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An Adaptive Trajectory Control of Manipulators

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로봇의 軌道 制御에 관한 硏究

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Key Words: Manipulator(메니퓰레이터), Adaptive Control(적응제어), Trajectory Control(궤도제어), Task-Oriented Space(작업공간)

抄 錄

작업 공간내에서 원하는 속도와 가속도로, 주어진 경로를 따라 이동하는 k차원 좌표계를 구성하고, 메니퓰레이터의 운동 방정식을 이 좌표계로 변환하여 운동 경로에 대한 선형화 식을 구하였다. 이 시스템의 입력을 변위와 속도의 함수로 정의한 후 安定性을 고려하여 이득을 결정하여 비례-적분제어 시스템을 구성하였다. 이와 같이 구한 적응제어 알고리즘은 메니퓰레이터의 동적 특성에 대한 정확한 지식을 요하지 않고 또 계산이 간단하여 實時間 응용이 가능하다. 예로서 3차원 공간상의 반경 10 cm의 원궤도에 적용하였을 때 최대 오차는 대략 1 mm이었으며 상황 변화에 무감각함을 보였다.

Nomenclature

- Λ_1, Λ_2 : $k\times k$ diagonal matrices
- λ_{1i}, λ_{2i} : i^{th} diagonal elements of Λ_1 and Λ_2, respectively
- γ_{1i}, γ_{2i} : i^{th} diagonal elements of Γ_1 and Γ_2, respectively
- K_p, K_v : Positional and velocity gain matrices
- $\underline{k}$: k-dimensional vector
- $K_p(0), K_v(0), \underline{k}(0)$: Initial Conditions of K_p, K_v, and $\underline{k}$, respectively
- P_{ij} : i,j^{th} $k\times k$ matrix element of P
- Q_1, Q_2, Q_3 : $k\times k$ positive definite matrices
- G_{pp}, G_{vp}, G_{kp} : Proportional gain matrices corresponding to K_p, K_v, and $\underline{k}$, respectively
- G_{pi}, G_{vi}, G_{ki} : Integral gain matrices corresponding to K_p, K_v, and $\underline{k}$, respectively
- γ_p, γ_v, γ_k : Gair factor corresponding to K_p, K_v, and $\underline{k}$, respectively
- r_{si}, r_{Mi} : i^{th} elements of $\underline{r}_s$ and $\underline{r}_M$, respectively
- $_i$: Ma ss cf i^{th} lirk
- l_i : Length of i^{th} link
- a_i, s_i, α_i, θ_i : Parameters of i^{th} link
- $\bar{x}_i$, $\bar{y}_i$, $\bar{z}_i$: Coordinate of the mass center of i^{th} link
- k_{ijj} : Radius of gyration of i^{th} link
- α, β, ϕ : Angles defining a trajectory(see Fig. 3)
- $\delta(t)$: Dirac delta function
- $u(t)$: Unit step function

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1. Introduction

The task of a manipulator control system is to enable the end effector of the manipulator to follow prescribed target trajectories. The target trajectories are usually planned in a coordinate frame characterizing the task-oriented space, different from the drive-oriented one. As a result, a coordinate transformation is necessary between task-oriented space and drive-oriented space.

In general, it is very difficult to control motions of mechanical manipulators by usual deterministic control methods in a wide range of motions, because of complex nonlinearities of the manipulator dynamics and uncertainty of parameters of the system. A number of control schemes[1~6] such as resolved motion control or model reference adaptive control, have been proposed. In (5) and (6) adaptive control laws in the task-oriented space are given, but it is difficult to apply their control laws when the target trajectory is given as a complex mathematical function. In this paper we give an adaptive control scheme for trajectory tracking in the task-oriented space, by considering a coordinate frame on the target trajectory. It is feasible to implement the control law for real time application since it needs only a simple calculation without requiring detailed knowledge about the system dynamics. Simulation results show that the proposed control scheme is effective to obtain uniformly high performances.

2. Equations of Motion

Consider an open kinematic chain with n joints, and let $q(t)$ be an n-dimensional vector representing the actual displacements of the joints. The coordinate system is assigned according to (7). The bodies in the chain are numbered, starting with the reference frame as body 0 and proceeding outward with bodies 1, 2, $\cdots$, n. The n variables q_i define the drive-oriented space Q. The dynamical model can be obtained by applying the Lagrange equation[8]:

$$I(\underline{q})\ddot{\underline{q}}=-\underline{c}(\underline{q},\ \dot{\underline{q}})+\underline{g}(\underline{q})+\underline{T}(\underline{q}), \tag{1}$$

where the inertia matrix $I(\underline{q})$ is $n\times n$ symmetric and positive definite for all $\underline{q}$, $\underline{c}(\underline{q},\dot{\underline{q}})$ represents Coriolis and centrifugal forces, $\underline{g}(\underline{q})$ represents gravitational force depending on the position of the links, and $\underline{T}(\underline{q})$ is generalized force assumed as the input to the system.

In general there exists an m-dimensional coordinate frame which is convenient to give performances and dynamical specifications of a manipulator. This coordinate system is called the task-oriented space X. Let $\underline{r}$ denote the position vector in X. The transformation between $\underline{q}$ and $\underline{r}$ is given by

$$\underline{r}=\underline{f}(\underline{q}). \tag{2}$$

$\underline{f}(\cdot)$ is a continuous function, with known structure and parameters. The inverse transformation

$$\underline{q}=\underline{f}^{-1}(\underline{r}), \tag{3}$$

being solution of the nonlinear equation (2), is not one-to-one, not always exists and hardly ever is an explicit function of $\underline{r}$.

From equation (2) it follows that

$$\dot{\underline{r}}=J(\underline{q})\dot{\underline{q}}$$
$$\ddot{\underline{r}}=J(\underline{q})\ddot{\underline{q}}+\frac{dJ}{dt}\dot{\underline{q}} \tag{4}$$

where $m\times n$ matrix $J(q)=\dfrac{\partial f(\underline{q})}{\partial \underline{q}}$ is the Jacobian matrix. Assuming $m=n$, the Jacobian matrix J is nonsingular almost everywhere in Q. From equations (1) and (4) it follows that

$$\ddot{\underline{r}}=\frac{dJ}{dt}\dot{\underline{q}}-J(\underline{q})I^{-1}(\underline{q})[\underline{c}(\underline{q},\ \dot{\underline{q}})-\underline{g}(\underline{q})]$$
$$+J(\underline{q})I^{-1}(\underline{q})\underline{T}(\underline{q}). \tag{5}$$

Now suppose a target trajectory C is given by

$$\underline{r}=\underline{r}(s) \tag{6}$$

where s is a parameter, and the end effector of a manipulator is required to follow the trajectory C with given velocity and acceleration. Let us consider a k-dimensional coordinate system on C which is moving along the trajectory with the desired velocity and acceleration. Let $\underline{r}_s$ denote a vector in this trajectory coordinate system. Dynamical specifications about the target trajectories are more easily given in $\underline{r}_s$ coordinate than in the task-oriented space, and accurate positional control can be expected if a control law is designed and implemented in terms of $\underline{r}_s$.

The transformation between the coordinate $\underline{r}$ in the task-oriented space and $\underline{r}_s$ in the trajectory coordinate system is given by

$$\underline{r}_s=\underline{f}_s(\underline{r}) \tag{7}$$

and hence it follows that

$$\dot{\underline{r}}_s=J_s(\underline{r})\dot{\underline{r}}$$

$$\ddot{\underline{r}}_s=J_s(\underline{r})\ddot{\underline{r}}+\frac{dJ_s}{dt}\dot{\underline{r}} \tag{8}$$

where $J_s(\underline{r})=\frac{\partial \underline{f}_s}{\partial \underline{r}}$ is the Jacobian matrix. Assuming $m=n=k$, we get from equations (5) and (8),

$$\ddot{\underline{r}}_s=\left(\frac{dJ_s}{dt}+J_s\frac{dJ}{dt}J^{-1}\right)J_s^{-1}\dot{\underline{r}}_s$$
$$-J_sJI^{-1}(\underline{c}-\underline{g})+J_sJI^{-1}\underline{T}. \tag{9}$$

Now let $\underline{F}_s$ be a force corresponding to a point $\underline{r}_s$. Then the equivalent force $\underline{T}$ which is applied to the joints can be obtained from the principle of virtual work(9): Since $\underline{F}_s$ balances with $-\underline{T}$, the total virtual work for the system becomes

$$\delta W=\underline{F}_s\cdot\delta\underline{r}_s-\underline{T}\cdot\delta\underline{q}$$
$$=\underline{F}_s\cdot J_sJ\delta\underline{q}-\underline{T}\cdot\delta\underline{q}$$
$$=0 \tag{10}$$

and hence

$$\underline{T}=(J_sJ)^T\underline{F}_s. \tag{11}$$

From equations (9) and (11) it follows that

$$\ddot{\underline{r}}_s=\left(\frac{dJ_s}{dt}+J_s\frac{dJ}{dt}J^{-1}\right)J_s^{-1}\dot{\underline{r}}_s$$
$$-J_sJI^{-1}(\underline{c}-\underline{g})+J_sJI^{-1}(J_sJ)^T\underline{F}_s. \tag{12}$$

In this equation the gravity term $\underline{g}$ is an explicit function of $\underline{q}$, and can be computed in real time. Note that the matrix $(J^TJ_s^T)^{-1}I(J_sJ)^{-1}$ exists almost everywhere in the task-oriented space and is positive definite.

Defining $2k$-vector $\underline{z}$ as

$$\underline{z}=[\underline{r}_s^T,\dot{\underline{r}}_s^T]^T, \tag{13}$$

we get from equation (12),

$$\dot{\underline{z}}=\underline{a}_p(\underline{z})+B_p(\underline{z})\underline{F}_s \tag{14}$$

where

$$\underline{a}_p(\underline{z})=\begin{bmatrix}\dot{\underline{r}}_s\\ \left(\frac{dJ_s}{dt}+J_s\frac{dJ}{dt}J^{-1}\right)J_s^{-1}\dot{\underline{r}}_s-J_sJI^{-1}(\underline{c}-\underline{g})\end{bmatrix}$$

$$B_p(\underline{z})=\begin{bmatrix}0\\ J_sJI^{-1}(J_sJ)^T\end{bmatrix}$$

For given reference inputs $\bar{\underline{r}}_s$, $\dot{\bar{\underline{r}}}_s$, and $\ddot{\bar{\underline{r}}}_s$, the open-loop control $\bar{\underline{F}}_s$ can be obtained as

$$\bar{\underline{F}}_s=-(J_sJ)^{-T}\underline{g}, \tag{15}$$

since $\bar{\underline{r}}_s=\dot{\bar{\underline{r}}}_s=\ddot{\bar{\underline{r}}}_s=\underline{0}$.

Let

$$\underline{z}=\bar{\underline{z}}+\delta\underline{z}=\begin{bmatrix}\bar{\underline{r}}_s+\delta\underline{r}_s\\ \dot{\bar{\underline{r}}}_s+\delta\dot{\underline{r}}_s\end{bmatrix} \tag{16}$$
$$\underline{F}_s=\bar{\underline{F}}_s+\delta\underline{F}_s.$$

Then we get

$$\frac{d}{dt}(\delta\underline{z})=A_p\delta\underline{z}+B_p\delta\underline{F}_s, \tag{17}$$

where

$$A_p=\left.\frac{\partial \underline{a}_p}{\partial \underline{z}}\right|_{\bar{\underline{z}}}+\left.\frac{\partial B_p}{\partial \underline{z}}\right|_{\bar{\underline{z}}}\bar{\underline{F}}_s$$
$$=\begin{bmatrix}0 & I_k\\ A_{p1} & A_{p2}\end{bmatrix}$$

and I_k is $k\times k$ identity matrix.

3. Adaptive Control for Manipulators

Let us define a matrix I_e as

$$I_e=(J_sJ)I^{-1}(J_sJ)^T \tag{18}$$

and choose $\underline{F}_s$ as

$$\underline{F}_s=-(J_sJ)^{-1}\underline{g}(\underline{q})-K_v\delta\dot{\underline{r}}_s-K_p\delta\underline{r}_s+\underline{k}. \tag{19}$$

Then it follows that

$$\frac{d}{dt}(\delta\underline{z})=A_s\delta\underline{z}+\underline{b}_s \tag{20}$$

where

$$A_s=\begin{bmatrix}0 & I_k\\ A_{p1}-I_eK_p & A_{p2}-I_eK_v\end{bmatrix}$$

$$\underline{b}_s=\begin{bmatrix}0\\ I_e\underline{k}\end{bmatrix}$$

Now consider the reference model:

$$\frac{d}{dt}(\delta \underline{z}_M) = A_M \delta \underline{z}_M \tag{21}$$

where

$$\delta \underline{z}_M = [\delta \underline{r}_M^T,\ \delta \dot{\underline{r}}_M^T]^T$$

$$A_M = \begin{bmatrix} 0 & I_k \\ -\Lambda_1 & -\Lambda_2 \end{bmatrix}$$

Defining a generalized state error $\underline{e}$ as

$$\underline{e} = \delta \underline{z} - \delta \underline{z}_M = (\underline{e}_1^T, \underline{e}_2^T)^T, \tag{22}$$

we get

$$\dot{\underline{e}} = A_M \underline{e} + (A_s - A_M)\delta \underline{z} + \underline{b}_s. \tag{23}$$

Define V as

$$\begin{aligned} V &= \underline{e}^T P \underline{e} \\ &\quad + tr\{(\Lambda_1 + A_{p1} - I_e K_p)^T Q_1^{-1} (\Lambda_1 + A_{p1} \\ &\quad - I_e K_p) + (\Lambda_2 + A_{p2} - I_e K_v)^T Q_2^{-1} (\Lambda_2 \\ &\quad + A_{p2} - I_e K_v)\} + (I_e \underline{k})^T Q_3^{-1} (I_e \underline{k}), \end{aligned} \tag{24}$$

where $2k \times 2k$ matrix P is chosen so that $A_M^T P + P A_M$ becomes negative definite. Then, differentiating along the solution of (20),

$$\begin{aligned} \dot{V} &= \underline{e}^T (A_M^T P + P A_M) \underline{e} \\ &\quad + 2tr\Big\{(\Lambda_1 + A_{p1} - I_e K_p)^T \\ &\qquad \Big[Q_1^{-1} \frac{d}{dt}(A_{p1} - I_e K_p) + \underline{v} \delta \underline{r}_s^T\Big] \\ &\quad + (\Lambda_2 + A_{p2} - I_e K_v)^T \\ &\qquad \Big[Q_2^{-1} \frac{d}{dt}(A_{p2} - I_e K_v) + \underline{v} \delta \dot{\underline{r}}_s^T\Big]\Big\} \\ &\quad + 2(I_e \underline{k})^T \Big[Q_3^{-1} \frac{d}{dt}(I_e \underline{k}) + \underline{v}\Big] \end{aligned} \tag{25}$$

where $\underline{v} = P_{21}\underline{e}_1 + P_{22}\underline{e}_2$.

Let

$$\begin{aligned} \frac{d}{dt}(A_{p1} - I_e K_p) &= -Q_1 \underline{v} \delta \underline{r}_s^T \\ \frac{d}{dt}(A_{p2} - I_e K_v) &= -Q_2 \underline{v} \delta \dot{\underline{r}}_s^T \\ \frac{d}{dt}(I_e \underline{k}) &= -Q_3 \underline{v} \end{aligned} \tag{26}$$

or

$$\begin{aligned} K_p &= I_e^{-1}\Big[\int_0^t Q_1 \underline{v} \delta \underline{r}_s^T d\tau + A_{p1} + K_p(0)\Big] \\ K_v &= I_e^{-1}\Big[\int_0^t Q_2 \underline{v} \delta \dot{\underline{r}}_s^T d\tau + A_{p2} + K_v(0)\Big] \\ \underline{k} &= -I_e^{-1}\Big[\int_0^t Q_3 \underline{v} d\tau + \underline{k}(0)\Big] \end{aligned} \tag{27}$$

then we get $\dot{V} < 0$.

Now consider the feedback system

$$\begin{aligned} \dot{\underline{e}} &= A_M \underline{e} + \begin{bmatrix} 0 \\ I_k \end{bmatrix} \underline{w} \\ \underline{v} &= D \underline{e} \\ \underline{w} &= (\Lambda_1 + A_{p1} - I_e K_p) \delta \underline{r}_s \\ &\quad + (\Lambda_2 + A_{p2} - I_e K_v) \delta \dot{\underline{r}}_s + I_e \underline{k}. \end{aligned} \tag{28}$$

Choosing D matrix as

$$D = [0 \vdots I_k] P \tag{29}$$

the transfer function

$$\begin{aligned} G(s) &= D(sI - A_M)^{-1} \begin{bmatrix} 0 \\ I_k \end{bmatrix} \\ &= (P_{21} + sP_{22}) \Big[\mathrm{diag}\Big(\frac{1}{s^2 + \lambda_{2i}s + \lambda_{1i}}\Big)\Big] \end{aligned} \tag{30}$$

is strictly positive real. Consider

$$\begin{aligned} \int_{0^-}^T \underline{v}^T \underline{w} dt &= \int_{0^-}^T \underline{v}^T [(\Lambda_1 + A_{p1} - I_e K_p) \delta \underline{r}_s \\ &\quad + (\Lambda_2 + A_{p2} - I_e K_v) \delta \dot{\underline{r}}_s + I_e \underline{k}] dt \\ &= -\int_{0^-}^T \underline{v}^T \Big\{\Big[\Big[\int_0^t Q_1 \underline{v} \delta \underline{r}_s^T d\tau + K_p(0) - \Lambda_1\Big] \delta \underline{r}_s \\ &\quad + \Big[\int_0^t Q_2 \underline{v} \delta \dot{\underline{r}}_s^T d\tau + K_v(0) - \Lambda_2\Big] \delta \dot{\underline{r}}_s \\ &\quad + \Big[\int_0^t Q_3 \underline{v} d\tau + \underline{k}(0)\Big]\Big\} dt \end{aligned} \tag{31}$$

Letting

$$K_p(0) = \Lambda_1,\quad K_v(0) = \Lambda_2,\quad \text{and}\quad \underline{k}(0) = \underline{0}, \tag{32}$$

equation (31) becomes

$$\begin{aligned} \int_{0^-}^T \underline{v}^T \underline{w} dt &= -\int_{0^-}^T \underline{v}^T \Big[\Big(\int_0^t Q_1 \underline{v} \delta \underline{r}_s^T d\tau\Big) \delta \underline{r}_s \\ &\quad + \Big(\int_0^t Q_2 \underline{v} \delta \dot{\underline{r}}_s^T d\tau\Big) \delta \dot{\underline{r}}_s \\ &\quad + \int_0^t Q_3 \underline{v} d\tau\Big] dt. \end{aligned} \tag{33}$$

Notice that[10]

$$-\int_{0^-}^T \underline{v}^T \Big[\int_0^t \Phi(\underline{v}, t, \tau) d\tau\Big] \underline{y} dt \leq \gamma^2, \quad \text{for all } T \geq 0 \tag{34}$$

where γ^2 is an arbitrary positive finite constant, if

$$\Phi(\underline{v}, t, \tau) = F(t-\tau) \underline{v}(\tau) [G \underline{y}(\tau)]^T, \quad \tau \leq t \tag{35}$$

where $F(t-\tau)$ is positive definite matrix kernels whose Laplace transform is positive real transfer matrix with a pole at $s=0$, and G is positive definite constant matrix. Hence from equations (33)$\sim$(35) it follows that

$$\int_{0}^{T}\underline{v}^{T}\underline{w}dt\leq\gamma^{2} \quad \text{for all } T\geq 0, \tag{36}$$

and the adaptation mechanism is asymptotically stable. Furthermore, proportional plus integral adaptation can be obtained if we set

$$\begin{aligned} Q_1&=G_{pp}\delta(t-\tau)+G_{pi}u(t-\tau)\\ Q_2&=G_{vp}\delta(t-\tau)+G_{vi}u(t-\tau)\\ Q_3&=G_{kp}\delta(t-\tau)+G_{ki}u(t-\tau)\end{aligned} \tag{37}$$

From equations (27), (33), and (37) we obtain an adaptation law:

$$\begin{aligned} K_p&=I_e^{-1}\{\gamma_p[G_{pp}\underline{v}(t)\delta\underline{r}_s^{T}(t).\\ &\quad+\int_0^t G_{pi}\underline{v}(\tau)\delta\underline{r}_s^{T}(\tau)d\tau]+A_{p1}+\Lambda_1\}\\ K_v&=I_e^{-1}\{\gamma_v[G_{vp}\underline{v}(t)\delta\dot{\underline{r}}_s^{T}(t)\\ &\quad+\int_0^t G_{vi}\underline{v}(\tau)\delta\dot{\underline{r}}_s^{T}(\tau)d\tau]+A_{p2}+\Lambda_2\}\\ \underline{k}&=-I_e^{-1}\gamma_k[G_{kp}\underline{v}(t)+\int_0^t G_{ki}\underline{v}(\tau)d\tau]\end{aligned} \tag{38}$$

Fig. 1 shows the block diagram of a control system described above.

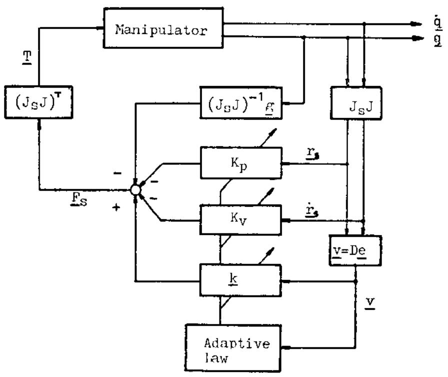


Fig. 1 Block diagram of the trajectory control system

4. Simplified Adaptive Law

Let us choose the reference model as

$$\begin{aligned}\Lambda_1&=[\mathrm{diag}(\lambda_{11},\ \lambda_{12},\ \cdots,\ \lambda_{1k})]\\ \Lambda_2&=[\mathrm{diag}(\lambda_{21},\ \lambda_{22},\ \cdots,\ \lambda_{2k})]\end{aligned} \tag{39}$$

The matrix P should be chosen so that $A_M^TP+PA_M$ is negative definite. Let

$$P=\begin{bmatrix}\Lambda_2^{-1}\Gamma_1+\Lambda_2\Gamma_1\Lambda_1^{-1}+\Lambda_2^{-1}\Gamma_2\Lambda_1 & \Gamma_1\Lambda_1^{-1}\\ \Lambda_1^{-1}\Gamma_1 & V_2^{-1}\Gamma_1\Lambda_1^{-1}+\Lambda_2^{-1}\Gamma_2\end{bmatrix} \tag{40}$$

where Γ_1 and Γ_2 are positive definite diagonal matrix, i.e.,

$$\begin{aligned}\Gamma_1&=[\mathrm{diag}(\gamma_{11},\ \gamma_{12},\ \cdots,\ \gamma_{1k})]\\ \Gamma_2&=[\mathrm{diag}(\gamma_{21},\ \gamma_{22},\ \cdots,\ \gamma_{2k})].\end{aligned} \tag{41}$$

Then

$$A_M^TP+PA_M=-2\begin{bmatrix}\Gamma_1 & 0\\ 0 & \Gamma_2\end{bmatrix} \tag{42}$$

and $\underline{v}$ can be simplified as

$$\begin{aligned}\underline{v}&=P_{21}\underline{e}_1+P_{22}\underline{e}_2\\ &=\Lambda_1^{-1}\Gamma_1(\delta\underline{r}_S-\delta\underline{r}_M)+\Lambda_2^{-1}(\Gamma_1\Lambda_1^{-1}+\Gamma_2)\\ &\quad(\delta\dot{\underline{r}}_S-\delta\dot{\underline{r}}_M)\\ &=\Big[\cdots,\ \frac{\gamma_{1i}}{\lambda_{1i}}(\delta r_{Si}-\delta r_{Mi})\\ &\quad+\frac{1}{\lambda_{2i}}\Big(\frac{\gamma_{1i}}{\lambda_{1i}}+\gamma_{2i}\Big)(\delta\dot{r}_{Si}-\delta\dot{r}_{Mi}),\cdots\Big]^T\end{aligned} \tag{43}$$

Hyperstability condition assures that all the trajectories reach subspace $\underline{v}=\underline{0}$ in a finite time and then $\underline{e}\to\underline{0}$ asymptotically. When the trajectory belongs to $\underline{v}=\underline{0}$, it follows that

$$\underline{e}_2=-P_{22}^{-1}P_{21}\underline{e} \tag{44}$$

and thus

$$\dot{r}_{si}=-\frac{\lambda_{2i}\gamma_{1i}}{\gamma_{1i}+\lambda_{1i}\gamma_{2i}}r_{si},\quad i=1,\ \cdots,\ k. \tag{45}$$

Hence the elements of Γ_1, Γ_2 and A_M determine the decaying speed of the generalized state error when the system operates on $\underline{v}=\underline{0}$. Considering the i^{th} component of the position error goes to zero with the time constant $\frac{\gamma_{1i}+\lambda_{1i}\gamma_{2i}}{\lambda_{2i}\gamma_{1i}}$, the elements γ_{1i} and γ_{2i} can be selected.

Now suppose that the matrices I_e, A_{p1}, and A_{p2} are constant, and let

$$\begin{aligned}Q_1&=I_e\{G_{pp}\delta(t-\tau)+G_{pi}u(t-\tau)\}\\ Q_2&=I_e\{G_{vp}\delta(t-\tau)+G_{vi}u(t-\tau)\}\\ Q_3&=I_e\{G_{kp}\delta(t-\tau)+G_{ki}u(t-\tau)\}\end{aligned} \tag{46}$$

Then the adaptation law (38) reduces to

$$\begin{aligned}K_p&=\gamma_p[G_{pp}\underline{v}(t)\delta\underline{r}_s^T(t)\\ &\quad+\int_0^t G_{pi}\underline{v}(\tau)\delta\underline{r}_s^T(\tau)d\tau]+\Lambda_1\\ K_v&=\gamma_v[G_{vp}\underline{v}(t)\delta\dot{\underline{r}}_s^T(t)\\ &\quad+\int_0^t G_{vi}\underline{v}(\tau)\delta\dot{\underline{r}}_s^T(\tau)d\tau]+\Lambda_2\\ \underline{k}&=-\gamma_k[G_{kp}\underline{v}(t)+\int_0^t G_{ki}\underline{v}(\tau)d\tau].\end{aligned} \tag{47}$$

Assuming that I_e is constant, the savings in computation time is great. On the other hand, as will be shown in example, the performance is not very different from the case when the adaptation law (38) is used.

5. Example

A manipulator with three degrees of freedom is considered. Its schematic diagram is shown in Fig. 2 and joint parameters are given in Table 1. Its equations of motion are obtained as

$$I(\underline{\theta})\ddot{\underline{\theta}}=-\underline{c}(\underline{\theta},\dot{\underline{\theta}})+\underline{g}(\underline{\theta})+\underline{T}(\underline{\theta}) \tag{48}$$

where

$$I_{11}=m_1k_{122}^2+m_2\{[l_2(l_2+2\bar{x}_2)+k_{222}^2]\cos^2\theta_2 +k_{211}^2\sin^2\theta_2\}+m_3[l_2^2\cos^2\theta_2 +2l_2\bar{z}_3\cos\theta_2\cos(\theta_2+\theta_3)+k_{311}^2\cos^2(\theta_2 +\theta_3)+k_{333}^2\sin^2(\theta_2+\theta_3)]$$

$$I_{22}=m_2[l_2(l_2+2\bar{x}_2)+k_{233}^2] +m_3[l_2^2+2l_2\bar{z}_3\cos\theta_3+k_{322}^2]$$

$$I_{23}=I_{32}=m_3[l_2\bar{z}_3\cos\theta_3+k_{322}^2]$$

$$I_{33}=m_3k_{322}^2$$

$$c_1=\{-m_2[l_2(l_2+2\bar{x}_2)+k_{222}^2-k_{211}^2]\sin 2\theta_2 +m_3[-l_2^2\sin 2\theta_2-2l_2\bar{z}_3\sin(2\theta_2+\theta_3) +(k_{333}^2-k_{311}^2)\sin 2(\theta_2+\theta_3)]\}\dot{\theta}_2\dot{\theta}_2 -2m_3[l_2\bar{z}_3\cos\theta_2+(k_{311}^2-k_{333}^2)\cos(\theta_2 +\theta_3)]\sin(\theta_2+\theta_3)\dot{\theta}_1\dot{\theta}_3$$

Table 1 Joint parameters of the manipulator

i	1	2	3
m_i(kg)	16.7	4.86	3.12
a_i	0	30	0
s_i	40	0	0
α_i	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$
θ_i	$\theta_1+\pi$	$\theta_2+\pi$	$\theta_3+\frac{\pi}{2}$
$\bar{x}_i$(cm)	0	−51.87	0
$\bar{y}_i$(cm)	−20	0	0
$\bar{z}_i$(cm)	0	0	10.8
k_{i11}(cm)	21	6.1	17.8
k_{i22}(cm)	10	53.6	17.8
k_{i33}(cm)	21	53.6	3.6

$$c_2=\{m_2[l_2(l_2+2\bar{x}_2)-k_{211}^2+k_{222}^2]\sin\theta_2\cos\theta_2 +m_3[l_2^2\sin\theta_2\cos\theta_2+l_2\bar{z}_3\sin(2\theta_2+\theta_3) +(k_{311}^2-k_{333}^2)\sin(\theta_2+\theta_3)\cos(\theta_2+\theta_3)]\}\dot{\theta}_1^2 -(m_3l_2\bar{z}_3\sin\theta_3)\dot{\theta}_3^2-(2m_3l_2\bar{z}_3\sin\theta_3)\dot{\theta}_2\dot{\theta}_3$$

$$c_3=m_3[l_2\bar{z}_3\cos\theta_2+(k_{311}^2-k_{333}^2)\cos(\theta_2 +\theta_3)]\sin(\theta_2+\theta_3)\dot{\theta}_1^2+(m_3l_2\bar{z}_3\sin\theta_3)\dot{\theta}_2^2$$

$$g_1=0$$

$$g_2=g\{m_2(l_2+\bar{x}_2)\cos\theta_2 +m_3[l_2\cos\theta_2+\bar{z}_3\cos(\theta_2+\theta_3)]\}$$

$$g_3=gm_3\bar{z}_3\cos(\theta_2+\theta_3)$$

The desired trajectory is a circle with radius of 10cm as shown in Fig. 3. Its reference velocity diagram is shown in Fig. 4.
Trajectory coordinate frame is selected so that $\underline{x}_S$ is in the tangential direction, $\underline{y}_S$ in the normal direction, and $\underline{z}_S$ in the binormal direction. J_S

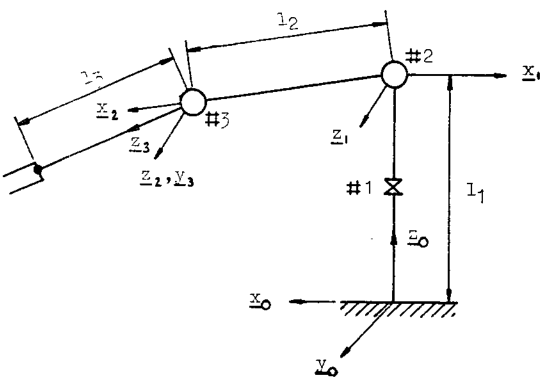
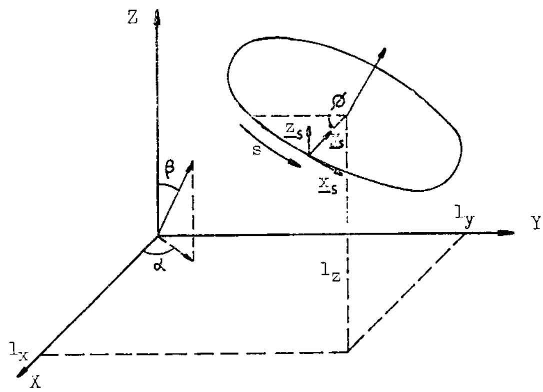


Fig. 2 Schematic diagram of a manipulator



$l_x=30,\ l_y=30,\ l_z=40,\ \alpha=30°,\ \beta=60°$

Fig. 3 Desired trajectory

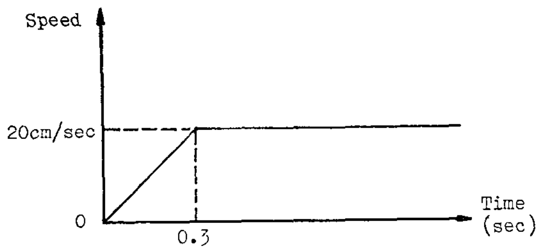


Fig. 4 Reference velocity

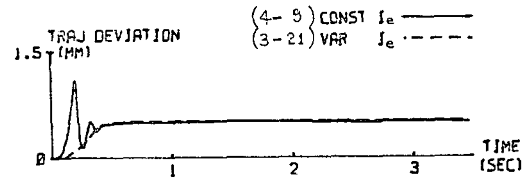


Fig. 5 Trajectory deviations of the adaptive law (38) and (47)

matrix is obtained as

$$J_S=\begin{bmatrix}\cos\phi\cos\alpha\cos\beta-\sin\phi\sin\alpha & \cos\phi\sin\alpha\cos\beta+\sin\phi\cos\alpha & -\cos\phi\sin\beta\\ -\sin\phi\cos\alpha\cos\beta-\cos\phi\sin\alpha & -\sin\phi\sin\alpha\cos\beta+\cos\phi\cos\alpha & \sin\phi\sin\beta\\ \cos\alpha\sin\beta & \sin\alpha\sin\beta & \cos\beta\end{bmatrix} \quad (49)$$

The matrices Λ_1, Λ_2, Γ_1, and Γ_2 are chosen as

$$\begin{aligned}\Lambda_1&=[\mathrm{diag}(400, 400, 400)]\\ \Lambda_2&=[\mathrm{diag}(40, 40, 40)]\\ \Gamma_1&=[\mathrm{diag}(400, 400, 400)]\\ \Gamma_2&=[\mathrm{diag}(1, 1, 1)]\end{aligned} \quad (50)$$

so that the eigenvalues of the reference model are at -20. Trajectory deviation is found minimum when the ratios of proportional gain and integral gain are

$$G_{jp}/G_{ji}=\frac{2}{100},\quad j=\mathrm{p, v, k} \quad (51)$$

and $\gamma_p=50$, $\gamma_v=10$, $\gamma_k=10$.

Trajectory deviation for the adaptation laws (38) and (47) are shown in Fig. 5. It is observed that the performances of the two adaptation laws are almost same except that the transient characteristics are better for the law (38). Since the savings in computation time is great by assuming I_e is constant, it is preferable to use the adaptation law (47). It is used in this example.

Fig. 6 shows the target and actual trajectories, and v and k are shown in Fig. 7, and torques

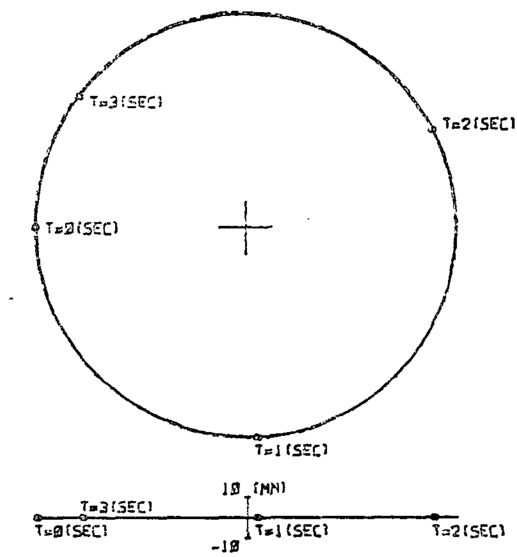


Fig. 6 Target and actual trajectories

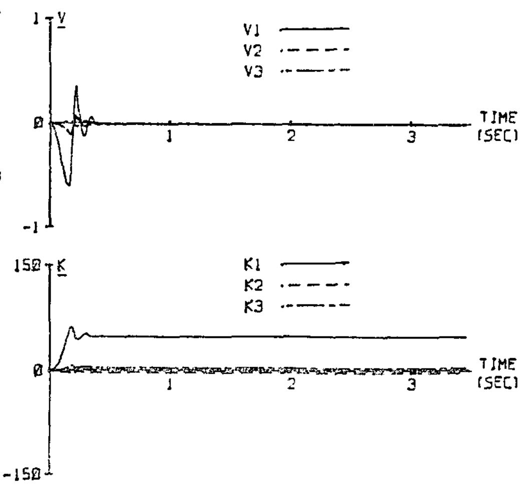


Fig. 7 Variations of v and k

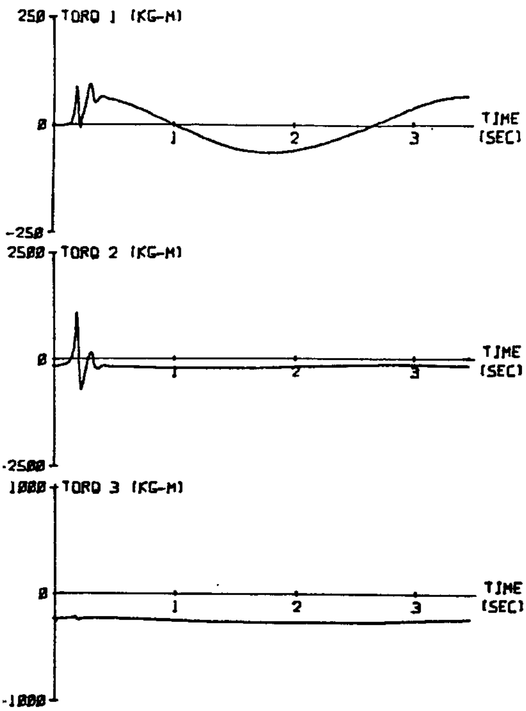


Fig. 8 Torques

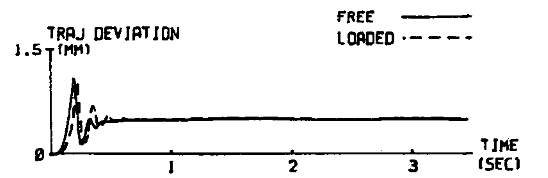


Fig. 9 Trajectory deviations when the manipulator is free and loaded

in Fig. 8. It is observed that $\underline{v}$ approaches to $\underline{0}$ rapidly. Fig. 9 compares the trajectory deviations when the hand of the manipulator is free and loaded with 3 kg load. It shows that the adaptation law proposed is insentive to parametric variations.

6. Conclusions

k-dimensional coordinate system which is moving with the desired velocity and acceleration is constructed on the target trajectory. Equation of motion is derived in the drive-oriented space and is transformed to this trajectory coordinate system by using the Jacobian matrix. This equation of motion is linearized about the desired trajectory which is the origin in this coordinate system.

The control force is expressed as a function of position, velocity, and a constant. An adaptation law is obtained from the hyperstability conditions.

This approach has the following advantages:

(1) Reference input is not needed. The desired trajectory with the desired velocity and acceleration is the origin in the trajectory coordinate system. Hence the reference input is not necessary to program.

(2) The exact knowledge about the dynamics of a manipulator is not necessary, and computational scheme is simple. It can be utilized for the real time application.

It is assumed that the dimensions of the drive-oriented space and the trajectory coordinate system are same. General case when the two dimensions are not equal will be investigated in the future work.

Acknowledgements

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