PUSAN KYŎNGNAM MATHEMATICAL JOURNAL

Vol. 1, 113~127, 1985

EQUIVALENT CONDITIONS IN TOTALLY UMBILICAL SUBMANIFOLDS OF A KAHLERIAN MANIFOLD

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0. Introduction

In the past seventies, Ako[1], Blair and Ludden([2]-[4]), Yano([2], [3], [26]-[33]), Okumura([4], [17], [18]), Chung[5], Eum[6], Ki([5]-[13], [31]-[33]), Kim [11], Pak([12], [13],[19]), Kwon[14], Lim and Choe[15], and Shin([20]-[22]), studided a structure induced on a hypersurface of an almost contact manifold or a submanifold of codimension 2 of an almost complex manifold. During the last 10 years, in spite of their vigorous efforts, the investigation about the submanifold of codimension 2 of a Kählerian manifold was not carried out successfully.

In 1967, Okumura[17] proved the following theorem A and in 1971, Ki[7] extended the theorem A under some conditions.

THEOREM A. *Let M be a complete, connected $2n$-dimensional totally umbilical submanifold with non-zero mean curvature vector μ of a $(2n+2)$-dimensional Kählerian manifold.*

(*) *Suppose that for any tangent vector X to M the covariant derivatives of the mean curvature vector $\tilde{\nabla}_X H$*

This research was supported by the Ministry of Education in Republic of Korea.

$$u^i = u_t g^{ti}, \quad v^i = v_t g^{ti},$$

g_{ji} being the Riemannian metric on M^{2n} induced from that of $\widetilde{M}^{2n+2}$

From the equation (1.2) and (1.4), we have

$$(1.5)\qquad \begin{cases} f_j{}^t f_t{}^h = -\delta_j{}^h + u_j u^h + v_j v^h, \\ f_j{}^t f_i{}^s g_{ts} = g_{ji} - u_j u_i - v_j v_i, \\ f_i{}^t u_t = \lambda v_i \text{ or } f_t{}^h u^t = -\lambda v^h, \\ f_i{}^t v_t = -\lambda u_i \text{ or } f_t{}^h v^t = \lambda u^h, \\ u_t u^t = v_t v^t = 1 - \lambda^2, \quad u_t v^t = 0. \end{cases}$$

Putting $f_{ji} = f_j{}^t g_{ti}$, we can easily find that f_{ji} is skew-symmetric, that is, M^{2n}admits an (f, g, u, v, λ)-structure.

We denote by $\left\{ {h \atop ji} \right\}$ and ∇_i the Christoffel symbols formed with g_{ji} and the operator covariant differentiation with respect to $\left\{ {h \atop ji} \right\}$ respectively.

Then the equations of Gauss and Weingarten are

$$(1.6)\qquad \begin{cases} \nabla_j B_i{}^\kappa = \partial_j B_i{}^\kappa + \widetilde{\left\{ {\kappa \atop \mu\lambda} \right\}} B_j{}^\mu B_i{}^\lambda - B_h{}^\kappa \left\{ {h \atop ji} \right\} \\ \qquad = h_{ji} C^\kappa + k_{ji} D^\kappa \\ \nabla_j C^\kappa = \partial_j C^\kappa + \widetilde{\left\{ {\kappa \atop \mu\lambda} \right\}} B_j{}^\mu C^\lambda = -h_j{}^i B_i{}^\kappa + l_j D^\kappa, \\ \nabla_j D^\kappa = \partial_j D^\kappa + \widetilde{\left\{ {\kappa \atop \mu\lambda} \right\}} B_j{}^\mu D^\lambda = -k_j{}^i B_i{}^\kappa - l_j C^\kappa \end{cases}$$

where h_{ji} and k_{ji} are second fundamental tensors of M^{2n} with respect to the normals C^κ and D^κ respectively, $h_j{}^i = h_{jt} g^{ti}$, $k_j{}^i = k_{jt} g^{ti}$ and l_j is the third fundamental tensor.

Differentiating (1.4) covariantly along M^{2n} and taking account of (1.3) and (1.6), we obtain

$$(1.7)\qquad \begin{cases} \nabla_j f_i^{\,t} = -h_{ji}u^h + h_j{}^h u_i - k_{ji}v^h + k_j{}^h v_i, \\ \nabla_j u_i = -h_j{}_t f_i^{\,t} - \lambda k_{ji} + l_j v_i, \\ \nabla_j v_i = -k_{jt} f_i^{\,t} + \lambda h_{ji} - l_j u_i, \\ \nabla_j \lambda = k_{ji}u^i - h_{ji}v^i. \end{cases}$$

The *mean curvature vector field* H^κ of $\widetilde{M}^{2n+2}$ is defined by $H^\kappa = \alpha C^\kappa + \beta D^\kappa$,

where $\alpha = \dfrac{1}{2n} h_t{}^t,\quad \beta = \dfrac{1}{2n} k_t{}^t.$

The *mean curvature* of M^{2n} in $\widetilde{M}^{2n+2}$ is the magnitude of the mean curvature vector field, that is,

$$\mu = \alpha^2 + \beta^2.$$

If the second fundamental tensors of M^{2n} are $h_{ji} = \alpha g_{ji}$, $k_{ji} = \beta g_{ji}$, then M^{2n} is said to be *totally umbilical* [17].

II. Equivalent conditions in totally umbilical submanifolds.

In this section we assume that M^{2n} is totally umbilical submanifold of $\widetilde{M}^{2n+2}$. Then we have from (1.7)

$$(2.1)\qquad \nabla_j f_i{}^h = -\alpha g_{ji}u_h + \alpha\delta_j{}^h u_i - \beta g_{ji}v^h + \beta\delta_j{}^h v^i,$$

$$(2.2)\qquad \nabla_j u_i = \alpha f_{ji} - \lambda\beta g_{ji} + l_j v_i,$$

$$(2.3)\qquad \nabla_j v_i = \beta f_{ji} + \lambda\alpha g_{ji} - l_j u_i,$$

$$(2.4)\qquad \nabla_j \lambda = \beta u_j - \alpha v_j,$$

respectively.

First of all we prove

LEMMA 2.1. *Let M^{2n} be a $2n$-dimensional totally umbilical submanifold of a $(2n+2)$-dimensional Kählerian manifold.*

Then the following conditions are equivalent to each other;

(1) *The covariant derivative of the mean curvature vector*

$$\tilde{\nabla}_j H^x \text{ is tangent to } M^{2n},$$

(2) $\nabla_k\alpha=\beta l_k,\ \nabla_k\beta=-\alpha l_k,$

(3) *The equations of Codazzi are*

$$\begin{cases}\nabla_k h_{ji}-\nabla_j h_{ki}=l_k k_{ji}-l_j k_{ki}\\ \nabla_k k_{ji}-\nabla_j k_{ki}=l_j h_{ki}-l_k h_{ji},\end{cases}$$

(4) $\nabla_k\nabla_j\lambda=-(\alpha^2+\beta^2)\ \lambda g_{kj}.$

PROOF. (1)$\Leftarrow\Rightarrow$(2) see[17].

(2)$\Rightarrow$(3): Since M^{2n} is a totally umbilical submanifold of $\widetilde{M}^{2n+2}$, we easily have

$$\nabla_k h_{ji}-\nabla_j h_{ki}=(\nabla_k\alpha)g_{ji}-(\nabla_j\alpha)g_{ki}=(\beta l_k)g_{ji}-(\beta l_j)g_{ki}$$
$$=l_k k_{ji}-l_j k_{ki}$$
$$\nabla_k k_{ji}-\nabla_j k_{ki}=(\nabla_k\beta)g_{ji}-(\nabla_j\beta)g_{ki}=(-\alpha l_k)g_{ji}-(-\alpha l_j)g_{ki}$$
$$=l_j h_{ki}-l_j h_{ki}.$$

with the aid of (2).

(2)$\Leftarrow$(3): Transvecting g^{ji} to the equation of Codazzi, we find the equations in (2).

(2)$\Rightarrow$(4): Differentiating (2.4) covariantly along M^{2n}, and using (2.2), (2.4), we obtain

(2.5) $$\nabla_k\nabla_j\lambda=-(\alpha^2+\beta^2)\lambda g_{kj}+(\nabla_k\beta+\alpha l_k)u_j-(\nabla_k\alpha-\beta l_k)v_j.$$

If the equations in (2) hold on whole M^{2n}, from(2.5) we obtain

$$\nabla_k\nabla_j\lambda=-(\alpha^2+\beta^2)\lambda g_{kj}.$$

(2)$\Rightarrow$(4): If $\nabla_k\nabla_j\lambda=-(\alpha^2+\beta^2)\lambda g_{kj}$ holds in M^{2n} we also have from (2.5)

$$(2.6)\qquad (\nabla_k\beta+\alpha l_k)u_j-(\nabla_k\alpha-\beta l_k)v_j=0$$

To begin with, let us consider the following two cases.

(Case I) If there is some open set in $\{P\in M^{2n}|\lambda^2(p)=1\}$, then $u_i=v_i=0$ holds on the open set because of $1-\lambda^2=0$.

Differentiating these covariantly and using (2.2) and (2.3), we find

$$\alpha f_{ji}-\lambda\beta g_{ji}=0,\quad \beta f_{ji}+\lambda\alpha g_{ji}=0$$

Since f_{ji} and g_{ji} are skew-symmetric and symmetric with respect to j and i respectively, we easily verify that $\alpha=\beta=0$.

Hence the equations in (2) hold on the open set.

(Case II) If $1-\lambda^2\neq 0$ $a.e.$, on an any open set $\{P\in M^{2n}|\lambda^2(p)\neq 1\}$, then transvecting (2.6) with u^j and v^j respectively yields

$$\nabla_k\beta+\alpha l_k=0,\ \nabla_k\alpha-\beta l_k=0.\quad (Q.E.D.)$$

According to the equations in (2), the mean curvature vector μ is constant.

Now, let us consider this result case by case

(i) $\mu=\alpha^2+\beta^2=0$ implies $h_{ji}=k_{ji}=0$.

(ii) $\mu\neq 0$ implies $\nabla_j\nabla_i\lambda=-(\alpha^2+\beta^2)\lambda g_{ji}$,

where we have used (2.4).

Summarizing the above results (i), (ii) and Obata's theorem[16], we obtain

Theorem 2.2. *Let M^{2n} be a ocomplete cnnected totally* *umbilical submanifold of codimension* 2 *of Kähelrián manifold* $\tilde{M}^{2n+2}$.

If one of the conditions in Lemma 2.1 *holds on* M^{2n}, *then* M^{2n} *is a to'ally geodesic submanifold or a sphere* S^{2n}.

Ⅲ. Submanifold of codimension 2 with normal (f, g, u, v, λ)-structure.

We now define a tensor field N of type (1,2) as follows:

$$N_{ji}{}^{h}=[f,f]_{ji}{}^{h}+(\nabla_j u_i-\nabla_i u_j)u^h+(\nabla_j v_i-\nabla_i v_j)v^h,$$

where $[f,f]_{ji}{}^{h}$ is the *Nijenhuis tensor* [26] formed with $f_j{}^h$ defined by

$$[f,f]_{ji}{}^{h}=f_j{}^t\nabla_t f_i{}^h-f_i{}^t\nabla_t f_j{}^h-(\nabla_j f_i{}^t-\nabla_i f_j{}^t)f_t{}^h$$

The (f, g, u, v, λ)-structure is said to be *normal* ([27]-[33]) if $N_{ji}{}^{h}$ vanishes identically.

In this section, we assume that the totally umbilical submanifold M^{2n} with (f, g, u, v, λ)-structure is normal. Then we have

$$(3.1) \qquad (l_j v_i-l_i v_j)u^h-(l_j u_i-l_i u_j)v^h=0$$

because of (2.1), (2.2) and (2.3).

Applying (3.1) with u_h and v^i successively, we obtain

$$(3.2) \qquad l_j(1-\lambda^2)^2=0$$

with the aid of (1.5).

If there is some open set in $\{P\in M^{2n}|\lambda^2(P)=1\}$, then we can easily verify that

$$\nabla_k\nabla_j\lambda=-(\alpha^2+\beta^2)\lambda g_{ji}$$

because of (2.5) and (1.5).

And if $1-\lambda^2\neq 0$ *a.e.*, then (3.2) gives $l_j=0$.

Now, differentiating (2.2) covariantly, we obtain

$$\nabla_k\nabla_j u_i-\nabla_k\nabla_i u_j=2(\nabla_k\alpha)f_{ji}+2\alpha(-\alpha g_{kj}u_i+\alpha g_{ki}u_j-\beta g_{kj}v_i+\beta g_{ki}v_j)$$

because of (2.1) and the fact $l_j=0$, from which, using Ricci identity,

$$(\nabla_k\alpha)f_{ji}+(\nabla_j\alpha)f_{ik}+(\nabla_i\alpha)f_{kj}=0. \tag{3.3}$$

Transvecting (3.3) with f^{ji}, we find.

$$(\nabla_k\alpha)(n-1+\lambda^2)-\nabla_k\alpha+(u^j\nabla_j\alpha)u_k+(v^j\nabla_j\alpha)v_k=0. \tag{3.4}$$

If we transvect u^k and v^k to this respectively, and make use of (1.5), it means

$$u^k\nabla_k\,\alpha=0,\ \ v^k\nabla_k\,\alpha=0 \tag{3.5}$$

provided that $\dim M^{2n}>2$.

Substituting (3.5) into (3.4), we see that α=constant. Similarly, we can verify that β=constant from (2.3).

Therefore, taking account of (2.5), α=constant, β= constant and $l_j=0$, we have

$$\nabla_k\nabla_j\lambda=-(\alpha^2+\beta^2)\lambda g_{kj}.$$

Combining theorem 2.2 and the above result, we conclude

Theorem 3.1. *Let M^{2n} be a complete connected totally umbilical submanifold of codimension 2 of a Kahlerian manifold $\tilde{M}^{2n+2}$.*

If the induced (f,g,u,v,λ)-structure is normal and dim $M^{2n}>2$, then M^{2n} is the same type of Theorem 2.2.

IV. Submanifold of codimension 2 in a locally Fubinian manifold

In this section, we consider a submanifold M^{2n} in a *locally Fubinian manifold*, that is, a Kählerian manifold of constant holomorphic sectional curvature. Then its curvature tensor is given by

$$(4.1)\quad \tilde{R}_{\nu\mu\lambda\kappa}=K(G_{\nu\kappa}G_{\mu\lambda}-G_{\mu\kappa}G_{\nu\lambda}+F_{\nu\kappa}F_{\mu\lambda}-F_{\mu\kappa}F_{\nu\lambda}-2F_{\nu\mu}F_{\lambda\kappa}),$$

where K is constant(see[8],[17]).

Substituting (4.1) into the equations of Gauss, Codazzi and Ricci,

$$\tilde{R}_{\nu\mu\lambda\kappa}B_k{}^\nu B_j{}^\mu B_i{}^\lambda B_h{}^\kappa=R_{kjih}-h_{kh}h_{ji}+h_{jh}h_{ki}-k_{kh}k_{ji}+k_{jh}k_{ki},$$

$$\tilde{R}_{\nu\mu\lambda\kappa}B_k{}^\nu B_j{}^\mu B_i{}^\lambda C^\kappa=\nabla_k h_{ji}-\nabla_j h_{ki}-l_k k_{ji}+l_j k_{ki},$$

$$\tilde{R}_{\nu\mu\lambda\kappa}B_k{}^\nu B_j{}^\mu B_i{}^\lambda D^\kappa=\nabla_k k_{ji}-\nabla_j k_{ki}+l_k h_{ji}-l h_{ki},$$

$$\tilde{R}_{\nu\mu\lambda\kappa}B_k{}^\nu B_j{}^\mu C^\lambda D^\kappa=\nabla_k l_j-\nabla_j l_k+h_{kt}k_j{}^t-h_{jt}k_k{}^t,$$

we have respectively

$$(4.2)\quad \begin{aligned} &K(g_{kh}g_{ji}-g_{jh}g_{ki}+f_{kh}f_{ji}-f_{jh}f_{ki}-2f_{kj}f_{ih})\\ &=R_{kjih}-h_{kh}h_{ji}+h_{jh}h_{ki}-k_{kh}k_{ji}+k_{jh}k_{ki}.\end{aligned}$$

$$(4.3)\quad \begin{cases}\nabla_k h_{ji}-\nabla_j h_{ki}-l_k k_{ji}+l_j k_{ki}=K(u_k f_{ji}-u_j f_{ki}-2u_i f_{kj}),\\ \nabla_k k_{ji}-\nabla_j k_{ki}+l_k h_{ji}-l_j h_{ki}=K(v_k f_{ji}-v_j f_{ki}-2v_i f_{kj}).\end{cases}$$

$$(4.4)\quad \nabla_k l_j-\nabla_j l_k+h_{kt}k_j{}^t-h_{jt}k_k{}^t=K(v_k u_j-v_j u_k-2\lambda f_{kj}).$$

Throughout this section, we assume the submanifold M^{2n} is βg_{ji}. Then the first equation in (4.3) can be transformed into

$$(4.5)\quad (\nabla_k\alpha-\beta l_k)g_{ji}-(\nabla_j\alpha-\beta l_j)g_{ki}=K(u_k f_{ji}-u_j f_{ki}-2u_i f_{kj})$$

Transvecting g^{ji} to (4.5) and using (1.5) yields

(4.6) $$(2n-1)(\nabla_k\alpha-\beta l_k)=-3\lambda Kv_k.$$

Substituting (4.6) into (4.5)×(2n−1), we easily have

(4.7) $$-3\lambda Kv_k g_{ji}+3\lambda Kv_j g_{ki}=(2n-1)K(u_k f_{ji}-u_j f_{ki}-2u_i f_{kj}).$$

Transvecting (4.7) with u_k and using (1.5) gives

$$3\lambda Kv_j u_i=(2n-1)K\{(1-\lambda^2)f_{ji}-u_j(-\lambda v_i)-2u_i(-\lambda v_j)\}.$$

Again, transvecting $v\,u$ to the above equation, we also obtain

$$3\lambda K(1-\lambda^2)^2=3(n-1)K\lambda(1-\lambda^2)^2.$$

If $\lambda(1-\lambda^2)\neq 0$ *a.e.*, we have $K=(2n-1)K$, which means $K=0$, if $n>1$.

Furthermore we have from (4.6)

(4.8) $$\nabla_k\alpha=\beta l_k.$$

On the other hand, from the second equation of (4.3), we get

(4.9) $$(\nabla_k\beta+\alpha l_k)g_{ji}-(\nabla_j\beta+\alpha l_j)g_{ki}=K(v_k f_{ji}-v_j f_{ki}-2v_i f_{kj}),$$

where we have used $h_{ji}=\alpha g_{ji}$ and $k_{ji}=\beta g_{ji}$.

Transvecting g_{ji} to (4.9), we also obtain

(4.10) $$(2n-1)\ (\nabla_k\beta+\alpha l_k)=3\lambda Ku_k.$$

Substituting (4.10) into (4.9)×(2n−1) yields

(4.11) $$(3\lambda Ku_k)g_{ji}-(3\lambda Ku_j)g_{ki}=(2n-1)K(v_k f_{ji} -v_j f_{ki}-2v_i f_{kj}).$$

Transvecting v^k to (4.11), we easily have

$$-3\lambda Ku_j v_i=(2n-1)K\{(1-\lambda^2)f_{ji}-v_j(\lambda u_i)-2v_i(\lambda u_j)\}.$$

And also, transvecting u^j with the above equation gives

$$-3\lambda(1-\lambda^2)Kv_i=-3(2n-1)\lambda(1-\lambda^2)Kv_i,$$

where we have used (1.5).

Now, if $\lambda(1-\lambda^2)\neq 0$ $a.e,$ then $K=0$.

This means

(4.12) $$\nabla_k\beta=-\alpha l_k.$$

To begin with, let us consider the following two cases.

(Case I) If there is some open set in $\{P\in M^{2n}|\lambda(P)=0\}$, we obtain from (4.6) and (4.10)

$$\nabla_k\mu=\nabla_k(\alpha^2+\beta^2)=2(\alpha\nabla_k\alpha+\beta\nabla_k\beta)=2(\alpha\beta l_k-\alpha\beta l_k)=0.$$

This means that $\mu=\alpha^2+\beta^2$ is constant.

(Case II) If $\lambda\neq 0$ $a.e,$ on any open set $\{P\in M^2|^n\lambda(P)\neq 0\}$,

then we also think the following two cases (A) and (B), that is,

(A) if $1-\lambda^2\neq 0$ $a.e.$, on the open set, from (3.8) and (3.12) we obtain that $\mu=\alpha^2+\beta^2$ is constant,

(B) if there is some open set in $\{P\in M^{2n}|(1-\lambda^2)(P)=0\}$, then $u_i=v_i=0$. Hence we deduce (4.8) and (4.12) from (4.6) and (4.10). By using the same method, we know that $\mu=\alpha^2+\beta^2$ is constant on M^{2n}. Hence, by making use of theorem 2.2, we have

Theorem 4.1. *Let M^{2n} be a complete connected totally umbilical submanifold of codimension 2 in a locally Fubinian manifold $\widetilde{M}^{2n+2}$. If dim $M^{2n}>2$, then the submanifold M^{2n} is a totally geodesic or a sphere S^{2n}.*

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