〈研究論文〉

Journal of the Korean Institute
of Industrial Engineers
Vol. 10, No. 1, June 1984

An Evaluation of Lot-sizing Rules under the Uncretainty of Demand and Lead Time

Hark Hwang*
Jae Ho Kim**

Abstract

This paper examines the influence of the uncertainty in demand and lead time on the relative performances of ten well-known single stage lot-sizing rules in a rolling schedule environment. Two other factors, coefficient of variation and time between orders, which may affect the performances of the rules are also considered. To compare the rules under an identical condition, 100% service level is set by introducing safety stocks.

The effects of various factor levels are checked statistically by the pairwise t-test and the results show that the uncertainty of the environment has a strong influence on the performance of the rules.

1. Introduction

Lot-sizing problem is basically to convert a forecast of component requirements into a series of replenishment orders. This involves determining how to group the time phased requirements into a schedule of replenishment orders which minimizes the inventory related costs. To solve this problem, a number of lot-sizing procedures have been proposed. Also, many comparative studies on these procedures have been proposed. Also, many comparative studies on these procedures have been undertaken. But most of these experiments assumed the constant lead time and so there is little guidance

* Korea Advanced Institute of Science and Technology
** Korea Telecommunication Authority

for an inventory manager who has to consider the uncertainty in lead time.

Thus this paper intends to compare the performance of 10 well-known single stage lot-sizing procedures under the uncertainty in both demand and lead time. For the experiment, performance data of 10 well-known rules are gathered for each case generated by combination of four factors, i.e., forecast error, lead time, time between orders, and forecast demand variation. From this experimental results, a manager could get some assistance in selecting a lot-sizing procedure under the uncertain environment of his industry.

Section 2 discribes the experimental factors and design. The results are analyzed in section 3. Finally section 4 discusses findings from this study.

Literature Survey

A number of comparative studies of single stage lot-sizing techniques in a discrete time system have been carried out (1, 2, 3, 4, 7, 8, 10, 11). In all cases, simulation has been a common research tool. Previous studies can be categorized as to whether a finite horizon or a rolling schedule environment has been used (static or dynamic situation), and whether demand uncertainty has been present or not. Note that all these studies assumed that the lead time is constant.

Berry (1) proposed a framework for comparing lot-sizing procedures based on two criteria, inventory related cost and computing time. And he gave an example of performance comparison of four lot-sizing procedures – Economic Order Quantity, Period Order Quantity, Part-Period Balancing (PPB), and Wagner-Whitin algorithm (WW). He considered two factors, the coefficient of variation of demand and the time between orders. Finally, Berry proposed three criteria, i.e., inventory cost performance, computational efficiency and procedural simplicity in choosing a lot-sizing procedure. Blackburn and Millen (2) investigated the effect of demand variance on cost performance of three lot-sizing techniques, WW, Minimum cost per period technique, and PPB. They showed that the performance of lot-sizing procedures studied are influenced by the lot-sizing index (i.e., cost parameters) and the demand variance. Besides these studies, Groff (4), Silver and Meal (8), respectively developed heuristic lot-sizing rules, and presented results of comparative experiment. Blackburn and Millen (3) studied the impact of a rolling schedule implementation on the performance of three single stage lot-sizing rules, PPB, Silver-Meal (SM), and WW. The main finding was that the SM heuristic outperforms the WW method in terms of average cost performance when the forecast horizon is less than 2 TB0-1 for all conditions examined and up to 4 TBO in some situations.

Wemmerlov and Whybark (11) presented results through a simulation evaluating fourteen lot-sizing procedures. Their study included the forecast errors, CV, TBO, and constant lead time as factors of the experiment. It was shown that the introduction of demand uncertainty not only changed the rank between the lot-sizing rules but also the character of this ranked relationship.

2. Experimental Design

The experimental design in this study is similar to those described in Wemmerlov and Whybark (11). The single-stage lot-sizing rules included in this study are as follows:

EOQ, Period Order Quantity (POQ), Discrete EOQ (DEOQ) (7),
Silver and Meal Procedure (SM) (8), Groff's
Marginal Cost Rule (GMR) (4), Part-Period Balancing (PPB) (6),

PPB with Look-ahead and Look-back Procedure (PPBLAB) (6),
Least Unit Cost (LUC) (6), Lot for Lot Ordering (LFL) (6),
Wagner and Whitin Algorithm (WW) (9).

Four factors which may affect the performance of the rules are considered and they are the coefficient of variation (CV), time between order (TBO), lead time (LT), and standard deviation of forecast error (σ_e).

CV is a device for measuring the lumpiness of data and is defined as the ratio between the standard deviation and the average of demand per period. The larger the value of CV is, the greater the variations among each period's demand are. In this study, the expected number of demand per period is assumed to be 100. Demand is forecasted by the use of uniform distribution with three levels of CV, and

level 1; CV = 0,
level 2; CV = 0.57,
and level 3; CV = 1.75.

For instance, to achieve 1.75 CV, a two step procedure is used to generate forecast demand as following;

1) $f(x) = \begin{cases} 0.3 & \text{for } X = 1 \\ 0.7 & \text{for } x = 0 \end{cases}$

2) $g(d_t \mid X = 1) = \begin{cases} 1/539 & \text{for } 64 \leqslant d_t \leqslant 603 \\ 0 & \text{otherwise} \end{cases}$

where d_t is the forecast demand for the t-th period.
Then, $E(d_t) = 100.5$
$Var(d_t) = 30629.7$
and CV = 1.75.

The relative performance of a rule can be influenced by the value of the cost parameters in the objective function. The important parameters are the unit holding cost h per period and the ordering cost S. The ratio between these two can be identified by $TBO = (2S/dh)^{1/2}$ where d is the average period demand. TBO also indicates the frequency of placing an order. Two levels of TBO are chosen and they are

level 1. TBO = 2 (S = 100 and h = 0.5)
and level 2. TBO = 6 (S = 900 and h = 0.5).

The uncertainty of lead time plays an important role in inventory management and the performance of a lot-sizing rule is expected to differ with the degree of the uncertainty of lead time.
Let L_t indicate the LT at the period t and σ_L^2 the variance of LT. Four different kinds of lead time distributions, each with $E(L_t) = 3$, are used and they are:

case 1. $\sigma_L^2 = 2$

$$\text{Prob}(L_t) = \begin{cases} 0.2 & \text{for } L_t = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

case 2. $\sigma_L^2 = 1$

$$\text{Prob}(L_t) = \begin{cases} 0.1 & \text{for } L_t = 1, 5 \\ 0.2 & \text{for } L_t = 2, 4 \\ 0.4 & \text{for } L_t = 3 \\ 0 & \text{otherwise} \end{cases}$$

case 3. $\sigma_L^2 = 0.5$

$$\text{Prob}(L_t) = \begin{cases} 0.25 & \text{for } L_t = 2.4 \\ 0.5 & \text{for } L_t = 3 \\ 0 & \text{otherwise} \end{cases}$$

case 4. $\sigma_L^2 = 0$

$L_t = 3$ for all periods.

Normal distributions with zero mean are used to generate forecast errors. These errors are used to find the actual demand of a particular period by subtracting the error from the forecasted value. Three levels of standard deviation of error σ_e are used for this study and

level 1; $\sigma_e = 0$ per period,
level 2; $\sigma_e = 30$ per period,
level 3; $\sigma_e = 60$ per period.

Therefore, a full factorial design is used with 4 factors, i.e., CV, TBO, LT, and σ_e, having 3, 2, 4, and 3 levels each and 72 cells are generated. Each rule is applied 10 times for each cell. Thus, a total of 7,200 total cost data are recorded. Each rule is applied to a same set of data in each replication.

Since shortages can occur due to the uncertainty of demand and lead time, different service levels might result from different lot-sizing procedures. The situation would make the comparison of the rules quite complicated and one way of avoiding this difficulty is maintaining an identical service level. In this study, almost 100% service level is pursued by introducing safety stocks and the performance measure becomes the sum of the inventory ordering and holding costs over a finite horizon.

The performance of each lot-sizing rule is expressed relatively in terms of those by the WW which gives an optimal solution in deterministic environment and finite planning horizon. Let PI (k, j) be the relative performance index of the rule k at j-th cell and TC (k, j, i) the total cost of the rule k at j-th cell in i-th replication.

Then

$$PI(k, j) = \sum_{i=1}^{10} [TC(k, j, i) - TC(WW, j, i)] \,/\, [10 \cdot TC(WW, j, i)].$$

The simulated lot-sizing takes place on the length of 330 periods for each experiment and the first 30 periods are taken as a start-up period to assure the stability of the simulation process. The data obtained from the remaining 300 periods are used for the calculation of the inventory cost. Following the suggestions of Ludin and Marton (5), 30 periods are taken as the forecast horizon. For every period, the net requirement r_t of a period t is calculated where

$$r_t = d_t + d_{t+1} + \dots + d_{t+E(L)} + \text{safety stock} - \text{inventory on hand and on order.}$$

When r_t becomes negative, no order takes place. Otherwise, lot-sizing takes place over the forecast horizon and only the first ordering is made. The order arrives at a predetermined period $t + L_t$. Figure 1 shows the experimental procedure taken in this study.

3. Results and Analysis

The results of the experiment are provided in table 1. These data are the average of PI in percent obtained for the case of $\sigma_L^2 = 0$, the average PI of each rule is based on N = 180 observations, i.e., (3 levels of CV) · (2 levels of TBO) · (3 levels of forecast error) · (10 replications). The introduction of the uncertainty in LT influences the relative performance of each rule. All the rules except POQ, LFL, PPBLAB, perform better than WW.

Table 2 shows the influence of the uncertainty of LT when forecast errors do not exist. The performance of LUC, PPB, GMR, and SM worsen relative to WW while POQ and LFL become improved as the variance of LT increases from 0.5 to 2.

Table 3 shows the results when uncertainty is not present, i.e., $\sigma_L = 0$ and $\sigma_e = 0$. The ranking of the first three rules, WW, GMR, and SM, are the same as those in Wemmerlov and Whybark's study.

Table 4 provides the ranked results for the case when uncertainty in demand and lead time are present. Comparison of the results in table 3 with those in table 4 indicates that the existence of the uncertainty in LT has a remarkable impact on the performance of DEOQ, LUC, EOQ, WW, and PPBLAB. DEOQ and LUC rank on the best two rules and WW and PPBLAB worsen.

The overall performance of each rule based on 720 observations are described in table 5. DEOO, GMR, PPB, and LUC are evaluated better than WW and compared to those in table 3, LUC, WW, SM, and PPBLAB have distinctive changes in ranking. In particular, the rank of DEOQ changes 1 from 7, LUC 4 from 8 and WW 5 from 1. Pairwise t-test with 5% singificance level are used to analyze the results statistically. Significance testing on the first five rules in table 4 based on 360 observations per rule indicates that DEOQ is different from the other rules and that no significant differences between the remaining four rules exist.

Comparison of this test results with those by Wemmerlov and Whybark suggests that DEOQ performs statistically better relative to the other rules when the uncertainty of lead time is introduced in addition to forecast error.

Table 6 shows the test results of the effects of various factor levels on the relative performance of the rules. As an example, for TBO with two levels, two groups with 360 observations each are used. The results indicate that varying the levels of TBO and CV have no significant effects while the introduction of demand uncertainty somewhat matters.

Lead time uncertainty has significant effect on the performances of the rules, while varying the levels of the uncertainty have not much effect.

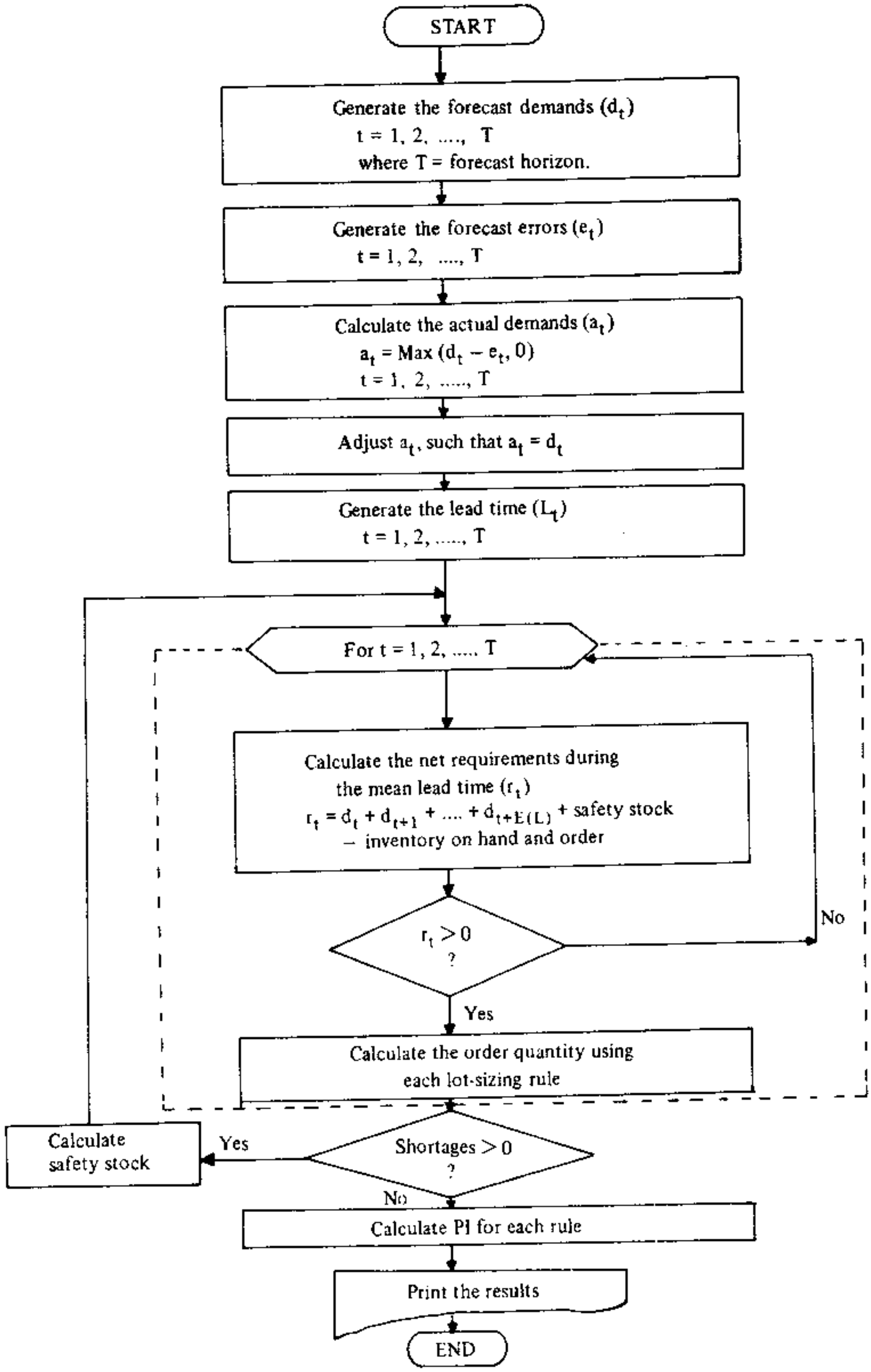

Figure 1. Flow of the experimental procedure.

Table 1. Average PI in percent

Factor	Level \ Rule	EOQ	DEOQ	POQ	LFL	LUC	PPB	PPBLAB	GMR	SM
LT	$\sigma_L^2 = 2$	–1.446	– 3.100	0.629	14,162	– 1.130	– 0.754	6.063	– 0.623	– 0.049
	$\sigma_L^2 = 1$	–0.068	– 2.327	3.206	18.812	– 0.773	– 1.152	7.911	– 0.695	– 0.794
	$\sigma_L^2 = 0.5$	–0.414	– 3.500	7.909	24.331	– 2.538	– 2.524	8.593	– 1.349	– 0.950
	$\sigma_L^2 = 0$	122.894	4.140	8.448	76.237	4.218	2.537	4.092	0.147	1.892
σ_e N = 240	0	73.756	– 0.014	4.961	41.109	2.416	0.492	6.754	– 0.427	1.323
	30	12.024	– 1.178	5.371	32.930	– 0.693	– 0.700	6.449	– 0.257	– 0.404
	60	4.945	– 2.400	4.812	26.118	– 1.890	– 1.206	6.792	– 1.207	– 0.843
CV N =240	0	0.0	0.0	2.974	54.390	0.0	0.084	0.0	0.0	0.197
	0.57	0.440	– 2.104	5.500	47.810	– 1.950	– 1.254	–0.135	– 1.490	– 0.078
	1.75	90.285	– 1.486	6.670	–2.003	1.782	– 0.244	20.130	– 0.400	– 0.044
TBO N = 360	2	59.385	– 1.190	9.077	–5.834	– 0.583	– 0.854	11.222	– 0.869	– 0.473
	6	1.089	– 1.203	1.019	72.605	0.477	– 0.089	2.109	– 0.391	0.524

Table 2. Average PI in percent when forecast errors are zero.

Factor	Level \ Rule	EOQ	DEOQ	POQ	LFL	LUC	PPB	PPBLAB	GMR	SM
LT N = 60	$\sigma_L^2 = 2$	–0.775	–3.177	0.747	14.397	–0.083	–0.281	6.266	–0.120	1.800
	$\sigma_L^2 = 1$	–0.048	–2.040	1.804	20.753	–1.905	–1.031	5.427	–0.770	–0.275
	$\sigma_L^2 = 0.5$	–1.696	–5.495	4.898	23.999	–3.466	–4.105	7.638	–2.684	–2.776
	$\sigma_L^2 = 0$	297.544	10.657	12.396	105.305	15.118	7.380	7.684	1.867	6.540

Table 3. Results based on N = 60 for the case of no forecast error and constant lead time.

Rank	Rule	Average	Std. dev.
1	WW	0.0	0.0
2	GMR	1.867	2.174
3	SM	6.540	7.372
4	PPB	7.395	5.624
5	PPBLAB	7.684	12.692
6	DEOQ	10.657	11.242
7	POQ	12.396	8.138
8	LUC	15.118	19.414
9	LFL	105.305	99.888
10	EOQ	297.544	642.036

Table 4. Results based on N = 360 when forecast errors are present and lead times are uncertain.

Rank	Rule	Average	Std. dev.
1	DEOQ	–2.678	8.006
2	LUC	–1.312	9.578
3	PPB	–1.308	8.245
4	GMR	–0.738	6.235
5	SM	–0.688	6.976
6	EOQ	–0.544	10.290
7	WW	0.0	0.0
8	POQ	4.631	14.325
9	PPBLAB	8.063	19.939
10	LFL	18.768	37.107

Table 5. Overall performance based on N = 720 observations.

Rank	Rule	Average	Std. dev.
1	DEOQ	–1.197	9.606
2	GMR	–0.630	6.895
3	PPB	–0.471	8.313
4	LUC	–0.056	11.465
5	WW	0.0	0.0
6	SM	0.025	8.056
7	POQ	5.048	13.677
8	PPBLAB	6.665	17.597
9	EOQ	30.242	199.987
10	LFL	33.386	62.431

Table 6. Effect of various factor levels on the cost performance.

Factor	Factor levels compared	DEOQ	LUC	PPB	GMR	SM
TBO	2 vs. 6	n.s.	n.s.	n.s.	n.s.	n.s.
CV	0 vs. .56	1%	5%	n.s.	n.s.	n.s.
	0 vs. 1.75	n.s.	n.s.	n.s.	n.s.	n.s.
	.58 vs. 1.75	n.s.	5%	n.s.	n.s.	n.s.
σ_e	0 vs. 30	n.s.	1%	n.s.	n.s.	5%
	0 vs. 60	1%	1%	5%	n.s.	1%
	30 vs. 60	n.s.	n.s.	n.s.	n.s.	n.s.
σ_L^2	2 vs. 1	n.s.	n.s.	n.s.	n.s.	n.s.
	2 vs. .5	n.s.	n.s.	5%	n.s.	n.s.
	2 vs. 0	1%	1%	1%	n.s.	1%
	1 vs. .5	n.s.	n.s.	n.s.	n.s.	n.s.
	1 vs. 0	1%	1%	1%	5%	1%
	.5 vs. 0	1%	1%	1%	5%	1%

n.s. = not significant with 5% level
1% = significant with 1% level, or less.

4. Conclusions

The simulation results provided the evidence that the introduction of lead time uncertainty has a strong influence on the relative performance of the lot-sizing rules as anticipated. WW was surpassed by other rules and the rank order of the overall performances was DEOQ, GMR, PPB, LUC, and WW. DEOQ emerged as number one and showed substantially better compared to the other rules. PPB and GMR proved effective when uncertainty exists and belong to the 5 best rules for all cases considered. DEOQ, PPB, and GMR are easy to understand and simple to use. Therefore, if the uncertainty in lead time and forecast demand exists, these rules could be used in actual single stage inventory system.

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