3}{h}, \quad (11)$$

which satisfies insulated boundary conditions at the upper and lower shell surfaces. These are appropriate to the vacuum of space, and one assumes no energy loss due to heat convection or radiation. Substituting Eqs. (8) and (11) into Eq. (6) and using the orthogonality property of Fourier series, one may solve for the coefficients a_p of Eq. (11) to find that $a_p\ll a_0$ for $p>1$. Therefore, a one-term approximation is acceptable with an error typically $<2\%$:

$$f(\alpha_3)\cong[f_R+if_I]\sin\frac{\pi\alpha_3}{h}, \quad (12)$$

where

$$f_R=\frac{1}{\pi^2}\frac{4h}{1-\nu}\Delta\frac{\omega^2\tau^2}{1+\omega^2\tau^2}$$

$$f_I=\frac{1}{\pi^2}\frac{4h}{1-\nu}\Delta\frac{\omega\tau}{1+\omega^2\tau^2}$$

Here we used the approximations $\theta\cong 1$ and $O[(h/L)^2]\cong 0$ for the shallow shells. τ is the characteristic time, which controlled by the choice of material, specimen shape and size, defined by

$$\tau\equiv\frac{\rho c_v h^2}{\pi^2 k}$$

Substituting Eq. (12) into Eq. (7) gives

$$\eta_t\cong\frac{96}{\pi^4}\left(\frac{1+\nu}{1-\nu}\right)\left[\Delta\frac{\omega\tau}{1+\omega^2\tau^2}\right] \quad (13)$$

Here the square-bracketed portion of Eq. (13) is called the Debye formula, η_D.

2.3 Free Vibration

Consider free vibration with forcing $F=0$ and assume harmonic motion, Eq. (8). Then, using the orthogonality property of Eq. (10), the equation of motion can be reduced in the form

$$\rho h(\omega^2-\omega_k^2)+i\eta_t(\Gamma_k-\rho h\omega_k^2)=0 \quad (14)$$

where

$$\Gamma_k=\Gamma_{mn}=Eh\frac{\nabla_k^4 W_k}{\nabla_1^4 W_k}$$

Because of small η_t, Γ_k are treated as a constant without causing significant error. In fact, Γ_k is found to be constant for the simply supported structure with harmonic motions considered in this study.

One measure of free-oscillation decay is the logarithmic decrement, which yields the modal damping loss factor, as follows:

$$\frac{\delta_k}{\pi}\cong\frac{96}{\pi^4}\left(\frac{1+\nu}{1-\nu}\right)\Delta\frac{\omega_k\tau}{1+\omega_k^2\tau^2}\left[\frac{\bar{\omega}_k}{\omega_k}\right]^2 \quad (15)$$

where

$$\bar{\omega}_k^2=\omega_k^2-\frac{\Gamma_k}{\rho h}.$$

2.4 Forced Vibration

Consider the vibration forced by a concentrated load acting at point $(\bar{\alpha}_1, \bar{\alpha}_2)$. In terms of modal modes governed by Eq. (9), Eq. (5) can be written

$$\rho h\sum_k^{\infty}(\omega_k^2-\omega^2)C_kW_k-i\eta_t\sum_k^{\infty}(\Gamma_k-\rho h\omega_k^2)C_kW_k$$
$$=P_0\delta(\alpha_1-\bar{\alpha}_1)\delta(\alpha_2-\bar{\alpha}_2) \quad (16)$$

Using again the orthogonality property of Eq. (10), one finds the amplification factors C_k, as follows:

$$C_k=\frac{P_0W_k(\bar{\alpha}_1,\bar{\alpha}_2)}{\rho hM_k[(\omega_k^2-\omega^2)+i\eta_t(\omega_k^2-\Gamma_k/\rho h)]} \quad (17)$$

As an estimate of damping, the half-power bandwidth $\Delta\omega=\omega_1-\omega_2$ is ready obtained from the amplication factor C_k. Then one measure of damping for the k^{th} mode of vibration is simply

$$\frac{\Delta\omega}{\omega_k}\cong\frac{96}{\pi^4}\left(\frac{1+\nu}{1-\nu}\right)\Delta\frac{\omega_k\tau}{1+\omega_k^2\tau^2}\left[\frac{\bar{\omega}_k}{\omega_k}\right]^2 \quad (18)$$

which proved identical to the logarithmic decrement, Eq. (15).

Another classical measure of damping is the loss factor η, defined as the ratio of energy dissipated in unit volume per radian of oscillation to the maximum strain energy per unit volume, that is

$$\eta=\frac{\Delta U}{2\pi U_{\max}} \quad (19)$$

Here

$$U\cong\frac{1}{2}\int_A\int_{-h/2}^{h/2}[\sigma_{11}\epsilon_{11}+\sigma_{22}\epsilon_{22}]d\alpha_3dA \quad (20)$$

$$\Delta U\cong\int_A\int_{-h/2}^{h/2}\int_0^{2\pi}[\sigma_{11}\dot{\epsilon}_{11}+\sigma_{22}\dot{\epsilon}_{22}]d\omega t\ d\alpha_3dA \quad (21)$$

In Eqs. (20) and (21), approximations have been made consistent with the foregoing derivations. Space limitations prevent reproducing detail of a consistent analysis, which leads to the expression

$$\eta=\frac{96}{\pi^4}\left(\frac{1+\nu}{1-\nu}\right)\Delta\frac{\omega\tau}{1+\omega^2\tau^2}\Pi \quad (22)$$

with

$$\Pi=\frac{\int_A[K_{11}+K_{22}]^2dA}{\int_A[K^2_{11}+2\nu K_{11}K_{22}+K^2_{22}]dA} \quad (23)$$

where K_{11} and K_{22} are bending strains(13). In any practical examples, $\Pi>1$ since $\nu<0.5$. However, one notes that $\Pi=1$ for the structures vibrating one-dimensionally like beam-plates. For the simply supported structures, considered in this paper, Π has the general form:

$$\Pi=\frac{\sum_k^{\infty}\bar{C}_k^2\left[\left(\frac{m\pi}{L_1}\right)^2+N^2\left(\frac{n\pi}{L_2}\right)^2\right]^2}{\sum_k^{\infty}\bar{C}_k^2\left[\left(\frac{m\pi}{L_1}\right)^4+2\nu N^2\left(\frac{m\pi}{L_1}\right)^2\left(\frac{n\pi}{L_2}\right)^2+N^4\left(\frac{n\pi}{L_2}\right)^4\right]} \quad (24)$$

where $\bar{C}_k$ is the magnitude of C_k, Eq. (17). L_1 and L_2 are the full dimensions of a structure along the coordinates α_1 and α_2, respectively. Note that $N=1$ for the rectangular flat plates and curved panels, and $N=2$ for the cylindrical shells and barrel-shaped shells(13).

Investigation of amplification factor C_k shows that the k^{th} normal mode predominates when the circular frequency ω is near the k^{th} natural frequency ω_k. Then one can use the approximation

$$\eta\cong\frac{96}{\pi^4}\left(\frac{1+\nu}{1-\nu}\right)\Delta\frac{\omega\tau}{1+\omega^2\tau^2}\Pi_k(\omega\cong\omega_k), \quad (25)$$

where the Π_k (or Π_{mn}) for simply supported structures are given by

$$\Pi_{mn}=\frac{\left[\left(\frac{m\pi}{L_1}\right)^2+N^2\left(\frac{n\pi}{L_2}\right)^2\right]^2}{\left(\frac{m\pi}{L_1}\right)^4+2\nu N^2\left(\frac{m\pi}{L_1}\right)^2\left(\frac{n\pi}{L_2}\right)^2+N^4\left(\frac{n\pi}{L_2}\right)^4} \quad (26)$$

The k^{th} modal loss factors for simply supported structures, without further approximation, are readily obtained from Eq. (22) in the form:

$$\eta_k = \frac{96}{\pi^4}\left(\frac{1+\nu}{1-\nu}\right)\Delta\frac{\omega_k\tau}{1+\omega_k^2\tau^2}\Pi_k \qquad (27)$$

3. Discussion of Results

Equations (15), (18) and (27) provide measures of modal damping at frequencies near natural frequencies. For structures vibrating one-dimensionally, such as beams and beam-plates, these equations give exactly identical results as the case of mass-spring-dashpot system. In general, however, there exists no unique expression suitable as a measure of damping, even at a natural frequency. One may therefore ask which measure of damping is the most meaningful and accurate. The author has concluded that this question has no definitive answer. As Jones[15] observed, this ambiguity is really not a serious problem. When comparing different materials and configurations, one must simply employ consistent, clearly defined measures. As far as small damping is concerned, every measure must provide the same useful information. For this reason, the author has adopted the loss factor η as a vehicle for further investigations.

How to maximize the loss factor seems to be the most interesting issue for damping analysis. Maximization of damping is not a simple matter, because of the complicated characteristics of vibration problem. Figures 1 through 3 have been calculated to illustrate factors, Π, and loss factors, η, for simply supported structures with same surface areas (i.e., $L_1=L_2=2m$). As a preliminary, Fig. 1 and study of C_k, Eq. (17), demonstrate that factor Π is nearly independent of circular frequency and structural thickness. These factors are clearly important for the part of η_D of Eq. (13). Figure 2 shows that loss factor increases at very low frequencies and decreases at high frequencies as the thickness increases. It also demonstrates that loss factor is almost proportional to the reference absolute temperature.

From earlier development, for a given material and structure, η_D and Π can be represented by

$$\eta_D \cong \eta_D(\omega, h)\,;\; \Pi \cong \Pi(L_1, L_2, m, n) \qquad (28)$$

Then it is obvious that η_D has its maximum value at frequency $\omega \cong 1/\tau$, which is called the Debye peak. The thickness for maximum η_D is readily obtained from

$$h \cong \pi\left[\frac{k}{\rho C_v \omega}\right]^{1/2} \qquad (29)$$

Since damping plays its most important role at frequencies near natural frequencies, it is valuable to maximize the modal factor Π_{mn} of Eq. (26). An optimal combination of geometry

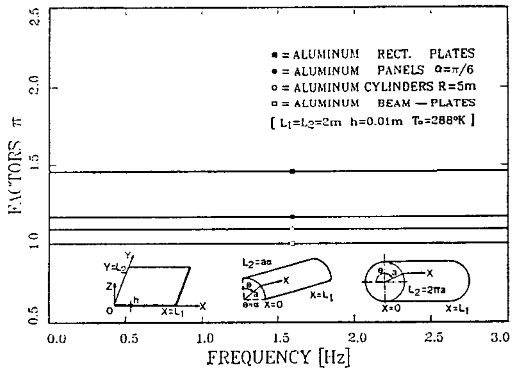

Fig. 1 Frequency dependence of factor Π

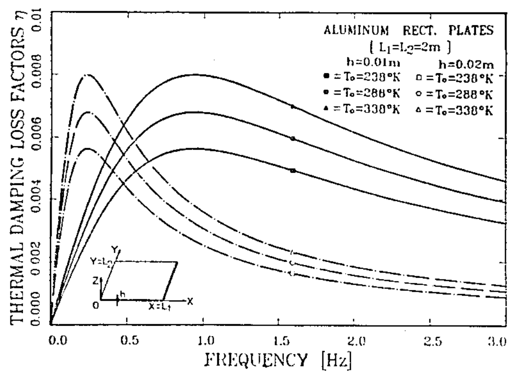

Fig. 2 Temperature and thickness dependence of loss factor η

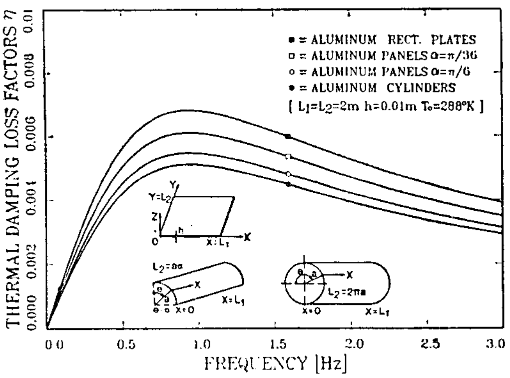

Fig. 3 Curvature dependence of loss factor η

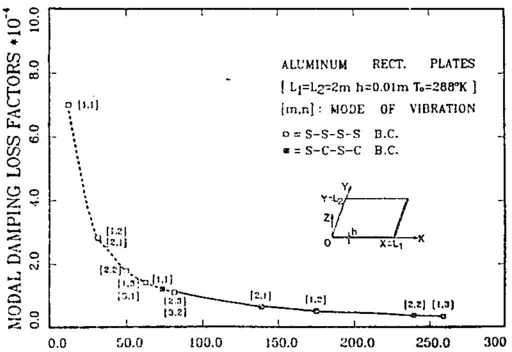

Fig. 4 Boundary condition dependence of modal loss factor η_k

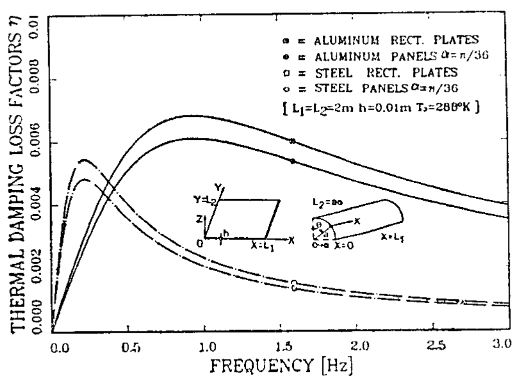

Fig. 5 Structural material dependence of loss factor η

and mode of vibration for the maximum value of Π_{mn} is found to be

$$\frac{L_2}{L_1} \simeq N\frac{n}{m} \tag{30}$$

Equations (29) and (30) will be useful for designers who wish to maximize the damping of vibration of the sort considered here.

Consider the modal loss factors of the plates and shells which are made of same material. At a natural frequency, when the same dimensions (L_1, L_2) and mode (m, n) are selected for the plate and shell, it follows that

$$\frac{\eta_s}{\eta_p} = \frac{\omega_s}{\omega_p} \frac{1+\omega_p{}^2\tau}{1+\omega_s{}^2\tau} \tag{31}$$

Since $\omega_p\omega_s > 1/\tau$ in general, the modal loss factor of a plate tends to be larger than that of a shell. Figures 1 and 3 show that plates do indeed have the largest damping, followed by panels, cylinders, beam-plates and simple beams. The damping of a panel gets closer to that of a plate as it gets flatter. Also the damping of a curved panel gets closer to that of cylinder as it approaches the shape of the cylinder (see Fig. 3). It is also found that the damping of a barrel shaped shell is larger than that of a cylinder, but again it gets closer to the other as the radius of barrel curvature increase. Figure 4 has been calcurated to illustrate the modal damping loss factors for simply supported and partially clamped plates at the first five fundamental frequencies. Simply supported plates achieve higher damping than partially clamped plates. Structural material dependence of loss factors is shown in Fig. 5. Aluminum structures achieve higher loss factor than steel structures at most frequency ranges except at very low

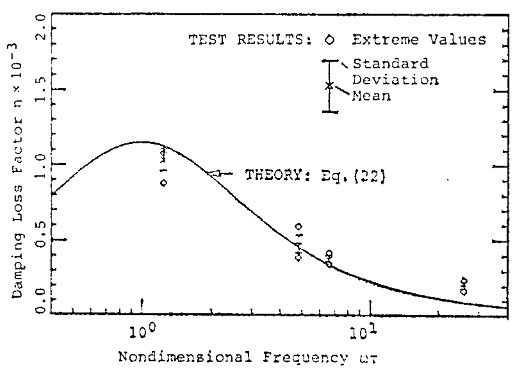

Fig. 6 Damping test results for aluminum beams

frequencies.

Recently, an experimental damping measurement was conducted by Edberg[16]. He developed a new testing method wherein possible every external influence on the test specimen was eliminated to the maximum extent possible, except for the telemetry package mounting. The test results for an aluminum beam at four different frequencies of vibration are shown in Fig. 6. Taken as a whole, the data agree well with the author's theoretical predictions, within experimental error. The author thus has confidence that the theoretical damping loss factor defined by Eq. (22) is also valid for the damping predictions of the other structures.

Without observation of what is really happening inside the material, it seems to be very difficult to clarify the foregoing results with a reasonable physical interpretation. From the heat conduction Eq. (2), however, one can conclude that the structure will experience higher damping when the rate of dilatation gets larger. When the geometry and boundary conditions for a particular structure are likely to increase the rate of dilatation, the structure will achieve great damping. Constraints on a structure seem to prevent increasing the rate of dilatation with increasing natural frequency. Curved and clamped structures have more constraints than flat and simply supported structures. For more detailed discussion, the reader is reffered to the author's earlier work[17].

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