〈研究論文〉

Journal of the Korean Institute of Industrial Engineers Vol. 10, No. 1, June 1984

An Optimization Procedure for a Multi-Item Multi-Source Materials Acquisition Problen

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Abstract

A materials acquisition planning (MAP) problem that involves the determination of how much to order of a number of different items from a number of different suppliers is considered. This particular problem is modelled as a nonlinear mixed integer programming problem. A solution procedure based upon the partition of variables is developed to handle the MAP problem. This solution procedure utilizes a modified Hooke-Jeeves Pattern Search procedure along with a linear programming simplex algorithm. An example problem is presented and the results of applying the suggested solution procedure to this problem are reported.

1. Introduction

The need for materials acquisition planning (MAP) arises when there is a number of sources from which a number of different types of items must be obtained. The problem is to determine when to order, how much to order, and from which supplier to order for each item on a procurement list. The demand for the individual items can be determined by exploding the bill of materials for the master production schedule. Thus, if the materials are ordered specifically to meet the needs of the master production schedule, then the demands for individual items can be treated as deterministic. In the MAP problem with deterministic demand the following will be pertinent to the questions of when, how much and from which supplier to order:

(1) the cost of acquiring ownership of the items;
(2) the cost of transporting the item from the supplier to the point where it will be stored or used;
(3) the cost of placing a purchase order;
(4) the cost of receiving which consists of unloading, inspecting, placing in inventory and making

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payment to the supplier;

(5) the cost of holding inventory.

The traditional approach to the MAP problem is to assume that there is only one source from which each item can be obtained [3, 4, 5]. This assumption eliminates the need to consider the cost of acquiring ownership. However, when there are multiple suppliers in a competitive market, this assumption will not be reasonable. The traditional approach also treats preparation cost, transportation cost, and receiving cost as part of order cost. Multi-item single source inventory models usually assume that order cost consists of a fixed cost of placing an order regardless of the number of line items and a cost which varies in proportion to the number of line items in the order. This assumption may be adequate for special situations in which transportation cost is either negligible or constant for all line items in an order. When these conditions are not met it will be more realistic to explicitly account for transportation cost in the analysis of MAP problems. One reason for an explicit accounting of transportation costs is because rising energy costs have increased the relative importance of transportation cost. At the same time computerization of the purchase order preparation process has greatly reduced the relative importance of the data processing cost associated with preparing a purchase order. In situations in which the purchase order generation system is run periodically the incremental cost of preparing a purchase order will consist of little more than the cost of the paper and postage. When the purchase order placement cost is trivial in relation to transportation cost, then purchase order placement cost can be ignored when analyzing the MAP problem. The cost of receiving will typically depend primarily on the number and kinds of units received during a given period and not on the number of units in each individual shipment. When this is the case the total receiving cost for a period will not appreciably influence the solution to the MAP problem and can therefore be ignored.

A mathematical model for the MAP problem with multi-item and multi-supplier is developed in section two. This particular model has as its objective the acquisition of a number of demanded items so as to minimize the sum of purchase cost, transportation cost and inventory holding costs. Section three describes a solution procedure for solving the MAP problem of interest. This procedure is based upon partitioning the origining problem into two subproblems and successively solving these problems a number of times. An example problem is presented in section four along wit the numerical results obtained when the suggested solution procedure is applied to the problem.

2. Model Formulation

The materials acquisition planning (MAP) problem considered in this paper is to determine how much to order of a particular item from each of a number of suppliers. The selected objective is to minimize the total relevant cost which consists of: (1) purchase cost; (2) inventory holding cost; and (3) transportation cost. This problem is to be solved subject to the following constraints:

(1) the total amount of any specific item acquired from all suppliers must equal the demand for that item

(2) the total amount acquired from any supplier cannot exceed the transportation capacity assigned to that supplier.

The following assumptions are utilized in formulating this model:

(1) the planning horizon is equal to one period

(2) demand for each of the items is known and occurs at a uniform rate throughout the planning period

(3) each item can be obtained from a number of different sources at different locations
(4) a fixed transportation cost dependent on source location is charged for each trip
(5) holding costs are proportional to the dollar value of inventory units
(6) the volume and weight of each item is known
(7) all transportation units (trucks in the following) have the same volumetric and weight capacity.

Constants

d_i = demand (consumption) of item i

h = holding cost per unit per planning period

m = number of suppliers

n = number of items

P_{ij} = price of unit item i offered from supplier j

TC = total cost

T_{Cj} = transportation cost of a truck from supplier j

T_v = volume of a truck

T_w = weight capacity of a truck

V_i = volume of one unit of item i

W_i = weight of one unit of item i.

Variables

U_j = number of trucks needed for supplier j

X_{ij} = number of units of item i purchased from supplier j.

The objective function for the MAP problem is given by the following expression:

$$\text{Minimize TC} = \sum_{j=1}^{m} \sum_{i=1}^{n} P_{ij} X_{ij} + \sum_{j=1}^{m} T_{Cj} U_j + \frac{h}{2} \sum_{j=1}^{m} \frac{1}{U_j} \sum_{i=1}^{n} P_{ij} X_{ij} \qquad (1)$$

The first term in this expression represents the total purchase cost for all items. The second term represents total transportation cost. The third represents inventory holding cost. This last term was formulated on the basis of the assumption that if more than one trip is required then the trips will be equally spaced throughout the planning period and will carry the same dollar value of cargo.

The above expression for the objective function can be expressed in a more compact form by combining terms

$$\sum_{j=1}^{m} \left(1 + \frac{h}{2U_j}\right) \sum_{i=1}^{n} P_{ij} X_{ij} + \sum_{j=1}^{m} T_{Cj} U_j \qquad (2)$$

A number of constraints must be observed in minimizing the above expression:

(1) Demand constraint

$$\sum_{j=1}^{m} X_{ij} = d_i, \quad i = 1, 2, ..., n. \qquad (3)$$

The purpose of this constraint is to assume that the total amount purchased will be adequate.

(2) Volumetric capacity constraint

$$\sum_{i=1}^{n} \frac{V_i X_{ij}}{T_v} \leqslant U_j, \quad j = 1, ..., m. \qquad (4)$$

This constraint specifies that the total volume shipped over any route must not exceed the total capacity of the truck trips assigned to that route.

(3) Weight capacity constraint

$$\sum_{i=1}^{n} \frac{W_i X_{ij}}{T_w} \leqslant U_j, \quad j = 1, 2, ..., m. \qquad (5)$$

The purpose of this constraint is to assume that the total weight shipped over any route does not exceed the weight capacity of the truck tripe assigned to that route.

(4) Nonnegativity and integer constraints

$X_{ij} \geqslant 0, U_j \geqslant 0$ and integer.

3. Solution Procedure

The model in section two is a mixed integer nonlinear programming problem. The number of transportation units (trucks, ships, planes, etc.) should be integer, while the number of units purchased can be either integer or continuous variables. If the quantities to be purchased are sufficiently large, then the decision variables can be treated as continuous. In this case we have a linear programming (LP) problem and the simplex algorithm can be utilized to determine the optimal value of the objective function for the unconstrained outer minimization problem. If the quantities to be purchased are relatively small then they should be treated as integers. In this case it would be necessary to utilize an integer programming (IP) algorithm to determine the optimal value of the objective function for the unconstrained outer minimization problem. If the problem is sufficiently small, then enumeration techniques could be employed to determine the optimal solution. However, the problem is combinatorial in nature and real world problems almost always will be too large to be solved by enumeration techniques. Consequently, it was necessary to develop a solution procedure which could efficiently solve realistic problems. The solution procedure developed utilizes the projection principle [1]. This principle utilizes a transformation that temporarily fixes the transportation units integer decision variables. The resulting problem, with integer variables fixed, will be referred to as the "inner minimization" problem. The problem which results from this transformation is imbedded in an "outer minimization" problem, the objective of which is to determine the optimal number of transportation units to assign to a particular route. The outer minimization problem is an integer optimization problem.

To solve the integer problem, a modified Hooke-Jeeves pattern search algorithm was employed. The conventional Hooke-Jeeves method [2] evaluates the objective function of the unconstrained optimization problem in order to determine how the values of the unconstrained optimization problem in order to determine how the values of the decision variables should be perturbed. In the present application the value of the objective function for the outer minimization problem is determined by solving the LP problem which characterizes the inner minimization problem. Based on the outcome of the inner minimization LP problem, the Hooke-Jeeves pattern search selects an integer point representing the number of trucks assigned to the suppliers, and the resulting LP problem is solved. This procedure is repeated until there is no further improvement in the objective function value. Thus the Hooke-Jeeves pattern search selects an integer point representing the number of trucks assigned to the suppliers, and the resulting LP problem is solved. This procedure is repeated until there is no further improvement in the objective function value. Thus the Hooke-Jeeves algorithm was modified so that the simplex algorithm or an integer programming algorithm could be used to determine the value of the objective function for the unconstrained optimization problem. (See the Appendix for a flow chart of the modified Hooke-Jeeves pattern search solution procedure.) The solution of the LP problem or ILP problem in the inner minimization problem specifies the number of units of each type product to be purchased from each supplier for a given assignment of trucks to routes. The Hooke-Jeeves pattern search specifies how the truck assignments should be varies so that total cost can be reduced. By alternately solving the LP or ILP inner minimization problem and the unconstrained outer minimization problem a local optimal solution will be reached. The stopping criterion for the Hooke-Jeeves algorithm indicates when this has occurred.

The efficiency of the solution procedure can be increased by identifying infeasible solutions to the LP and ILP in the inner minimization problem without having to explicitly solve the inner minimization problem. Two ways for doing this have been developed:

(1) If the number of trucks available is less then the minimum number required to transport the required volume assuming that each truck is filled to capacity, then the solution to the LP or ILP in the inner minimization problem will be infeasible regardless of how the trucks are assigned. The minimum number of trucks (M) necessary to transport all the demands can be calculated as follows:

M is the smallest integer equal to or exceeding.

$$\frac{\sum_{i=1}^{n} d_i v_i}{\text{truck capacity}}$$

(2) No route can be assigned a negative number of trucks.

The computational effort required to solve the original optimization problem can be reduced by judiciously selecting starting values for the number of trucks. One procedure for doing this would be to use the optimal solution for a prior time period with similar demand characteristics. Another procedure would be to tentatively assign the various items to be purchased to the suppliers on the basis of the lowest purchase price.

A disadvantage of all search produres is that they do not guarantee convergence to a global optimal solution. However, the risk of converging to a non-global optimal solution can be reduced by using a variety of different starting points.

4. Example Problem

A problem was generated and solved by the solution technique described in the previous section. This example has five suppliers and each of the suppliers is offering ten itms with the prices shown in Table I. Demands of the goods (D), volume of each item (V) and the transportation cost (T_{cj}) of each supplier are also shown in Table I. A truck's volume capacity is 100 units. Weight considerations were deemed insignificant when compared with volume restrictions.

Table I. Example Problem Information

Supplier \ Item	1	2	3	4	5	6	7	8	9	10	T_{cj}
1	2.5	1	10	5	3	1	20	99999*	9	5	40
2	3	.9	8	6	2.5	1.3	19	3	8	4.5	50
3	2	1.2	99999	4	3.5	.9	25	4	10	5	20
4	3.2	1	11	99999	4	1	15	2	12	99999	70
5	99999	.8	10.5	7	2.7	1.1	20	3	9	6	100
D	500	400	1000	300	700	1500	200	500	300	100	
V	3	2	5	5	2	1	5	2	10	7	

*99999 represents that the supplier does not offer the item (h=0.1)

Two different starting points were tested and resulted in the following local optimal solutions.

Starting point One

Starting vector = (40, 40, 40, 40, 40) where ith element represents the number of trucks that ith supplier can utilize.

Supplier \ Item	1	2	3	4	5	6	7	8	9	10	Trucks Allocated
1		400							240		32
2			1000		700			200			68
3	500			300		1500			60	100	60
4							200	300			16
5											0
D	500	400	1000	300	700	1500	200	500	300	100	

Total cost = 28,213.45

Starting Point Two

Starting vector = (10, 90, 20, 10, 80)

Supplier \ Item	1	2	3	4	5	6	7	8	9	10	Trucks Allocated
1						100			10	1000	9
2			1000						60		56
3	500			300		1400					44
4							200				10
5		400			700			500	230		56
D	500	400	1000	300	700	1500	200	500	300	100	

Total cost = 31,320.96

In both problems the initial step size was 8 trncks and it was reduced by half until it reached one truck.

5. Conclusion

An optimization procedure for a specific multitem multi-source materials acquisition problem has been presented. The solution precedure developed for the problem utilizes a modified Hooke-Jeeves pattern search and linear programming simplex method althernately. This solution method may be used to solve problems of moderate size even though the problem as formulated is complexanonlinear mixed integer problem.

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APPENDIX

Flow Chart for Modified Hooke-Jeeves Pattern Search