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STATISTICS RELATED TO DE LURY-RASTOGI TYPE DISTRIBUTION

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1. Suppose $(X_i,\ Y_i)$, $i=1, 2, \cdots, N$ is a random sample of size N from a bivariate normal population with the covariance matrix

$$\sigma^2\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix},$$

and finite means EX, EY. Suppose we wish to test the hypothesis that $\rho=\rho_0$ ($\rho_0\neq 0$ and $|\rho_0|<1$).

This paper purports to show that, for the purpose of testing the indicated hypothesis, several useful statistics can be derived from De Lury-Rastogi type distribution [6].
Then the paper indicates how the derived result can be applied. The statistics are derived as in the following.

Let the transformation be $U_i=X_i+Y_i$ and $V_i=X_i-Y_i$ ($i=1. 2, \cdots, N$). Let $\bar{X}=(1/N)\Sigma X_i$ and $\bar{Y}=(1/N)\Sigma Y_i$. Then $\bar{U}=\bar{X}+\bar{Y}$ and $\bar{V}=\bar{X}-\bar{Y}$. Under the indicated transformation, U and V are bivariate normal with the covariance matrix

$$2\sigma^2\begin{pmatrix}1+\rho & 0\\ 0 & 1-\rho\end{pmatrix}.$$

In particular, U and V are independent because Cov $(U,\ V)=0$. Therefore, if we form

$$Q_U=\frac{1}{2\sigma^2(1+\rho)}\sum_{i=1}^{N}(U_i-\bar{U})^2$$

and

$$Q_V=\frac{1}{2\sigma^2(1-\rho)}\sum_{i=1}^{N}(V_i-\bar{V})^2,$$

then the statistics Q_U and Q_V are both chi-square variates with $N-1$ degrees of freedom. Further define two statistics,

$$F=\frac{Q_U/(N-1)}{Q_V/(N-1)}=\frac{Q_U}{Q_V}=\frac{(1-\rho)\Sigma(U_i-\bar{U})^2}{(1+\rho)\Sigma(V_i-\bar{V})^2}$$

and

$$F^*=F^{-1}=\frac{(1+\rho)\Sigma(V_i-\bar{V})^2}{(1-\rho)\Sigma(U_i-\bar{U})^2}.$$

Then both F and F^* are F variates with $(N-1,\ N-1)$ degrees of freedom, and with the following probability densities:

$$g(F)=\frac{\Gamma(N-1)}{\left\{\Gamma\left(\frac{N-1}{2}\right)\right\}^2}F^{(N-3)/2}(1+F)^{-(N-1)}$$

$$g(F^*)=\frac{\Gamma(N-1)}{\left\{\Gamma\left(\frac{N-1}{2}\right)\right\}^2}F^{*(N-3)/2}(1+F^*)^{-(N-1)}$$

$$0<F,\ F^*<\infty.$$

2. The derived statistics may be applied in the following manner. Suppose we wish to test the null hypothesis that $\rho=\rho_0$ against the alternative hypothesis that $\rho\neq\rho_0$ where $\rho_0\neq 0$. Let α be the level of significance and assume that σ^2 is unknown. We may form the statistic

$$F=\frac{(1-\rho_0)\Sigma(U_i-\bar{U})^2}{(1+\rho_0)\Sigma(V_i-\bar{V})^2},$$

and then apply the two-tailed test with the decision rule to reject the null hypothesis if $F>F_{\alpha/2}(N-1,\ N-1)$ or if $F<1/F_{\alpha/2}(N-1,\ N-1)$. In an analogous manner we may use the test statistic

$$F^*=\frac{(1+\rho_0)\Sigma(V_i-\bar{V})^2}{(1-\rho_0)\Sigma(U_i-\bar{U})^2}.$$

Now assume that σ is known. As a test statistic, we may use either

$$Q_U=\frac{1}{2\sigma^2(1+\rho_0)}\Sigma(U_i-\bar{U})^2$$

or

$$Q_V=\frac{1}{2\sigma^2(1-\rho_0)}\Sigma(V_i-\bar{V})^2,$$

where the computation is now simpler than forming F or F^*.

If the hypothesis is of the form that $\rho=0$ against $\rho\neq 0$, there are several alternative test statistics applicable, including the one developed by the author [3, 4, 5]

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