

Optimal Policy for a Regional Water Distribution System

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Abstract

This paper presents optimum policy of water supply distribution of the Osaka Prefectural Waterworks System located in the midwest of Japanese Islands. Owing to the ever increasing demand for water, the Osaka Prefectural Government endeavors to expand potable and industrial water distribution system to satisfy the growing water demand of the constituents under its jurisdiction. In this regard, the paper discusses a problem of establishing an efficient and effective water distribution system. The criteria to be considered are stability of water level at the reservoirs, stability of flow in the network, and the water treatment and distribution cost. These objective functions may be combined to form a multiple objective optimization problem or may be used independently and formulated into single objective optimization problems.

1. INTRODUCTION

Due to the recent remarkable progress of the Japanese economy, urbanization has accelerated in main areas in Japan. The Osaka prefecture is located midwest of Japanese Islands, and has the second largest population, just next to the Tokyo Metropolis. Owing to the ever increasing demand for water, the Osaka Prefectural government endeavors to expand the potable and industrial water supply system to satisfy the growing water demand of the many cities and towns under its jurisdiction, except Osaka City which has its own water supply system.

At present the Osaka Prefectural Waterworks can supply water at rate 2,000,000 cubic meters

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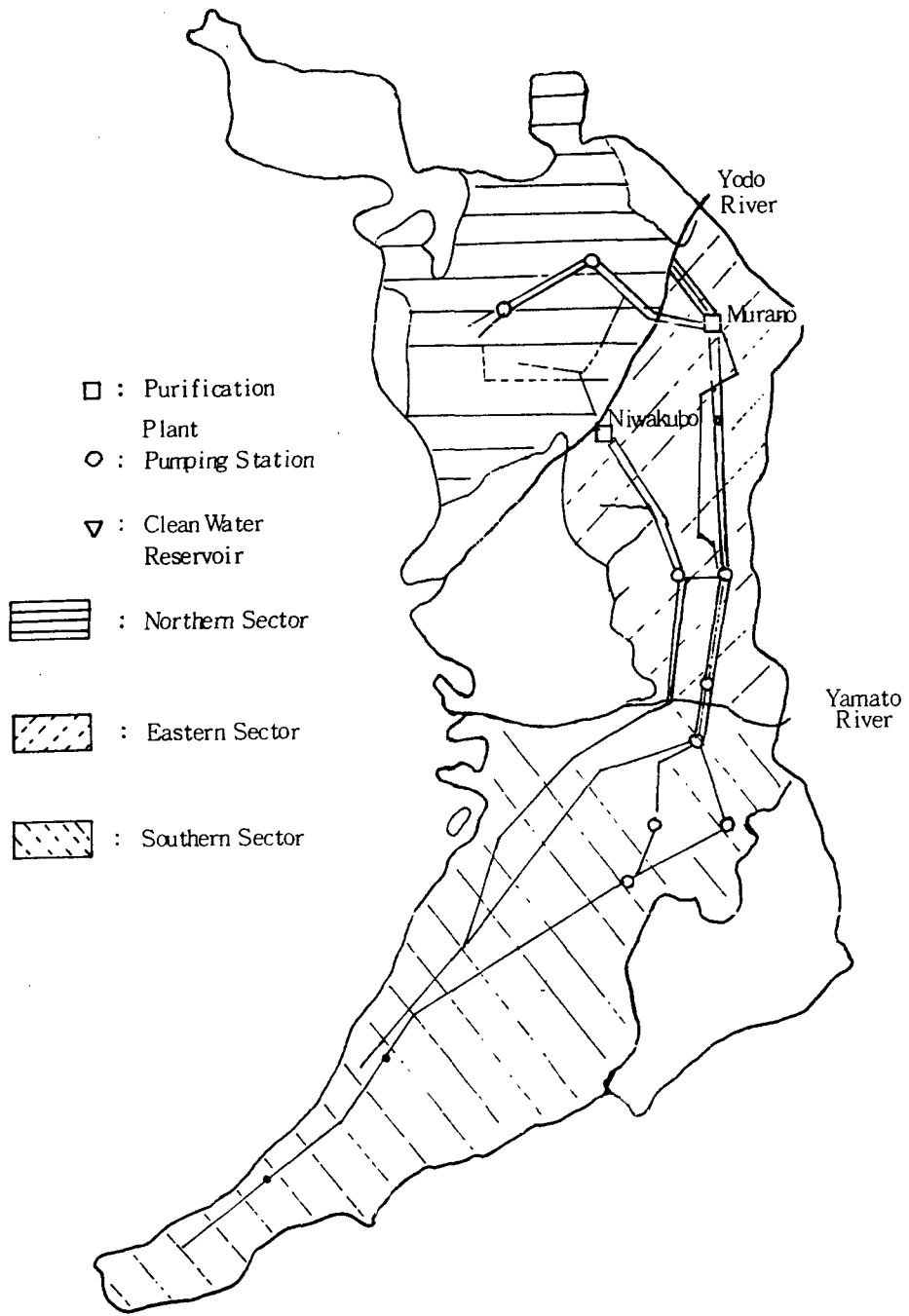


Fig. 1. Osaka Prefectural Water Distribution System

per day to 28 cities and 7 towns by means of its two purification plants at Murano and Niwakubo, 414 kilometer network of water conveyance and supply pipes, 3 main clean water reservoirs, and 7 main pumping stations with buffer reservoirs. That is, the Osaka Prefectural Waterworks have a large scale water supply network system composed of many facilities (see Fig. 1).

The water source for both purification plants is the surface flows of the Yodo River. The water from the purification plants is sent to intermediate pumping stations located along the way for additional pressure and distributed to respective water reservoirs supplying the demand.

Up to now, they kept on distributing water steadily despite of the increasing water demand, shortage of water in the source, and raising cost of water distribution. In the process of drawing, purifying, transporting and distributing water, the Osaka Prefectural Waterworks has adopted the policy of determining the water level of the reservoirs and the amount of water flow from the viewpoint of the past practical experiences in supplying the water demand. It is noted that the policy has not been the optimal water supply with respect to the total system.

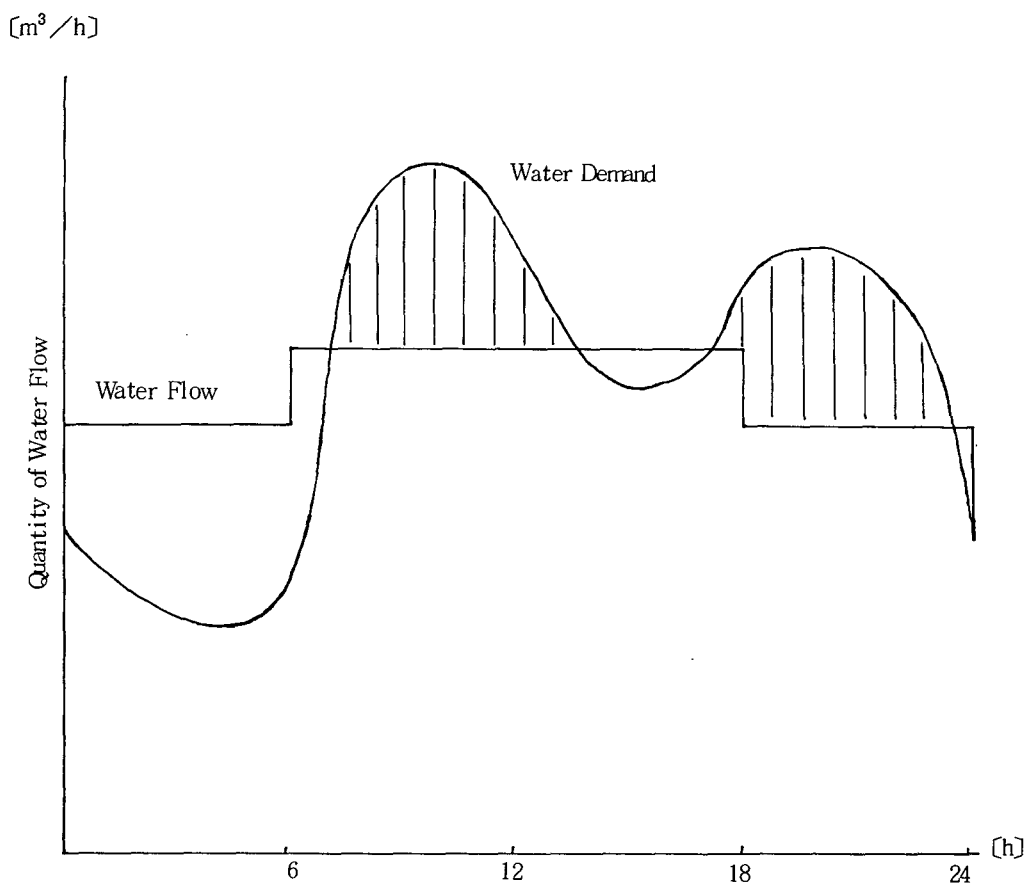


Fig. 2. Water Flow Model

In order to respond the ever growing water demand under limited amount of water resources and restricted facility capacities, it is necessary to develop and efficient water distribution control system.

For efficient utilization of the water resources, it is necessary to equalize amount of drawing water to stabilize water flow in the network, and to lessen the loss of water through leaks due to high sending pressure. It is also required to distribute water according to the schedule planned for the total water supply system with a small number of operators.

It is expected that improvement of water supply efficiency can be attained, since the purification cost at Niwakubo and Murano Plants depends heavily on the quality of water drawn from the source river, and also the optimal policy is greatly affected by changes in water demand because of the special structure of the network.

Reduction of contract price for electric power, and savings in electric power consumption at the intermediate pumping stations are achievable by equalization of water flow (see Fig. 2).

This paper discusses the optimal policy of water distribution of the Osaka Prefectural Waterworks System, and applies the parametric nonlinear programming by deformation method to the optimization in water distribution. In Section 2 the present situation of the Osaka Prefectural Water Supply System is described and the objective functions are given. Results of the computational experiments are given in Section 3. Summary of the results is presented in Section 4. Finally the chapter concludes with a brief summary results together with a discussion on a possible development of the method to meet the application needs in the field of water resources management.

2. OSAKA PREFECTURAL WATER DISTRIBUTION PROBLEM

This Section introduces the sets of constraints with respect to the amount of flows in the network and the water level of the reservoirs. Although the whole waterworks system should be treated as one model, owing to geographical structure of the system and administrative considerations, the system is partitioned into three Sectors – The Northern, the Eastern, and the Southern (see Fig. 1).

Let us consider the equilibrium equation for the quantity of water flowing in and out of reservoir i in the time interval $(t, t+1)$ (see Fig. 3). Note that water is sent to the reservoir by the pumping station, and is distributed for demand. Then the equilibrium equation is given as follows:

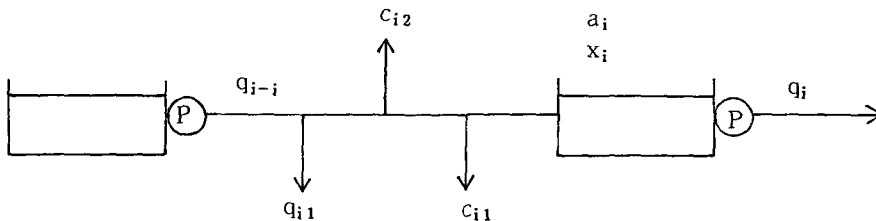


Fig. 3. Water Flow from pumping station $i-1$ to station i

$$a_i (x_{i,t+1} - x_{it}) = q_{i-1,t} - c_{it} - q_{it}, \quad (1)$$

where

- a_i : the area of reservoir i ,
- x_{it} : the water level of reservoir i at time constant t ,
- $q_{it}, q_{i-1,t}$: the amounts of water transported in the time interval $(t, t+1)$ at the i th and $i-1$ th pumping stations, respectively,

and

- c_{it} : the total amount of demands in the time interval $[t, t+1]$ at nodes which are located between $i-1$ th and i th pumping stations.

It is noted that the amount of water demand in the time interval $[t, t+1]$ is assumed to be known by certain forecasting method (see Fig. 4) and practical bounds are imposed on the capacity of the water transported and the water level of the reservoir. A constraint of the amount of water transported is given as

$$q_j \leq q_j \leq \bar{q}_j$$

where q_j and \bar{q}_j mean the minimum and maximum capacity of water transported at the j th pumping station, respectively.

It is natural to consider that water level is limited within the range between the bottom and the brim of the reservoir. However, from the viewpoint of control and operational stability of the waterworks system, water level x_i in the reservoir i is assumed to be constrained within a narrow range:

$$\underline{x}_i \leq x_i \leq \bar{x}_i,$$

where the values of \underline{x}_i and \bar{x}_i are determined from the experience in operating the waterworks system.

2.1 Constraints

The equilibrium equation of the network at the Northern Sector is given as (see Fig. 5):

$$\begin{cases} a_1 (x_{1,t+1} - x_{1t}) = q_{1t} - c_{1t} - q_{2t} \\ a_2 (x_{2,t+1} - x_{2t}) = q_{2t} - q_{3t} - c_{3t} \\ a_3 (x_{3,t+1} - x_{3t}) = q_{3t} - c_{2t} - q_{4t} \\ a_4 (x_{4,t+1} - x_{4t}) = q_{4t} - c_{4t} - c_{5t} \end{cases} \quad (2)$$

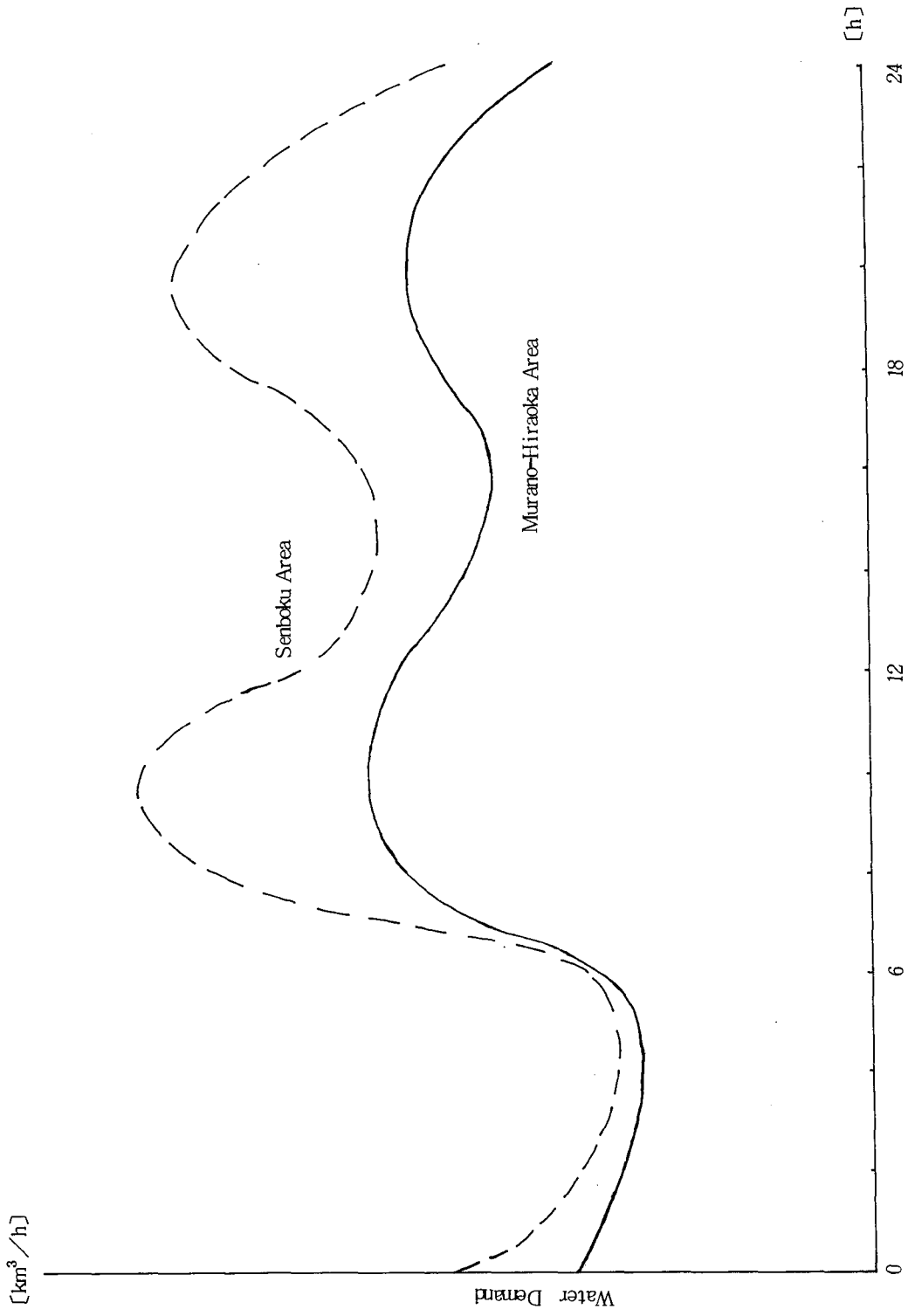


Fig. 4. Water Demand Pattern

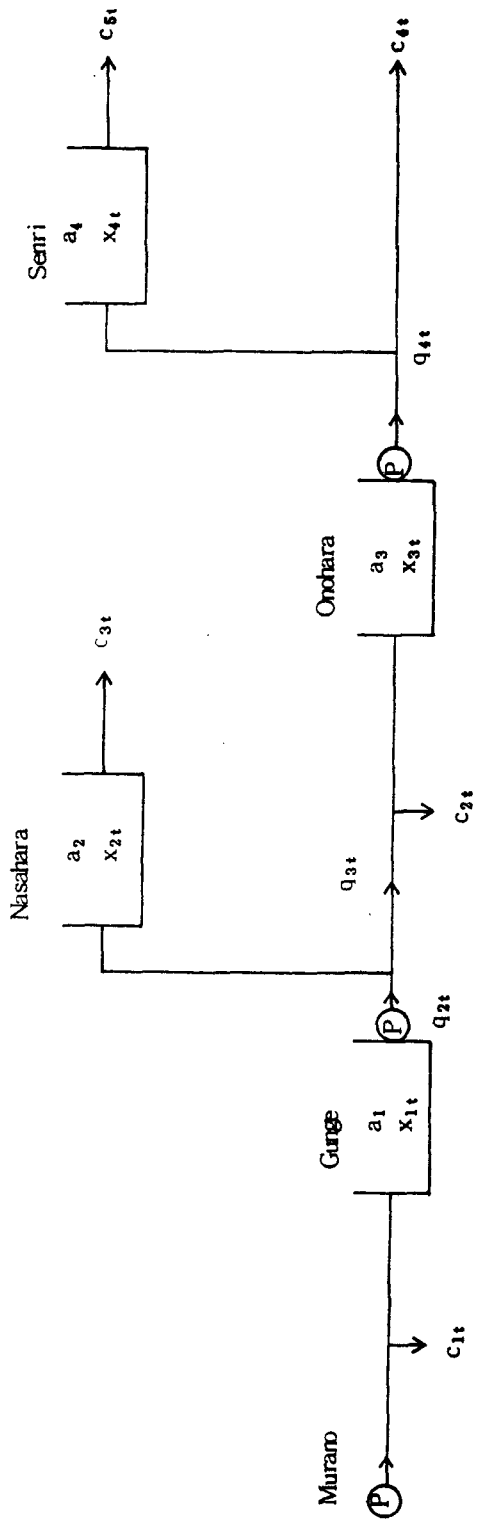


Fig. 5. Northern Sector Water Distribution Network

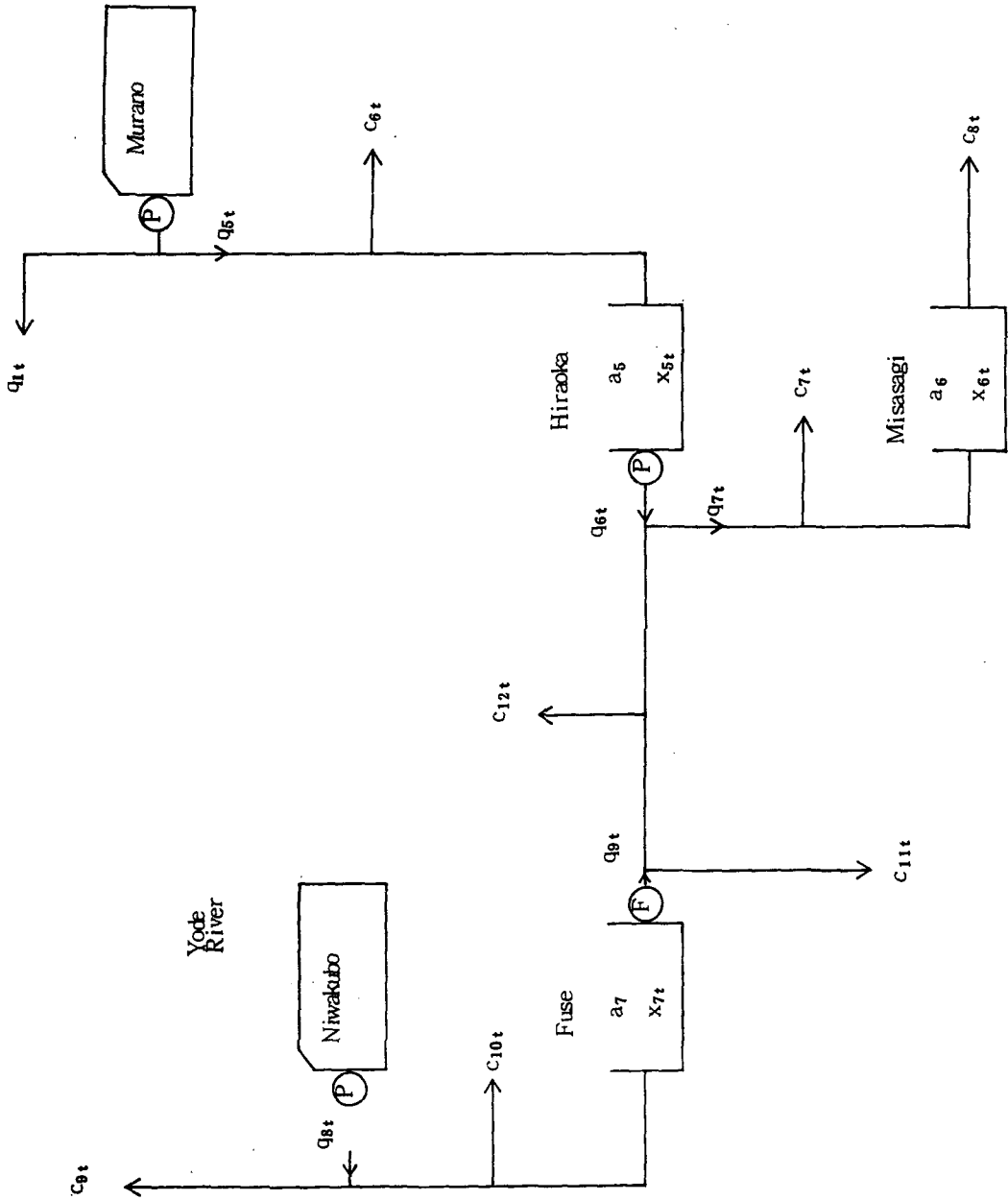


Fig. 6. Eastern Sector Water Distribution Network

where

- a_i : the area of a reservoir i ,
- q_{it} : the amount of water supply,
- c_{it} : total demand of each waterwork at the time constant t , and
- t : time parameter.

The equilibrium equation of the network at the Eastern Sector is modeled as (see Fig. 6):

$$\begin{cases} a_5(x_{5t+1} - x_{5t}) = q_{5t} - c_{6t} - q_{6t} \\ a_6(x_{6t+1} - x_{6t}) = q_{7t} - c_{7t} - c_{8t} \\ a_7(x_{7t+1} - x_{7t}) = q_{8t} - c_{9t} - c_{10t} - q_{9t} \\ q_{9t} - c_{11t} - c_{12t} + q_{6t} - q_{7t} = 0 \end{cases} \quad (3)$$

Similarly, the equilibrium equation of the network at Southern Sector is expressed as (see Fig. 7):

$$\begin{cases} a_8(x_{8t+1} - x_{8t}) = q_{11t} - q_{13t} \\ a_9(x_{9t+1} - x_{9t}) = q_{12t} - q_{14t} \\ a_{10}(x_{10t+1} - x_{10t}) = q_{13t} + q_{14t} - c_{14t} - c_{15t} \\ q_{10t} - q_{11t} - q_{12t} - c_{13t} = 0. \end{cases} \quad (4)$$

2.2 Objective Functions

In general, optimization problems of waterworks distribution system are formulated into multicriteria programming problem (Hwang and Masud (1979), Keeney and Raiffa (1976)) because of the complexity of the system. From the viewpoint of operating waterworks system, a very important criterion is to distribute water at the minimum cost. It is noted, however, that the distributing cost is greatly affected by changes in water demand which can be hardly anticipated, beforehand (see Fig. 4). As usual, to meet the maximal demand of the following day, the reservoir may be filled close the maximum reservoir level before the increase of water demand due to early morning. Since the filling policy of the maximum reservoir level is not always economical, target reservoir level is introduced. That is, the reservoir is to be filled close to the target reservoir level every night. However, if the reservoir is not filled to the target level because of the large amount of water demand of the previous day. This causes a failure to supply the full amount of the current demand. The attainability of the target reservoir level is an important criterion for the safety of the water distribution system. Also the stability of flow in the network are important for continuity in the operation of pumps and for

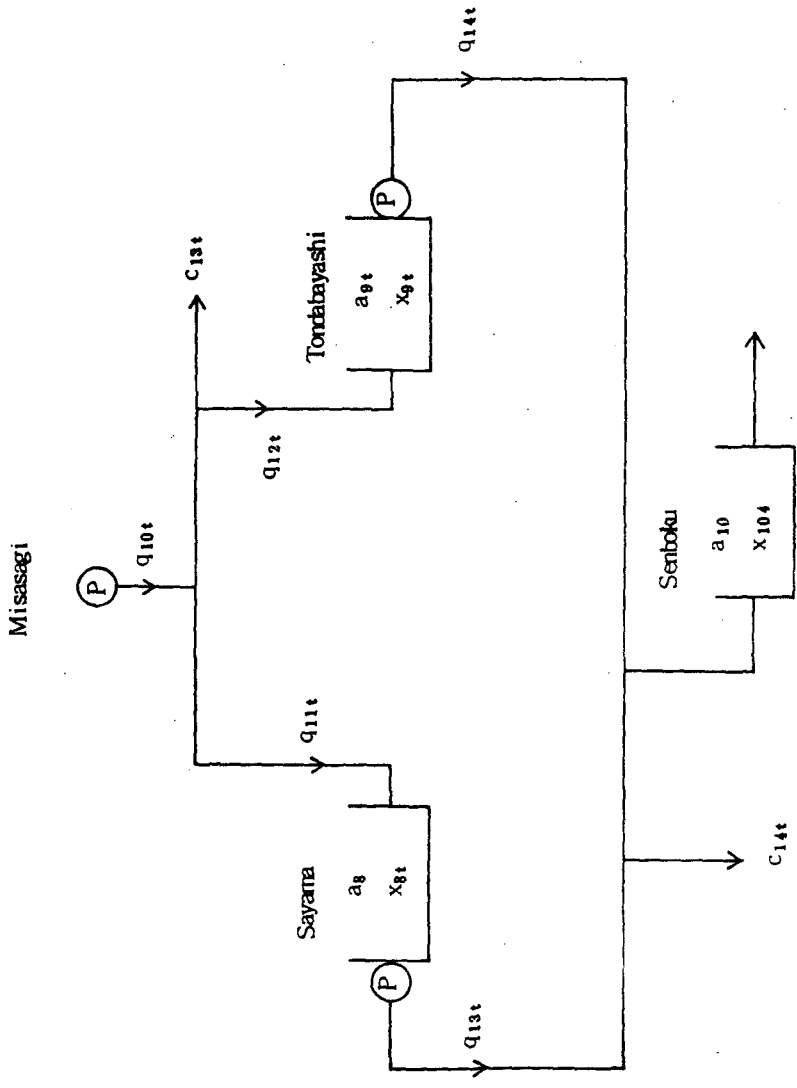


Fig. 7. Southern Sector Water Distribution Network

meeting water supply contingencies like changes in water intake quantities and variation in the demand.

Thus three different objective function formulations are introduced. The first and second objective functions involve the attainability of the target reservoir level and the stability of the flow rate in the network. The third objective function is concerned with the cost of purification and distribution of the water supply.

The objective function for the attainability of the target reservoir level is as follows :

$$f_1 = \sum_i v_{li}(x_{i0} - x_{ig})^2 \quad (5)$$

where

- x_{ig} : target water level for reservoir i,
- x_{i0} : water level in reservoir i at six o'clock in the morning, and
- v_{li} : weight assigned to the relative importance of water level stability at reservoir i.

The objective function for the stability of the flow rate in the network is as follows:

$$f_2 = \sum_j \sum_t v_{2j}(q_{j,t+1} - q_{jt})^2, \quad (6)$$

where

- v_{2j} : weight assigned to the relative importance of stability of flow through pipeline j and
- q_{jt} : the amount of water flow through pipeline j at t the the time interval [t, t+1].

The objective function involving the cost of purification and distribution of the water supply is given as :

$$f_3 = \sum_i \sum_t v_{3i}(p_{it} + k_{it})q_{it}, \quad (7)$$

where

- k_{it} : purification cost per unit quantity of water
- p_{it} : transportation cost per unit quantity of water at pumping station i.

The purification treatment cost k_{it} per unit quantity of water is given on the empiric basis as follows:

$$k_{it} = b_1 NH_i + b_2 + b_3 (OT_i)^{b_4} + b_5 (AL_i - b_6),$$

where

- NH_i : ammonia nitrogen density of raw water,
- OT_i : turbidity of raw water,
- AL_i : alkalinity of raw water, and
- b_1, b_2, \dots, b_6 : coefficients.

The transportation cost p_{it} consists of the cost of drawing water from the river and distributing it through the network. This cost, however, is highly nonlinear function of q_{it} due to many factors like water head loss through the pipelines, cavitation in pumping operations, efficiency of pumps, and rate of pumping, among many others. The practical expression of p_{it} is given by:

$$p_{it} = uR_i,$$

where

- u : cost per unit electric power [Yen/kwh] and
- R_i : the power consumption per unit quantity of water transported.

It is shown in the sequel that R_i is a nonlinear function of the electric power for the pumping operation, the pumping height and the efficiency rate of the pump and the motor at pumping station. The following is a derivation of an expression of R_i with respect to the variables at Pumping station i .

When a waterworks system is connected by only one water distribution pipeline, the loss in water head due to the friction loss is a function of the rate of flow and the diameter and length of the pipe. Thus the water head loss equation on average flow is given by Hazen-Williams as follows:

$$H_a = 10.666 \sum_{j=1}^{n+1} q_j^{1.85} r_j^{-4.87} d_j^{-1.85}, \quad (8)$$

where

- q_j : amount of water flowing per unit time into the pipe between node $j-1$ and j ,
- r_j : length of th pipeline from node $j-1$ to node j ,

- d_j : diameter of the pipe connecting nodes j-1 and j,
- e_j : coefficient of current velocity between nodes j-1 and j, and
- H_a : loss of water head.

From equation (8), the loss of pipeline has the relations of nonlinear with respect to the amount of water flowing. Let's assume here that r_j , d_j and e_j are fixed. Moreover, the average of a pressures at the pumping stations a and b is given as follows (see Figure 8):

$$P_a = H_a + h_a + p'_b, \quad (9)$$

where

- P_a : pumping pressure at pumping station a,
- P'_b : pressure of water supply in pumping station b, and
- h_a : difference in water level from reservoir b to reservoir a with the condition $p_b \geq 0$ between pumping station a and b.

Equation (9): means that the pressure p_a of the water distribution at pumping station a is a function of receiving pressure at b, water head loss, and the difference in the water levels between the two reservoirs. Moreover, when the waterwork operates at optimal pressure, it is expected that p_b is almost zero. Since the water head loss is a nonlinear function of the diameter of the pipe, the velocity, and the amount of water following, the optimal distribution pressure at pumping station is also expected to be a nonlinear function of the same variables.

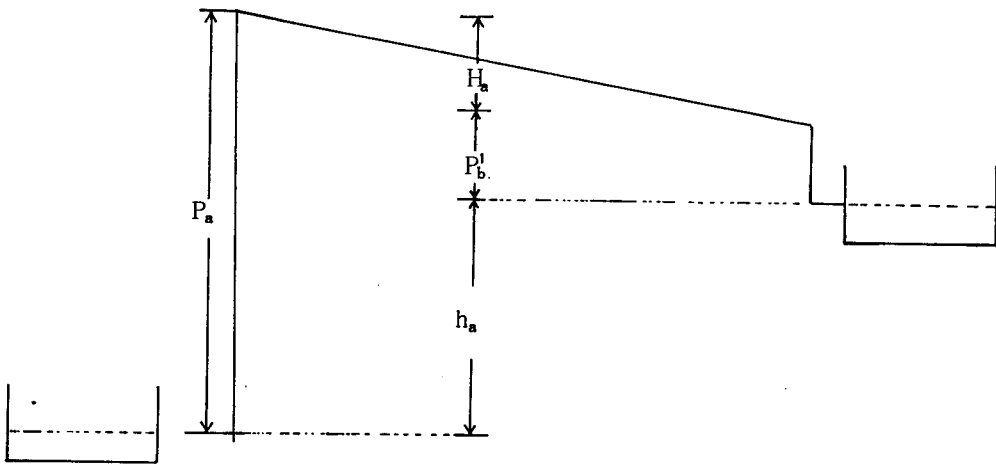


Fig. 8. Loss in Pipeline of Water Flow

The price of transporting water through a waterwork system is mainly concerned with the cost of operating pumps due to electric power consumption. Thus, an important problem in operating a waterwork system is the minimization of the cost of pumping water due to electric power used in the operation. The power consumption per unit quantity of water transported is given as follows:

$$R_i = \frac{E_i}{q_i} = \frac{9.8q_i h_{pi}}{y_{pi} y_{mi} q_i} = \frac{9.8 h_{pi}}{y_{pi} y_{mi}} \quad (10)$$

where

- E_i : amount of electric power for the pumping operation,
- h_{pi} : pumping height at pumping station i, and
- y_{pi}, y_{mi} : the efficiency rate of the pump and the motor in operation, respectively.

The minimum value of k_i in the above equation may be obtained by differentiating with respect to h_{pi} . However, it must be noted the efficiency rates y_{pi} and y_{mi} are in general function of q_i .

The optimal pumping pressure necessary to deliver water through a waterwork system is related to the water head loss. Moreover, due to cavitation in the network, the lift height should not go beyond certain minimum limits. In general, a pumping station has n branches of pipelines in the water distribution and the lift height at station i should satisfy the following:

$$h_{pi} = \max \quad h_{miL}, h_{qi_1}, h_{qi_2}, \dots, h_{qi_n} \quad (11)$$

where

- h_{qi_j} : the lift height at station i necessary for pumping branch j and
- h_{miL} : the minimum lift height for pumping station i.

Figure (7) shows the relation between the lift height at the pumping station and the amount of water distributed. It will be assumed that the minimum lift height h_{miL} is constant.

Although our problem is to minimize any of the objective functions given above (depending on the goals intended), subject to the constraints given earlier, the objective functions may be combined to form a multiple objective optimization problem or may be used independently and formulated into single objective optimization problems. In this paper the following weighted sum of the three objective functions is introduced for each sector:

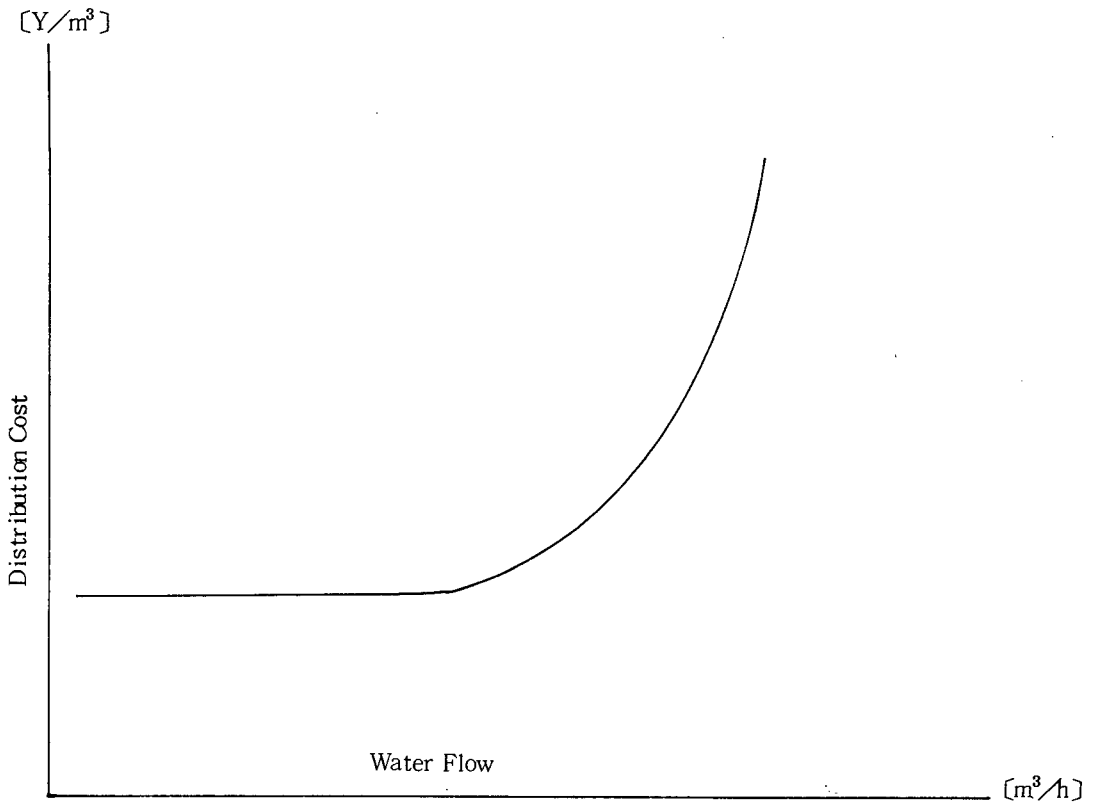


Fig. 9. Relation between Distribution Cost and Water Flow Rate

$$f = V_1 f_1 + V_2 f_2 + V_3 f_3, \quad (12)$$

where f_1, f_2, f_3 are the objective functions given by (5), (6), (7), and the corresponding weights $V_1, V_2,$ and V_3 are to be determined exgenously by assessment procedure. It is necessary to formulate the model ofr each of the three sectors and to intergrate the resulting expressions into the whole system. Thus the above discussion yields the entire expression of the optimization problem to be solved for the whole system consisting of Northern, Eastern and Southern Sectors, as follow:

$$\begin{aligned} \text{minimize} \quad & f = f_N(X_1, Q_1) + f_E(x_2, Q_2) + f_S(x_3, Q_3) \\ & (13) \end{aligned}$$

subject to

$$g = g_N(X_1, Q_1) + g_E(x_2, Q_2) + g_S(X_3, Q_3)$$

$$\underline{X}_1 \leq X_1 \leq \bar{X}_1,$$

$$\underline{Q}_1 \leq Q_1 \leq \bar{Q}_1,$$

$$\underline{X}_2 \leq X_2 \leq \bar{X}_2,$$

$$\underline{Q}_2 \leq Q_2 \leq \bar{Q}_2,$$

$$\underline{X}_3 \leq X_3 \leq \bar{X}_3,$$

$$\underline{Q}_3 \leq Q_3 \leq \bar{Q}_3,$$

where f_N , f_E , f_S , g_N , g_E , g_S , X_1 , X_2 , X_3 , Q_1 , Q_2 , and Q_3 are given in Tables 2.1, 2.2, and 2.3 corresponding to the Northern, the Eastern, and the Southern Sectors, respectively.

Table 2.1 (Northern Sector)

minimize

$$f_N = \sum_{i=1}^4 v_{1i} (x_{i0} - x_{ig})^2 + \sum_{j=1}^4 \sum_{t=0}^3 v_{2j} (q_{j,t+1} - q_{jt})^2$$

subject to

(14)

$$g_N = A_1 X_1 + B_1 Q_1 + C_1 = 0,$$

$$\underline{X}_1 \leq X_1 \leq \bar{X}_1,$$

$$\underline{Q}_1 \leq Q_1 \leq \bar{Q}_1,$$

where

$$X_1 = (x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, \dots, x_{44})^T,$$

$$Q_1 = (q_{11}, q_{12}, q_{13}, q_{14}, q_{21}, \dots, q_{44})^T,$$

$$C_1 = (c_{11} - a_1 x_{10}, c_{12}, c_{13}, c_{14},$$

$$c_{31} - a_2 x_{20}, c_{32}, c_{33}, c_{34},$$

$$c_{21} - a_3 x_{30}, c_{22}, c_{23}, c_{24},$$

$$c_{41} - c_{51} - a_4 x_{40}, c_{42} + c_{52}, c_{43} + c_{53}, c_{44} + c_{54})^T,$$

Table 2.2 (Eastern Sector)

minimize

$$f_E = \sum_{i=5}^7 v_{1i} (x_{i0} - x_{ig})^2 + \sum_{j=5}^9 \sum_{t=0}^3 v_{2j} (q_{j-t+1} - q_{it})^2 + \sum_{k=5}^9 \sum_{t=0}^3 v_{3k} (p_{it} + k_{it}) q_{it}$$

subject to

(15)

$$g_E = A_2 X_2 + B_2 Q_2 + C_2 = 0,$$

$$\underline{X}_2 \leq X_2 \leq \bar{X}_2,$$

$$Q_2 \leq Q_2 \leq \bar{Q}_2,$$

where

$$k_{6t} = 0, k_{7t} = 0, k_{9t} = 0,$$

$$X_2 = (x_{51}, x_{52}, x_{53}, x_{54}, x_{61}, \dots, x_{74})^T,$$

$$Q_2 = (q_{51}, q_{52}, q_{53}, q_{54}, q_{61}, \dots, q_{94})^T,$$

$$C_2 = (c_{61} - a_5 x_{50}, c_{62}, c_{63}, c_{64}, c_{71} + c_{81} - a_6 x_{60}, \dots, -c_{114} - c_{124})^T,$$

3. COMPUTATIONAL EXPERIMENTS

This section gives observations on some computational results of the waterworks distribution problem. Data on water demand, cost of transporting water, volumes of reservoirs, etc. were taken from the Waterworks Department of the Osaka Prefectural Government. The optimization problem was solved by means of the parametric nonlinear programming by deformation method given in (Mine and Ryang (1979)) using FACOM M200 Computer of the Kyoto University Computing Center.

The planning horizon consisted of one whole day divided into four planning periods of 6 hours each. Thus, the number of constraints was multiplied by four, corresponding to each planning period. First, the entire optimization problem was solved, assuming proper values of the weights for the objective functions. Next, the optimization problem was decomposed into three subproblems each of which corresponds to one of the three Sectors. Then each subproblem was solved by using the same constraints and bounds on the variables as those of the preceding entire analysis. Both the results seemed to coincide, if reasonable values of the weights were determined by sensitivity analysis, considering the actual data derived from the experiences in operating the actual system. Subproblems for North, East, and Southern Sector are given by tables 2.1, 2.2, and 2.3, respectively.

In the Northern Sector the objective function consists of both the stability of the daily water flow and the security in the water level of reservoir. Water distribution cost is not included because the pumping cost is almost proportional to the pump flow in the Northern Sector.

In the Eastern Sector the objective function consists of both the purification cost and transportation cost. It is noted that although not much cost is incurred in transporting water from Yodo River to the purification plant at Niwakubo when it is compared to that at Murano, the water quality at Niwakubo in general is inferior to that at Murano. Since the Southern Sector and the Northern Sector use Purified water from the two purification plants in the Eastern Sector, ideally the three sectors should be considered as one system. In order to treat the Eastern Sector independently we assume that the amount of water used by the Southern Sector is approximately the intake quantity derived from the water demand data of the Southern Sector. Similarly, the water intake quantity of the Northern Sector is assumed from the water demand data of the Northern Sector.

In the Southern Sector the objective function consists only of the transportation and distribution cost. Water purification cost is not included because the water distributed in this Sector has already been purified at Niwakubo and Murano of the Eastern Sector.

4. SUMMARY OF THE RESULTS

- 4.1 The computational experiments assure that only one or two changes of water flow are required per day at each of the purification plants, while five or six changes were required by the conventional

operating method. This improvement has been achieved by equalization of drawing water amount from the source, stabilization of water purification process, and equalization of transported water amount.

- 4.2 The computational experiments improve the safety of waterworks system due to the planned control of water levels at the reservoirs by introducing target reservoir level.
- 4.3 From the economic viewpoint, a reasonable reduction of electric power consumption can be achieved by systematic operation of purification plants, efficient water transportation through the network, equalization of varying demand, and clipping of peak demand. At the same time, this implies that contract for cheap electric power can be made.
- 4.4 The successful decomposition of the optimization problem yields the optimization of localized systems involving three Sectors. This means much reduction in computation time and flexibility of the computer problem.

5. CONCLUSION

Recently, works on water distribution systems have been studied by Fallside and Perry (1975), Alperovits and Shamir (1977), Rao and Bree (1977a), Rao-Markel-Bree (1977b), Sakaguchi-Nakahori-Ozawa (1978), and Joalland and Cohen (1978). In this chapter, we have proposed optimal policy for water distribution of the Osaka Prefectural Waterworks System and its applicability has been verified. Daily control policy of the waterworks system established here is based upon the daily anticipation of water demand. Large amount of time dependent variation in water demand has been absorbed and shared between the water reservoirs. As a result, enough stability of daily water flow has been achieved at each of pumping stations operational water level of each reservoir ranges between feasible high and low levels without exhaustion. It is noted that water level in every reservoir at six o'clock in every morning almost surely attain the target water level, and a higher safety of the water supply safety can be achieved than the one of the conventional water supply. Since efficient operation of reservoirs in common and efficient operation of the network system are attained, 2-7% cost reduction is expected by the optimal policy in the case of the anticipated demand of 1,200,000-1,9580,000m³/day. In the case of the low quality of raw water, the amount, of water to be purified at Niwakubo decreases to a great extent, because of the poor purification ability of Niwakubo Plant.

Summarizing these results, the optimization method has been successfully applied to the daily operation of the Osaka Prefectural Waterworks System. Implementation of the policies obtained from the analysis has led to reasonable savings in the operating costs, and at the same time has attained sufficient control of reservoir water level stability and network flow stability.

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