〈硏究論文〉

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Optimal Design of a Branched Pipe Network with Multiple Sources

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Abstract

This paper is concerned with a branched pipe network system which transports some fluids or gas from multiple sources to multiple demand nodes. A nonlinear programming model is proposed for determining junction locations simultaneously with selection of pipe sizes and pump capacities such that the capital and operating costs of the system are minimized over a given planning horizon. To solve the model, a hierarchical decomposition method is developed with the junction location being the primary variable. With some values fixed for the primary, the other decision variables are found by linear programming. Then, using the postoptimality analysis of LP, junction locations are adjusted. We repeat this process until an optimum is approached. A simple example of designing a water distribution network is solved to illustrate the optimization procedure developed.

1. Introduction

Pipe networks are generally classified into two kinds of system, a distribution system which transports some materials from sources to demand nodes, and a gathering system in which a node collects materials from multiple sources. However, design methods of those systems are almost same because the latter, having only one demand node, is a special case of the former. In this paper, the network under consideration is a branched distribution system (i.e. a tree structure). The other system could be easily handled by slight modifications of the method presented.

The costs of a network consist of the capital cost of pipes connecting nodes and pump operating

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cost (or energy cost) over a planning period. The pipe diameter is an important decision variable since both junction location and pump capacity are closely related to its size. Smaller diameter of pipe causes large friction head (pressure) loss, requi: ıg larger pumping head and higher operating cost as shown in Figure 1. It shows a trade-off relationship between the pump capacity and the pipe size.

In reality, designing a new pipe network involves several decision making problems. Firstly, one has to determine how to insert junction nodes to suitably connect demand nodes to sources. A junction node is the place where incoming flows either merge or divide into more than one flow, and each of them flows in defferent pipe links. And also, the exact position of each junction node has to be chosen. Finally, pipe sizes of each link and pump capatiy of each source have to be determined. To obtain an optimal design, the above problems should be considered simultaneously, not separately.

Many sophisticated mathematical methods related to the pipe network were developed based on non-linear programming [1, 2, 3, 4, 5, 9, 12], linear programming [7, 8, 11] and dynamic programming [13]. However, all of these works assumed that the network layout is always given. Only a few works can be found that treat junction location problems in the network design. Bhaskaran et al. [2] firstly considered the junction location as a decision variable in a gas pipe network. However, he treated the pipe sizes as continuous variables, which requires rouding-off of the pipe diameters obtained in the final solution to commercial sizes. Also, energy cost was not incorporated in the objective function. Rothfarb et al. [14] treated an offshore pipeline design problem. But his work can not be considered as a junction location problem, since the junctions have to be made at the wells. And in Vidal's work [15], the design of a single source and single terminal system was considered with the predetermined junction nodes.

This paper presents an optimal design procedure for a water distribution system in which junction locations, pipe sizes, and pump capacities are determined simultaneously.

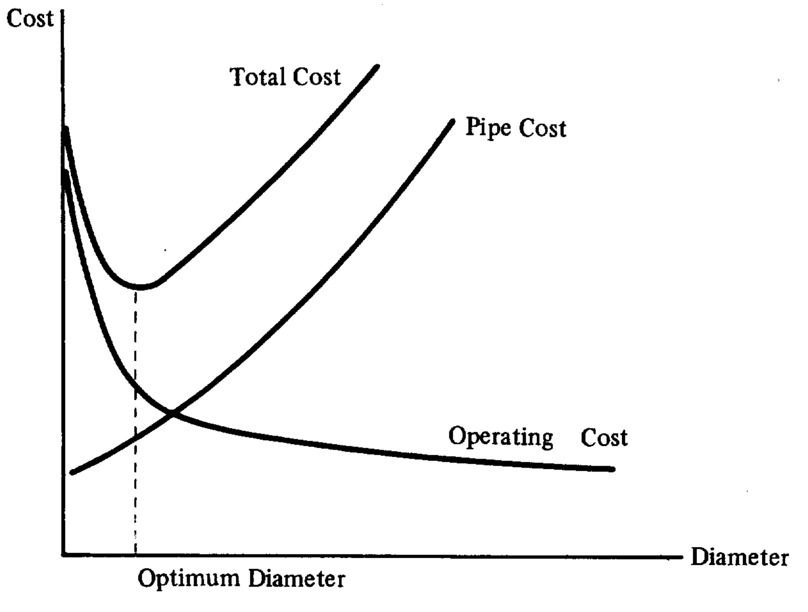

Figure 1. Variation of costs with pipe diameter

2. The Problem

A pipe network is to be built to transport water from NS different sources to ND demand nodes as depicted in Figure 2.

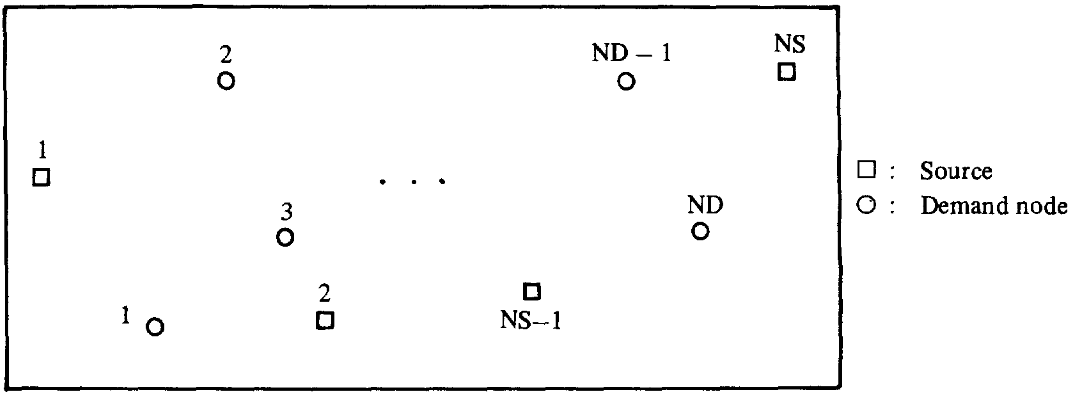

Figure 2. Sources and demand nodes being connected.

Between each source and each demand node, there may exist at least one junction node. The maximum number of junction nodes that has to be considered in the branched network is NS + ND - 2, as proved in the well known Steiner's problem [6]. The network configuration having NS + ND - 2 junction nodes is called a full configuration. Otherwise, it is called reduced.

The i-th demand node requiring Q_i rate of flow, i = 1, 2, ..., ND has to be connected to the j-th source having S_j supplying capacity, j = 1, 2, ..., NS, by a network of pipes so as to satisfy the supply-demand constraint given as,

$$\sum_{i=1}^{NS} S_i = \sum_{j=1}^{ND} Q_j .$$

A link of pipe between two nodes can have one or more pipe sections with different diameters in a discrete value. The diameters of each link are selected from a finite set, $D = \{d_1, d_2, ..., d_K\}$ and a unit length pipe cost for each $d_i \in D$ is known. The pressure of the flow arriving at a demand node i is required to be greater than or eaqual to the given P_i.

To calculate friction head loss in a pipe section of a link, the widely used Hazen-Williams equation is adopted:

$$S = \alpha \, (Q/CHW)^{1.852} / D^{4.875}$$

where S = head loss per unit length of pipe
Q = flow through the pipe per unit time
CHW = Hazen-Williams coefficient
D = pipe diameter
α = a constant depending on the units used.

Given this information, the problem is how to select the junction nodes and its locations, the diameters of each of link, and pump capacities such that total cost of the network is minimized.

3. Model Formulation

If we choose the NJ ($\leqslant$ NS + ND − 2) junction nodes to connect the NS sources and the ND demand nodes, and then construct a branched network with them (see Figure 5), the flow in each link, q is easily found by the flow conservation law where the amount of inflows in a node should be equal to that of outflows. That is,

$$\sum_i q_{ij} = \sum_k q_{jk} + Q_j.$$

Then for a single source system, if we denote the position variables of the junction nodes by (x_i, y_i) for i = 1, 2, ..., NJ, a nonlinear programming model for optimizing our problem network is formulated as below.

$$\text{Minimize } TC = \sum_{\substack{(i,j)\\ \in A}} \sum_{k=1}^{K} CP_k l_{ijk} + CH \cdot y \quad \text{.......... (1)}$$

(P) subject to

$$\sum_{k=1}^{K} l_{ijk} = L_{ij} \text{ for all link } (i, j) \quad \text{.......... (2)}$$

$$\sum_{\substack{(i,j)\\ \in PT_n}} \sum_{k=1}^{K} S_{ijk} l_{ijk} + y \geqslant P_n \text{ for } n = 1, 2, ..., ND \quad \text{.......... (3)}$$

$$l_{ijk} \geqslant 0 \qquad \text{for all } (i, j) \text{ and } k = 1, 2, ..., K$$

$$y \geqslant 0$$

where $L_{ij} = ((x_i - x_j)^2 + (y_i - y_j)^2)^{1/2}$ for all (i, j) .... (4)

(x_i, y_i) = coordinate location of node i on 2-dimensional Euclidean space

TC = total cost of the system

CP_k = capital cost per unit length of pipe having k-th diameter including installation and salvage values

CH = present worth cost of unit pump-head at the source

l_{ijk} = length of k-th diameter pipe in link (i, j)

S_{ijk} = friction head loss of k-th diameter pipe in link (i, j)

y = pumping head at the source

P_n = pressure requirement at demand node n.

A = a set of links in the network
PT_n = a path linking the source and demande node n.

The first set of constraints (2) assures that the sum of the lengths of the pipe section in the link (i,j) is equal to the prespecified length of the link (i, j). The second set of constraints, (3) assures that remaining head (source head – friction head loss) should exceed the minimum head required at each demand node.

In this model, Euclidean distance which is widely used in this area [2], is assumed for measuring the distance between any two nodes. The variable (x_i, y_i) in (4) becomes a known constant if the node i is either a source or a demane node.

For the problem with multiple sources, the set of constraints(3) are changed as follows, using electrical circuit theory developed by Gupta et al. [8]:

$$\sum_{\substack{(i,j) \\ \in L_p}} \sum_{k=1}^{K} S_{ijk}\, l_{ijk} < P_s - P_e \quad \text{for} \quad P = 1, 2, ..., NP$$

where L_p = a set of links in loop p with starting node s and terminal node e
NP = number of pseudo loops in the system modified by electrical circuit theory.

Also, the cost function (1) becomes,

$$TC = \sum_{\substack{(i,j) \\ \in A}} \sum_{k=1}^{K} CP_k l_{ijk} + \sum_{s=1}^{NS} CH_s y_s.$$

where y_s is the pumping head required at the source s.

4. Optimization Method

The outline of the optimization procedure is that, firstly, set up and solve the problem (P) for the full configuration of the problem network. If the lengths of some links are found to be smaller than the prescribed value, then the optimal network becomes a reduced configuration.

To solve (P) which contains nonlinear terms at the righthand side in the first set of constraints (2), a two-level hierachically integrated method is developed as depicted in Figure 3. In the first level, the usual optimization of the design is considered when the junction locations are assumed to be known. Since all L_{ij} are constant, (P) becomes a linear programming problem where the length of each pipe section of each link and the pump operating head are the decision variables. Then, the optimal total cost obtained from the LP problem can be considered as a function of junction locations (X, Y) which is a pair of ND-dimensional vectors:

$$TC = LPC\,(X, Y)$$

In the next level, systematic changing of (X, Y) with the aim of improving cost is carried out using a rule suggested below.

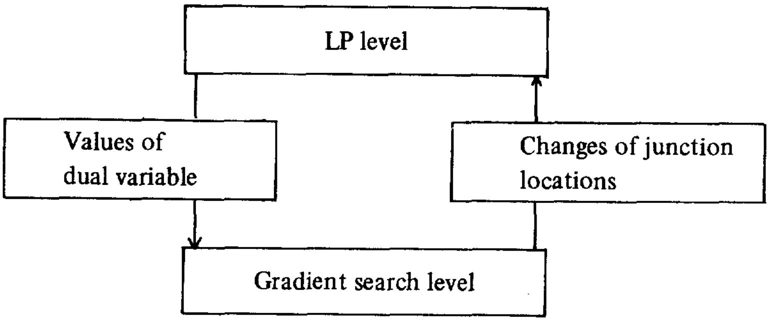

Figure 3. Hierarchy of the optimization method

The rule is based on the dual variables of the constrains (2), which guide a gradient move, $(\Delta X, \Delta Y)$, a vector of changes in the junction locations. $(\Delta X, \Delta Y)$ is sought such that,

$$LPC\,(X + \Delta X, Y + \Delta Y) < LPC\,(X, Y).$$

The first partial derivative of the total cost function with respect to x_i becomes,

$$\frac{\partial TC}{\partial x_i} = \frac{\partial TC}{\partial L_{ij}} \cdot \frac{\partial L_{ij}}{\partial x_i} = u_{ij} \cdot \frac{\partial L_{ij}}{\partial x_i}$$

From equation (4),

$$\frac{\partial L_{ij}}{\partial x_i} = (x_i - x_j) / L_{ij}$$

Hence

$$\frac{\partial TC}{\partial x_i} = u_{ij}\,(x_i - x_j) / L_{ij}$$

Because u_{ij} is the value of dual variable of the constraints (2), the gradient, $\partial TC/\partial x_i$ can be readily obtained. Similarly, for the variable y_i,

$$\frac{\partial TC}{\partial y_i} = u_{ij}\,(y_i - y_j) / L_{ij}$$

The number of constraints containing the variables (x_i, y_i) is always 3, since 3 links are joined at the junction node i. Therefore, to find the feasible direction of changes $(\Delta x_i, \Delta y_i)$ which brings cost reduction, the following heuristic rule given in Figure 4 is adopted. There Δ implies a step sizé of change.

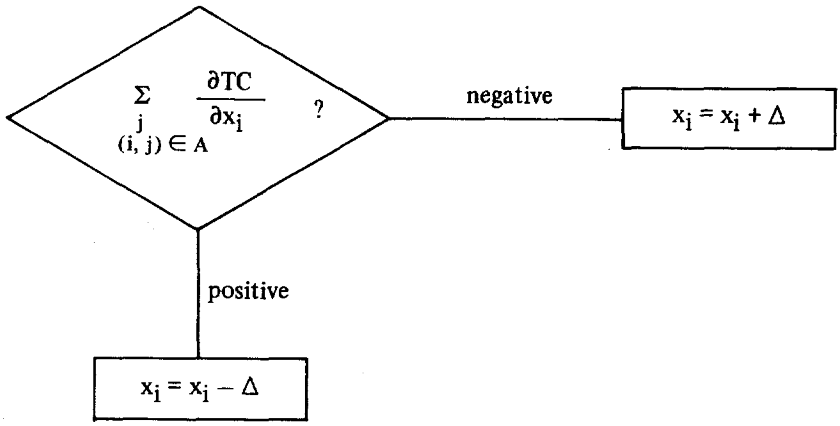

Figure 4. A heuristic rule to determine the direction of change.

Then, with the new values of junction variables, it goes back to the level 1. In this way the process is repeated until a predetermined amount of cost decrease is not possible in the objective function of the problem (P). The step size may be adjusted in each iteration using a simple heuristic rule such as the method of rotating coordinates [10].

5. An Example Problem

We considered the problem with 4 demand nodes and 2 sources given in Figure 5. An initial configuration with 4 junction nodes is arbitrarily chosen. The maximum number of junction nodes is

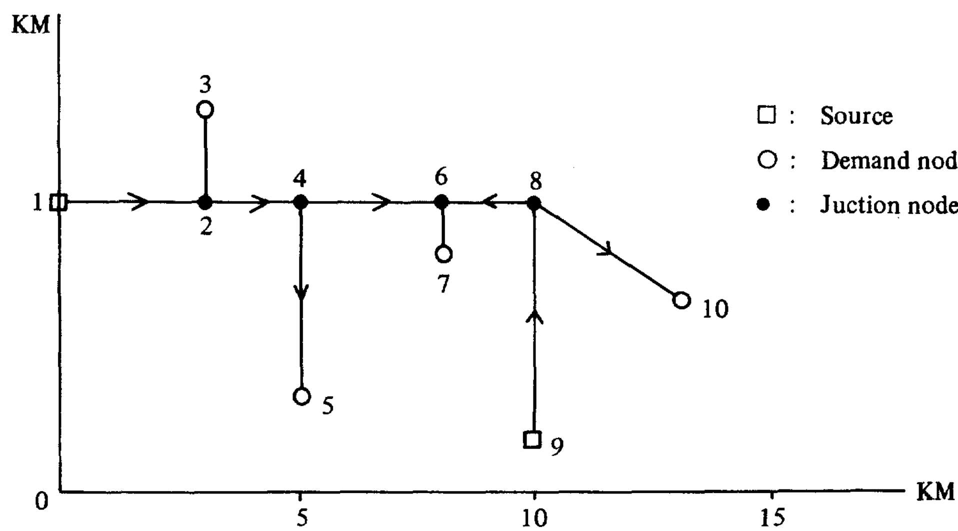

Figure 5. An initial configuration of the problem network consisting of 2 sources and 4 demand nodes

$$NS + ND - 2 = 2 + 4 - 2$$
$$= 4,$$

and the initial one becomes a full configuration. Details of the data are listed in Table 1 and 2. Also, the additional assumptions are made as follows:

1) The minimum required residual head is 10.0 M for each demand node.
2) The planning horizon is 15 years.
3) Pump operating cost is $0.1 per KWH.
4) The Hazen-Williams coefficient, CHW is 100 for all links.
5) The discount rate is 10%.

The present value of energy cost resulted from 15 year operation is represented in dollars as a function of flow (M^3/hr), q and pump head, y expressed in meters:

$$CH = 21.62\ qy.$$

To show the sensitivity of the optimization method to initial junction location, the computer runs are carried out with six different initial values. The results are summarized in Table 3, from which the final costs are found to be approximately same regardless of the initial values.

Since the final location of the junction node 6 turns out to be the same as that of the demand node 7, the optimal network depicted in Figure 6 becomes a reduced configuration having only 3 junction nodes. Among the 6 runs, the fourth run gives an optimal design and the optimal values of other decision variables are summarized in Table 4.

Table 1. Given data for the fixed nodes

Node	Flow Req't. (M^3/hr)	Location
1	–800	(0, 6)
3	350	(3, 7)
5	100	(5, 2)
7	520	(8, 5)
9	–420	(10, 1)
10	250	(13, 4)

Table 2. Pipe sizes and costs

Diameter		Cost/M ($)
No.	Size (in)	
1	6	16
2	8	23
3	10	32
4	12	50
5	14	60
6	16	90
7	18	130
8	20	170

Table 3. Final junction locations with different initial values

Run No.	Location Initial	Location Final	Junction No.	Location Initial	Location Final	Run No.
1	(1, 3)	(0.2, 6.0)	2	(3, 6)	(0.2, 6.0)	4
	(5, 6)	(1.8, 5.6)	4	(5, 6)	(2.2, 5.4)	
	(7, 2)	(8.0, 5.0)	6	(8, 6)	(8.0, 5.0)	
	(10, 5)	(10.6, 2.8)	8	(10, 6)	(10.6, 2.8)	
	4136820	2265060	Total-cost ($)	2869700	2264730	
2	(1, 7)	(0.2, 6.0)	2	(2, 6)	(0.2, 6.0)	5
	(2, 3)	(3.2, 5.2)	4	(4, 3)	(3.2, 5.2)	
	(6, 3)	(8.0, 5.0)	6	(7, 3)	(8.0, 5.0)	
	(10, 4)	(10.6, 2.8)	8	(9, 3)	(10.6, 2.8)	
	3196990	2266310	Total-cost ($)	2852620	2266310	
3	(1, 1)	(0.2, 6.0)	2	(1, 1)	(0.2, 6.0)	6
	(6, 1)	(2.0, 5.6)	4	(6, 1)	(1.4, 5.6)	
	(8, 1)	(8.0, 5.0)	6	(8, 6)	(8.0, 5.0)	
	(9, 1)	(10.6, 2.8)	8	(10, 6)	(10.6, 2.8)	
	4227410	2265030	Total-cost ($)	4360470	2266380	

Table 4. Details of Optimal solutions

link (i-j)	Flow (M^3/hr)	Lenght (KM)	Diameter-Length (in) (KM)	Pump-head (M)
(1, 2)	800	0.20	20-0.20	source 1:
(2, 3)	350	2.97	12-2.56, 10-0.41	43.92
(2, 4)	450	2.09	16-2.09	
(4, 5)	100	4.40	10-0.81, 8-3.59	
(4, 6)	350	5.81	14-5.81	source 2:
(6, 7)	520	0.0	–	31.01
(8, 6)	170	3.41	12-1.07, 10-2.34	
(9, 8)	420	1.90	14-1.90	
(8, 10)	250	2.68	14-2.68	

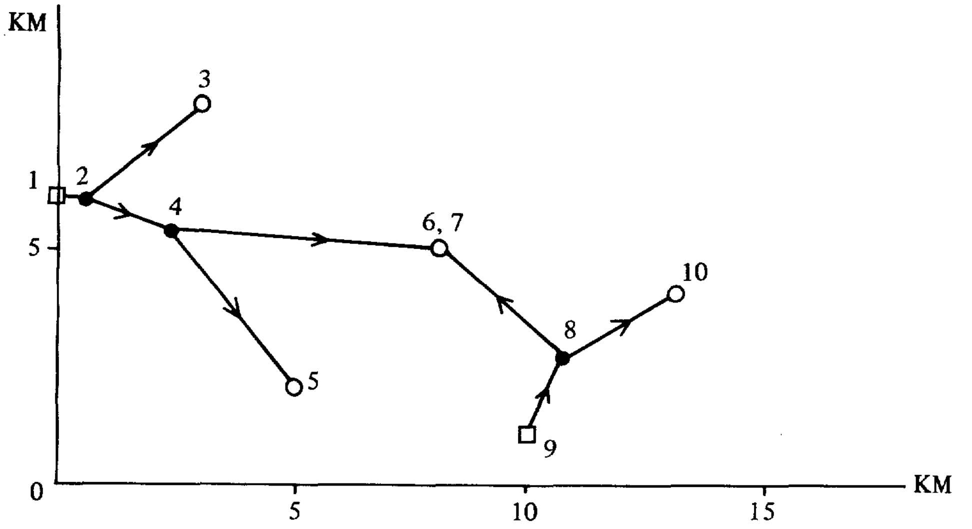

Figure 6. An Optimal Network Configuration

Computations for each run took less than 3 sec. on a Cyber 835 and we think a more complex system could be designed with reasonable computing time.

6. Summary and Conclusions

A NLP model is formulated for the least cost design of a pipe network system. Compared with the previous studies on this problem, this model includes the junction location as a decision variable with treating the pipe diameters as discrese variables. A solution procedure utilizing a gradient search technique is developed which can be easily applied to distribution systems as well as gathering systems for oil and gas. An example is solved for a simple water distribution system and the results show that the final costs of several runs having different initial values are almost same.

However, it should be noted that the optimizing procedure presented is suitable only for branched networks. For the case of looped systems, futher studies are recommended.

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