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□論 文□

An Integrated Urban Land Use-Transportation Model

都市部 土地利用과 交通의 複合模型

金 昌 浩*

(美 Illinois 大 敎授)

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要 約

都市部의 活動이 地方部의 그것과 크게 다른點은 土地利用의 集約度가 높은 데 있다 하겠다. 지금까지 都市部 土地利用의 密度變位等에 關聯되는 事項은 잘 認識되어 오기는 하였으나 이를 實用的模型으로 다룬 例는 극히 드물다. 本 論文은 多樣한 輸送手段을 包容하는 交通網에서의 混雜費用이 土地利用의 集約度에 따라 效果的으로 決定될 수 있도록 하는 土地利用과 交通의 複合模型을 提示하였다. 이를 위하여 多品種交通流 및 I／O模型의 범주에서 非線形計劃의 接近方法이 採擇되었다.

I. Introduction

Perhaps the single most distinctive characteristic of urban activity that distinguishes it from rural and regional activities is the intensive use of urban land. While the nature of density variations of urban land uses and their associated land rents are well understood, there have been few operational urban models that explicitly address the density variations of land uses.

Clearly, urban land use patterns and their associated density variations are the results of complex interactions among private sectors and between private and public sectors. Location of these activities, intensity of land uses, means of production and origins and destinations are affected by the provision and pricing of transportation facilities, particularly by congestion pricing. Conversely, locations and intensity of private activities strongly affect the demand placed on the transportation system.

* 本學會正會員・工學博士

The purpose of this paper is to present a combined land use-transportation model, in which the realtionship among network congestion, land uses and intensity of such uses are simultaneously and explicitly identified.

This is a further extension of original works by Mills (1972, 1974, 1975), extended works by Kim (1979, 1983) and Kim Boyce and Hewings (1983). More specifically, the original Mills model is modified in a non linear programming framework so as to assess the impact of network congestion on the intensity of urban land uses.

Literature reviews are omitted here since an extensive review on the state-of-the-art of modelling combined land use transportation problems can be found in Kim (1983).

II. The Model

1. Postulates

1) Export Requirements of Urban Goods: An exogenously determined amount of several goods must be exported outside the urban area. These goods may be produced in any zones in the urban area and are used not only for export but also as inputs in production of other goods and for final consumption in urban areas. This postulate is different from the assumption in the Lowry model framework in which the amount and location of "basic" employees for each zone are exogenously given.

2) Factor Substitution: Input used in the production of each good include outputs of other goods, labor, land and capital. It is postulated that the most important kind of substitution in urban areas is between land and non-land inputs. The proportion between land and non-land inputs determines locations and densities of resident and employment and building heights.

3) Postulate of Equal Journey Costs: Wardrop (1952) stated the concept of equal journey costs between an origin-destination pair which results in an equilibrium condition in an urban transportation newwork. A network is in equilibrium if:

a. all routes which are used between an origin-destination pair have equal travel costs, and

b. no unused route has a lower travel cost.

In other words, at equilibrium, no driver can reduce his/her travel cost by switching routes.

4) Postulate of Spatial Interaction: The spatial interaction postulate arises from the observation that the locational choices (origin and/or destination) observed in urban travel reflect an equilibrium between a desire spatial interaction among sets of activities and the cost of travel.

2. Exogenous Variables

Exogenous variables are defined as follows:

E_r: rotal export of commodity r from the urban area as a whole.

a_{qrs}: the amount of input q required per unit output r with the s production technique when production takes place in an area at s-intensity of land use (i.e. s-story building). q ranges 1 to $r+2$ in which 1 to $r-1$ represents input of produced goods, r = labor input, $r+1$ represents

land inputs r+2 represents capital inputs. r takes integer values from 1 to r. Although there is almost no limit to the number of urban sectors one could identify, r=1 to r−1 can specify typical urban production sectors such as service, retail and manufacturing.
manufacturing.

The sector r is the household sector each of which consumes some of each good produced plus housing. Households may also consume goods imported into the urban area, but they are not included in the model presented here implying that this model does not have the problems associated with a closed Leontief system. Substitution between land input and other input is represented by these coefficients in which s represents production technology that identifies various intensities of land uses. Goods and services are produced in tall build ings by using smaller land-output ratios and higher capital-land ratios, as typically observed in the service sector in urban areas.

d_r^i: unit cost of exporting commodity r from each zone i if i belongs to the set of export zones ($i \in e$)

g_r: passanger car equivalent of road space occupancy required for shipping commodity r.

δ_{ar}^{ijp} : the incident matrix; it takes an integer value of 1, if route p from zone i to j includes link a for shipping r, otherwise it takes 0.

ℓ^i: available land in zone i.

s_r: level of spatial interaction for commodity r.

L: the opportunity cost of land at urban periphery. It is assumed that as much land as needed can be rented by expanding the urban area, i.e., by increasing the number of zones.

R: the rental rate of unit amount of capital. It is assumed that unlimited amounts of capital can be acquired at this rental rate.

3. Endogenous Variables

x_r^i: output of commodity r in zone i.

x_{rs}^i: output of commodity r produced with s-intensity of land input at zone i.

x_r^{ij}: units of r shipped from zone i to zone j. $\sum_j x_r^{ji}$ represents total amount of commodity r shipped to zone i from all other origins and $\sum_j X_r^{ij}$ represents total amount of commodity r shipped from i to all other destinations.

$c_a^k(x)$: generalized cost of travel (shipment) by mode k on link a at flow volume of x.

4. Model Development

The following subsections present sequential procedures for modelling urban activities in a general equilibrium framework in which the relationship between transportation costs and the intensity of land uses is explicitly identified. We begin the model development with a case in which a single transportation mode with fixed transportation user costs are assumed (M1). Assumption on the fixed user cost is relaxed and explicit transportation network is introduced in the model presented in the subsequent subsection (M2). The final model (M3) specifies the interrelationship between congestion costs of alternative transportation modes and the intensity of land uses.

1) Combined Land Use and Density with Fixed Transportation User Costs (M1): Given input-output production function (A) as specified above for an urban area and assuming that total amount of each commodity to be exported from the urban area is given (E), the total amount of each good to be produced (X) from the urban area is:

$$X = AX + E$$
$$X = [I-A]^{-1} E \tag{1}$$

where X is an r by 1 vector, A is an r by r matrix and E is an r by 1 vector.

Further assuming that the urban area is subdivided into zones, 1=1 .... N, and that export zones such as CBD are designated from which goods are exported at given cost (d_r^i). Once the transportation cost between zone i and zone j is also given (c_r^{ij}), the cost minimization problem becomes to find amounts of commodity r to be produced in each zone (x_r^i) and exported from export zone (E_r^i) in such a way that total transportation costs, export costs, land and capital costs are minimized subject to constraints specified below·

Min
$$M1 = \sum_i \sum_j \sum_r c_r^{ij} x_r^{ij} + \sum_{i \in e} \sum_r d_r^i E_r^i + \sum_i \sum_r \sum_s [L(a_{r+1,s,r} x_{rs}^i) + R(a_{r+2,r,s} x_{rs}^i)] \tag{2}$$

s.t.
$$\sum_i E_r^i \geqslant E_r \quad \forall_r \tag{3}$$

$$\sum_j x_r^{ji} + x_r^i = \sum_j x_r^{ij} + \sum_q \sum_s a_{rqs} x_{qs}^i + E_r^i \quad \forall \text{ i and r} \tag{4}$$

$$\sum_r \sum_s a_{r+1,r,s} x_{rs}^i \leqslant \ell^i \qquad \forall i \tag{5}$$

$$x_{rs}^i, E_{rs}^i \geq 0 \qquad \forall \text{ i, r and s} \tag{6}$$

The equation (4) represents the conservation of flow introduced in Leontief and Strout (1963). In it, $\sum_q \sum_s a_{rqs} x_{qs}^i$ represents both intermediate and final consumption since sector r includes household sector. The equation (5) is the land use constraint. Formulating the problem in this way, equation (1) is now redundant since equation (4) becomes equation (1) when both sides of equation (4) are summed over i and r. Models in this framework can be found in Mills (1972, 1974, 1975). Hartwick and Hartwick (1974) and Ikm (1978a, 1978b, 1979). These authors have shown that minimization of this linear programming problem yields an efficient assignment of activities to locations with optimal intensity of land uses, when tranaportation user costs are given.

2) Combined Land Use and Density, and Shipment Route Choice with Network Congestions (M2): In this section, the assumption of the fixed transportation user cost is relaxed. Instead, the congestion cost is endogenously determined as a function of shipment volume on each link.

Given that a generalized shipment cost function on link a at flow x ($c_a(x)$) is known, and assuming that the cost on each link is a strictly increasing function of total flow on that link, total flow in passenger equivalent terms on link a (f_a) is:

$$f_a = \sum_r g_r \sum_i \sum_j \sum_p x_r^{ijp} \delta_{ar}^{ijp} \qquad \forall\ a \qquad (7)$$

$$x_r^{ij} = \sum_p x_r^{ijp} \qquad \forall\ i,\ j \text{ and } r \qquad (8)$$

where f_a = flow volume on link a.

δ_{ar}^{ijp} = 1, if route p from zone i to j includes link a for shipping r,
= 0, otherwise.

x_r^{ijp} = units of r shipped from i to j via route p.

The coefficients g_r converts the amount of commodity and passenger flow into passenger vehicle equivalent. Equation (8) defines total shipment flow between zones i and j (x_r^{ij}) in terms of volume on all paths connecting the two zones.

Now let us suppose that the patterns of observed urban goods movement and passenger travel reflect an equilibrium between a desire for spital interaction among sets of activities and the cost of travel as was postulated above. Then the following constraint can be added:

$$-\sum_i \sum_j \left(\sum_p x_r^{ijp}\right) \ln \left(\sum_p x_r^{ijp}\right) \geq S_r \qquad \forall\ r \qquad (9)$$

where S_r represents an observed measure of the spatial dispersion of flow distribution for commodity r.

The objective function to be minimized is:

$$\min \quad M2 = \sum_a \int_0^{f_a} c_a(x)\, dx + \sum_{i \in e} \sum_r d_r^i E_r^i$$

$$+ \sum_i \sum_r \sum_s [L(a_{r+1,\, r,\, s}\, x_{rs}^i) + R(a_{r+2,\, r,\, s}\, x_{rs}^i)] \qquad (10)$$

subject to equations 3, 4, 5, 7, 8, 9 and a non-negativity constraint.

This is a nonliner programming problem with a strictly convex function. The problem is to find x_{rs}^{i}, x_{r}^{ijp}, E_r^i and f_a given the cost function ($c_a(x)$), the incident matrix (δ_{ar}^{ijp}), the input-output coefficients (a_{qrs}) and export amounts (E_r). Florian et al (1975), Evans (1976) and Boyce et al (1983) have shown that the equilibrium flows (x_i^{ijp}) can be found by Frank-Wolfe method (1956), when trip volume regarding origin zones ($\sum_j x_r^{ij}$) and destinatin zones ($\sum_i x_r^{ij}$) is a priori known. This is conventionally known as a "fixed demand" transportation problem and the problem M2 is certainly not a "fixed demand" problem.

However, given the similarity between the problem M2 and the "fixed demand" problems that are formulated in a non-linear programming framework, it is conceivable that Evans' algorithm (1976) can be adapted to solve the problem M2.

3) Combined Land Use and Density Shipment Route and Mode Choice with Network Congestions (M3): The basic models derived above are extended to the combined land use, density of land use, route and mode choices when alternative transportation betworks are given. To do so, it is necessary to assume that each mode's link cost is independent of the flow of vehicles of other modes on the same link. The additional parameter values are defined as follows:

$c_a^k(x)$: generalized cost of travel on mode k on link a at flow volume of x.

f_a^k : flow volume of mode k on link a if $a \in A^k$, the set of links used by mode k.

δ_{ar}^{ijkp} = 1, if route p of mode k from zone i to zone j include link a for shipment of r,
= 0, otherwise.

The extended model (M3) now becomes:

$$\text{Min} \quad M3 = \sum_k \sum_a \int_0^{f_a^k} c_a^k(x)dx + \sum_{i\epsilon e} \sum_r d_r^i E_r^i + \sum_i \sum_r \sum_s [L(a_{1+2,r,s} x_{rs}^i) + R(a_{r+2,r,s} x_{rs}^i)] \quad (11)$$

$$\text{s.t.} \quad f_a^k = \sum_r g_r \sum_i \sum_j \sum_p x_r^{ijkp} \delta_{ar}^{ijkp} \qquad \forall a \text{ and } k \quad (12)$$

$$\sum_i E_r^i \geq E_r \qquad \forall r \quad (13)$$

$$\sum_j x_r^{ji} + x_r^i \geq \sum_j x_r^{ij} + \sum_q \sum_s a_{rqs} x_{qs}^i + E_r^i \qquad \forall r \text{ and } i \quad (14)$$

$$x_r^i = \sum_s x_{rs}^i, \quad x_r^{ij} = \sum_k \sum_p x_r^{ijkp} \qquad \forall i, j \text{ and } r \quad (15)$$

$$-\sum_i \sum_j \sum_k (\sum_p x_r^{ijkp}) \ell n (\sum_p x_r^{ijkp}) \geq s_r \qquad \forall r \quad (16)$$

$$\sum_r \sum_s a_{r+1,\,r,\,s}\; x^i_{rs} \leqslant \ell^i \qquad \forall\, i \qquad (17)$$

$$x^{ijkp}_r,\; x^i_{rs},\; E^i_r \geqslant 0 \qquad \forall\, i,j,k,p,r \text{ and } s \qquad (18)$$

The Lagrangian for M3 is

$$L = M3 + \sum_r \sigma_r (E_r - \sum_i E^i_r)$$

$$+ \sum_i \sum_r \gamma^i_r (\sum_q \sum_s a_{rqs}\; x^i_{qs} + \sum_j x^{ij}_r + E^i_r - \sum_j x^{ji}_r - x^i_r)$$

$$+ \sum_r 1/\mu_r \; [Sr + \sum_i \sum_j \sum_k (\sum_p x^{ijkp}_r) \ln (\sum x_r^{ijkp})]$$

$$+ \sum_i \lambda^i (-\ell^i + \sum_r \sum_s a_{r+1,\,r,\,s}\; x^i_{rs})$$

To determine the optimality conditions, when $X^{ijkp}_r > 0$, $X^i_{rs} > 0$, $E^i_r > 0$, differentiate L with respect to each of these variables,

$$\frac{\partial L}{\partial x^{ijkp}_r} = \sum_a c^k_a\, (f^k_a)\, \delta^{ijkp}_{ar}\; g_r + \gamma^i_r - \gamma^j_r + \frac{1}{\mu_r}\, [1 + \ln (\sum_p x^{ijkp}_r)] = 0 \qquad (19)$$

$$\frac{\partial L}{\partial x^i_{rs}} = L\, (a_{r+1,\,r,\,s}) + R(a_{r+2,\,r,\,s}) + \sum_q \gamma^i_q (a_{qrs}) - \gamma^i_r + \lambda^i\, (a_{r+1,\,r,\,s}) = 0 \qquad (20)$$

$$\frac{\partial L}{\partial E^i_r} = d^i_r - \sigma_r + \gamma^i_r = 0 \qquad (21)$$

The optimality conditions may be interpreted as follows:

(1) Defining $\sum_a c^k_a\, (f^k_a)\, \delta^{ijkp}_{ar}\; g_r \equiv c^{ijkp}_r$ and since the transportation cost of all routes chosen between i and j pair for the shipment of r by each mode k is the same at equilibrium, c^{ijkp}_r equal c^{ijk}_r at optimum solution. Furthermore, for all i≠export zones, equation (21) becomes irrelevant. Thus, for all non-export i, equation (19) can be expressed as

$$\sum_p x_r^{ijkp} = \text{Exp}\,[\mu_r\,(\gamma_r^j - \gamma_r^i - c_r^{ijk})] \tag{22}$$

Writing equation (20) as

$$\gamma_r^i = L(a_{r+1,\,r,\,s}) + R(a_{r+2,r,s}) + \sum_q \gamma_q^i\,(a_{qrs}) + \lambda^i\,(a_{r+1,\,r,\,s}) \tag{23}$$

and thus equation (22) becomes:

$$\sum_p x_r^{ijkp} = \text{Exp}\,[\mu_r\,(-c_r^{ijk})]\ \text{Exp}\{\mu_r\,[\gamma_r^j - L(a_{r+1,\,r,\,s}) - R(a_{r+2,\,r,\,s}) - \sum_q \gamma_q^i\,(a_{qrs}) - \lambda^i\,(a_{r+1,\,r,\,s})]\} \tag{24}$$

Defining terms in { } as W and summing over k in equation (24),

$$\sum_k \sum_p x_r^{ijkp} = x_r^{ij} = \sum \text{Exp}\,[\mu_r\,(-c_r^{ijk})] \cdot \text{Exp}\,\{W\} \tag{25}$$

$$\text{Exp}\,\{W\} = x_r^{ij} \,/\, \sum_k \text{Exp}\,[\mu_r\,(-c_r^{ijk})] \tag{26}$$

Now equation (24) can be expressed as

$$\sum_p x_r^{ijkp} = x_r^{ijk} = x_r^{ij}\ \frac{\text{Exp}\,[\mu_r\,(-c_r^{ijk})]}{\sum_k \text{Exp}\,[\mu_r(-c_r^{ijk})]} \tag{27}$$

Equation (27) states that, at equilibrium, amount of commodity r shipped from i to j by mode k(x_r^{ijk}) is the product of x_r^{ij} and the share defined in terms of the relative cost of shipping r from i to j by mode k given in logit function. Further discussion on such share models can be found in Williams (1977); detailed equations that have a similar function as equation (27) are described in Fisk (1980) and Boyce et al.(1983).

(2) Equation (20) can be expressed as

$$\frac{[\gamma_r^i - \sum_q \gamma_q^i (a_{qrs})]}{(a_{r+1,\,r,\,s})} = L + R\left(\frac{a_{r+2,\,r,\,s}}{a_{r+1,\,r,\,s}}\right) + \lambda^i \tag{28}$$

in which Lagranger multipliers may be interpreted as

γ_r^i : the location surplus arising from producing unit r at i.

$\Sigma_q \gamma_q^i$: the locational surplus arising from consuming r at i for the production of other goods

λ^i : land rent at i.

Equation (28) may be interpreted as follows: If all land available at i is used, the only way to increase production amount of r at i is to use land more intensively. More intensive use of land means less land input ($a_{r-1,r,s}$) per unit output (see Mills, 1972 and Kim, 1979 for illustrative coefficients for land-capital substitution for various intensive use of land). As the values of ($a_{r+1,r,s}$)become smaller and($a_{a+2,r,s}$)become larger, net surplus arising from producing r at i in the left hand side of equation (28) increases. If, however, the increased amount of the surplus is not just offset by capital cost increase of producing r at higher intensity of land use (i.e., $a_{r+2,r,s}/a_{r+1,r,s}$), then land owner at i will maximize his/her profit by increasing the land rent, λ^i. (Note that L at the right hand side of equation (28) is constant).

If $\sum_r \sum_s a_{r+1,r,s} \, x_{rs}^i < \ell^i$ which implies that vacant land is available in zone i, then land rent, λ^i, will be zero as Kuhn-Tucker optimality conditions imply.

(3) Equation (21) can be expressed as equation (29) and Lagrange multipliers may be interpreted as

$$d_r^i = \sigma_r - \gamma_r^i \tag{29}$$

where d_r^i : unit export cost of r from export zone i (exogenously given).

σ_r : the opportunity cost of exporting an additional unit of r from the urban area ($=\partial M3^*/\partial E_r$).

γ_r^i : the locational surpluses arising from producing r at i export zone.

The equation (29) implies that, at the optimality, d_r^i should at most equal the differences between the opportunity cost of exporting an additional unit of r(σ_r) and the locational surpluses arising from producing r at i (γ_r^i), if commodity r is to be exported through i. If the unit export cost from i (d_r^i) is larger than the differences, i.e., $d_r^i > \sigma_r - \gamma_r^i$, then the complementary slackness theorem implies that exports should take place elsewhere, since, at optimality, $(\partial L/\partial E_r^{i*}) E_r^{i*} = 0$

III. Concluding Remarks

A combined transportation-land use model is proposed in this paper in which the relationship between the transportation congestion costs and the intensity of land uses is explicitly expressed. Unlike other existing urban land use and transportation planning models, zonal travel demand is endogenously determined together with link congestion costs, optimal amounts of production and resulting efficient densities of land uses, once alternative transportation networks are given.

Few suggestions have been made for solving such problems as implied in M2 and M3. Gartner (1980) formulated a route choice model with interzonal trip demand function in the manner of Beckman et al. (1956) and suggested a solution algorithm based on Frank-Wolfe method. LeBlanc and Farhangian (1981) compared Frank-Wolfe and Evans' algorithms in solving such problems as are implied in M2 and M3 and concluded that Evans' algorithm is superior to the Frank-Wolfe technique.

The proposed model represents progress over previous efforts in combining land use-transportation problems since the travel choice as to origin, destination and routes as well as amounts of goods to be produced at the optimal density of land uses are integrated into a consistent mathematical programming framework.

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