

## ON THE WEAK COMPACTNESS IN THE SPACES OF BOCHNER INTEGRABLE FUNCTIONS

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The purpose of this paper is to find the necessary sufficient conditions for weak compactness of sets in the spaces of Bochner integrable functions.

In this paper, in section I, we prove that if  $K \subset L_E^1(\mu)$  is bounded and  $E^*$  has RNP, then  $K$  is conditionally weakly compact if and only if

(1) the set  $\langle K, g \rangle = \{ \int_{(\cdot)} \langle f, g \rangle du : f \in K \}$  is uniformly  $\sigma$ -additive for every function  $g \in L_{E^*}^\infty(\mu)$ ,

(2)  $K(A) = \{ \int_A f du : f \in K \}$  is conditionally weakly compact in  $E$ , for each  $A \in \Sigma_f$ ,

In section II, we prove that  $E^*$  has the RNP if and only if for every finite measure space  $(\Omega, \Sigma, \mu)$ , any bounded and uniformly  $\sigma$ -additive subset  $K \subset L_E^1(\mu)$  is conditionally weakly compact whenever for every  $B \in \Sigma_f$ , the set  $\{ \int_B f du : f \in K \}$  is conditionally weakly compact in  $E$  and  $E^*$  is separably complementable.

Finally, in section III, we prove that Banach space  $E$  and  $E^*$  have RNP if and only if for every finite measure space  $(\Omega, \Sigma, \mu)$ , any bounded and uniformly integrable subset  $K \subset L_E^1(\mu)$  is relatively compact whenever for every  $B \in \Sigma_f$ , the set  $\{ \int_B f du : f \in K \}$  is relatively weakly compact in  $E$ .

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