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GENERALIZED METRIC SPACES DEFINED BY SEPARATION PROPERTIES

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1. Introduction

Many of the generalized metric spaces can be characterized by "separation properties". We shall unify and reformulate some known spaces by use of separation properties of *COC-maps*. The major results are visualized by a table at the end of this section. Also we shall define two classes of new spaces, called γ^*-spaces and $k\gamma^*$-spaces, which are in between semistratifiable spaces and stratifiable spaces. We investigate properties that are enjoyed by these classes of spaces. Important results are: (1) a k-semistratifiable space is a γ^*-space and (2) there exists a γ^*-space which is not k-semistratifiable.

Let X be a space. A function g from $N\times X$ (N=the positive integers) to the topology of X such that

$$x\in g(n,x)$$
$$g(n+1,x)\subset g(n,x)$$

for every $(n,x)\in N\times X$, is called a COC-map (=*countable open covering map*) [8]. Note that if we let $G_n=\{g(n,x):x\in X\}$, then $G_1, G_2, G_3 \ldots$ is a sequence of open covers of X such that G_{n+1} refines G_n. For any subset S of X, we define

$$g(n,S)=U\{g(n,x):x\in S\}$$
$$g^2(n,S)=g(n,g(n,S)).$$

As usual, S^- denotes the closure of S in X. Throughout this paper, all spaces will be T_1-spaces.

Let $\mathcal{A},\mathcal{B}$ be families of subsets of X. Consider the following separation properties on a COC-map g.

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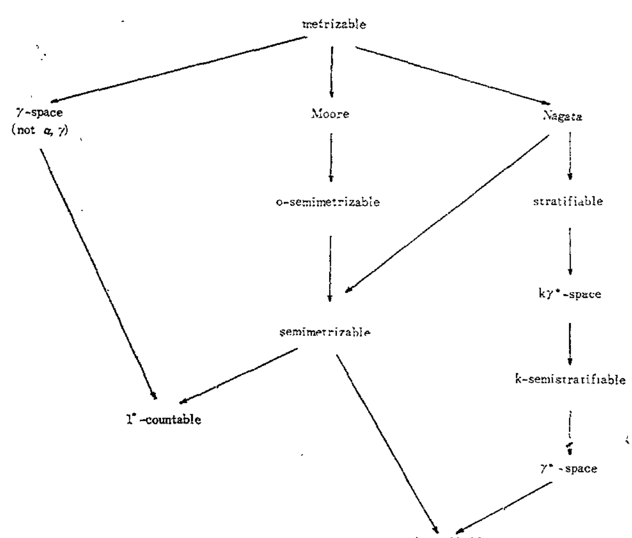
⟨**fig. 1**⟩ A regular space is $\square$ if and only if it has a COC-map which separates $\mathscr{B}$ from $\mathscr{A}$ (doubly, regularly, disjointly, starly).

	$\mathscr{B}$	$\mathscr{A}$	—	doubly	regularly	disjointly	starly
I	points	closed	1°-countable	γ-space	1°-countable	Nagata	Moore
I′	compacta	closed	γ-space	γ-space	γ-space	metrizable	metrizable
II	closed	points	semistratifiable	γ^*-space	stratifiable	Nagata	Moore
II′	closed	compacta	k-semistratifiable	$k\gamma^*$-space	stratifiable	metrizable	metrizable
I+II			semimetrizable		Nagata	Nagata	Moore
$g=g^*$ and I			o-semimetrizable	metrizable		metrizable	metrizable
$g=g^*$ and II			o-semimetrizable	metrizable	metrizable	metrizable	metrizable
$g=g^*$ and I′ or $g=g^*$ and II′			metrizable				

⟨**fig. 2**⟩



DEFINITION. For each $A \in \mathcal{A}$, $B \in \mathcal{B}$, if there exists $n \in N$ such that

(1) $A \cap g(n, B) = \phi$, then g *separates* $\mathcal{B}$ from $\mathcal{A}$
(2) $A \cap g^2(n, B) = \phi$, then g *separates doubly* $\mathcal{B}$ from $\mathcal{A}$
(3) $A \cap g(n, B)^- = \phi$, then g *separates regularly* $\mathcal{B}$ from $\mathcal{A}$
(4) $g(n, A) \cap g(n, B) = \phi$, then g *separates disjointly* $\mathcal{B}$ from $\mathcal{A}$
(5) $A \cap st(B, G_n) = \phi$, then g *separates starly* $\mathcal{B}$ from $\mathcal{A}$.

Our main results can be tabulated as fig. 1:
Interrelations between various spaces can be depicted as fig. 2:

2. Separating points (or compacta) from closed sets

In this section we consider the spaces with a COC-map g which separates points (or compacta) from closed sets.

A space is a *γ-space* [7] if it has a COC-map g satisfying: if $x_n \in g(n, y_n)$ and $y_n \in g(n, x)$ for each $n \in N$, then x is a cluster point of $\{x_n\}$. It is also called a *co-Nagata space* [9], or a *co-convergent space* [11].

Let P be a collecion of ordered pairs of subsets of a T_1-space X such that, for each $p = (p_1, p_2) \in P$, p_1 is open and $p_1 \subset p_2$, and such that, for every $x \in X$ and every neighborhood U of x, there is a $p \in P$ for which $x \in p_1 \subset p_2 \subset U$. Then P is called a *pair base* for X. Moreover P is called *cushioned* if, for every $Q \subset P$,

$$[\cup \{p_1 : p \in Q\}]^- \subset \cup \{p_2 : p \in Q\}$$

and P is *σ-cushioned* if it is a union of countably many cushioned collections. An *M_3-space* [2] is a T_1-space with a σ-cushioned pair base.

A T_1-space X is called a *stratifiable space* [1] if, to each open $U \subset X$, one can assign a sequence $\{U_n | n \in N\}$ of open subsets of X such that

(a) $U_n^- \subset U$
(b) $\cup U_n = U$
(c) $U_n \subset V_n$ whenever $U \subset V$.

It is well-known that M_3-spaces are exactly the stratifiable spaces [1].

A first countable stratifiable space is called a *Nagata space.* There is another characterization due to Hodel. A T_0-space is Nagata if and only if it has a COC–map g satisfying: if $g(n, x) \cap g(n, x_n) \neq \phi$ for $n \in N$, then x is a cluster point of $\{x_n\}$.

A space is *developable* if there exists a sequence of open covers $\{U_n : n \in N\}$ such that $\{st(x, U_n) : n \in N\}$ is a local base at x. A regular developable space is a *Moore space*. A developable space can be charac terized by having a COC–map g satisfying: if $x, x_n \in g(n, y_n)$ for $n \in N$, then x is a cluster point of $\{x_n\}$.

Now we characterize the spaces mentioned above via separation properties on a COC–map.

THEOREM 2.1. *A space is*

(1) *first countable if and only if it has a COC–map which separates points from closed sets.*

(2) *a γ–space if and only if it has a COC–map which separates doubly points from closed sets.*

(3) *a first countable regular space if and only if it has a COC–map which separates regularly points from closed sets.*

(4) *a Nagata space if and only if it has a COC–map which separates disjointly points from closed sets.*

(5) *a developable space if and only if it has a COC–map which separates starly points from closed sets.*

Proof. (1) It is clear.

(2) See [9; Theorem 2.1].

(3) Suppose that X is first countable and regular. Each $x \in X$ has a local base. We denote it by $\{g(n, x) \mid n \in N\}$. We may assume that g is a COC–map. Let F be a closed set and $x \notin F$. Since X is regular, there is a neighborhood U of x such that $U^- \cap F = \phi$. Choose $n \in N$ so that $g(n, x) \subset U$. Then $g(n, x)^- \cap F = \phi$. Therefore, g separates regularly x from F. The converse is clear.

(4) Suppose that X is Nagata. We use Hodel's characterization of Nagata spaces. Let g be a Nagata function for X. We claim that g separates disjointly points from closed sets. Suppose the contrary. Then there exist a closed set F and a point $x \notin F$ such that $g(n, x) \cap g(n, F) \neq \phi$ for every n. This means that there exists $x_n \in F$ so that $g(n, x) \cap g(n, x_n) \neq \phi$ for each n. Since g is a Nagata function, x is a cluster point of $\{x_n\}$. As before, $\{x_n\}$ is a sequence in the closed set F clustering to x. However, $x \notin F$, which is a contradiction.

For the converse suppose that $g(n,x)\cap g(n,x_n)\neq\phi$ for every n and that x is not a cluster point of $\{x_n\}$ · If we take $F=\{x_n\}^-$, then $x\notin F$. This is contradictory to the assumption that there exists n_0 such that $g(n_0,x)\cap g(n_0,F)=\phi$, because $g(n,x_n)\subset g(n,F)$ for every n. Therefore x must be a cluster point of $\{x_n\}$.

(5) Let g be a COC-map and F be a closed set not containg x. Then $st(x,G_n)\cap F\neq\phi$, where $G_n=\{g(n,x):x\in X\}$, if and only if there exist x_n and y_n such that $x_n, x\in g(n,y_n)$. By the same argument as employed in (4) above, we are done.

COROLLARY 2.2. *A space is metrizable if and only if it has a COC-map separating disjointly and doubly (or disjointly and starly) points from closed sets.*

Proof. Let X be a metric space. Take $g(n,x)=S\left(x,\frac{1}{n}\right)$, open $\frac{1}{n}$-ball centered at x. Then g is a desired COC-map. For the converse, note that a Nagata space is metrizable if it is a γ-space or a developable space. See [7; (6.1)].

THEOREM 2.3. *For a regular space X, the following are equivalent:*

(1) *X is a γ-space,*

(2) *X has a COC-map which separates compacta from closed sets,*

(3) *X has a COC-map which separates doubly compacta from closed sets, and*

(4) *X has a COC-map which separates regularly compacta from closed sets.*

Proof. We will prove only the equivalence of (2) and (4). For the rest, see [9; Theorem 2.1].

(2⇔4) As X is regular, disjoint compact set K and a closed set F can be separated by open sets, say $U\supset K$, $V\supset F$ with $U\cap V=\phi$. Let $n\in N$ be such that $g(n,K)\subset U$. Then $g(n,K)^-\subset U^-\subset V^c$. Thus $g(n,K)^-\cap F=\phi$. The converse is trivial.

THEOREM 2.4. *For a regular space X, the following are equivalent;*

(1) *X is metrizable,*

(2) *X has a COC-map which separates disjointly compacta from closed sets, and*

(3) *X has a COC-map which separates starly compacta from closed*

sets.

Proof. (1⇒2) and (1⇒3). Let X be a metric space. Take $g(n, x)=S\left(x;\frac{1}{n}\right)$, open $\frac{1}{n}$-ball centered at x. In a metric space, for any disjoint compact K and closed F, $d(K, F)>0$. Therefore, g satisfies (2) and (3).

(2⇒1) Such a g satisfies the condition in Theorem 2.1 (4) and Theorem 2.3 (2). Therefore the space is Nagata and γ-space, which is metrizable. See [7; (6.1)].

(3⇒1) Such a g separates starly points from closed sets, so that X is developable. Also g separates starly closed sets from compacta so that X is k-semistratifiable (See Theorem 3.4 (1)). By [10; Theorem 3.2], a first countable k-semistratifiable space is stratifiable. Therefore X is a developable stratifiable space, which is metrizable.

Q. E. D.

REMARK. Let g be a COC-map. If g^2 separates $\mathcal{A}$ from closed sets, then g^k separates $\mathcal{A}$ from closed sets for all $k\geqslant 1$.

3. Separation of closed sets from points (or compacta)

In this section we study spaces which have a COC-map which separates closed sets from points (or compacta). First we need some definitions.

A space X is *semistratifiable* [3] if, to each open set $U\subset X$, one can assign a sequence $\{U_n : n\in N\}$ of closed subsets of X such that

(a) $\bigcup_{n\in N} U_n=U,$

(b) $U_n\subset V_n$ if $U\subset V.$

A correspondence $U\rightarrow\{U_n : n\in N\}$ is a semistratification for the space X whenever it satisfies the conditions (a) and (b). A semistratifiable space can also be characterized as a space having a COC-map g satisfying: if $x\in g(n, x_n)$ for every $n\in N$, then x is a cluster point of $\{x_n\}$.

A space X is *k-semistratifiable* [10] if it has a semistratification $U\rightarrow\{U_n : n\in N\}$ with the property that whenever $K\subset U$ with K compact and U open in X, there is an $n\in N$ with $K\subset U_n$. Clearly, stratifiable ⇒k-semistratifiable⇒semistratifiable. It is known that a first countable

k-semistratifiable space is stratifiable (and hence Nagata).

DEFINITION 3.1. A space is a *γ^*-space* if it has a COC-map satisfying: if $x \in g(n, y_n)$ and $y_n \in g(n, x_n)$ for each n, then x is a cluster point of $\{x_n\}$.

THEOREM 3.2. *A space is* (1) *semistratifiable if and only if it has a COC-map which separates closed sets from points.*

(2) *γ^*-space if and only if it has a COC-map which separates doubly closed sets from points,*

(3) *stratifiable if and only if it has a COC-map which separates regularly closed sets from points,*

(4) *Nagata if and only if it has a COC-map which separates disjointly closed sets from points,*

(5) *developable if and only if it has a COC-map which separates starly closed sets from points.*

Proof. (1) The proof is evident.

(2) Let g be a COC-map satisfying the condition in (3.1). Let F be a closed set not containing x. Suppose $x \in g^2(n, F)$ for every $n \in N$. Then there are $x_n \in F$ and $y_n \in X$ so that $x \in g(n, y_n)$ and $y_n \in g(n, x_n)$. By hypothesis, x is a cluster point of $\{x_n\}$. This contradicts $x \notin F$. Conversely, let g separate doubly closed sets from points and let $x \in g(n, y_n)$ and $y_n \in g(n, x_n)$. If x is not a cluster point of $\{x_n\}$, $F = \{x_n\}^-$ is a closed set not containing x. Since g separates doubly closed sets from points, there must exist $k \in N$ so that $x \notin g^2(k, F)$. However $x \in g(k, y_k)$ and $y_k \in g(k, x_k)$ with $x_k \in F$, which is a contradiction.

(3) See [8].

(4) A COC-map separates disjointly closed sets from points if and only if it separates disjointly points from closed sets. Now apply Theorem 2.1 (4).

(5) This is a trivial result of Theorem 2.1 (5), which can be obtained by the same argument as employed above. Namely, $st(A, G_n) \cap B \neq \phi$ if and only if $A \cap st(B, G_n) \neq \phi$ for any $A, B \subset X$. Take A = a closed set and B = a point.

COROLLARY 3.3. *For a space, Nagata $\Rightarrow$ stratifiable $\Rightarrow$ γ^*-space $\Rightarrow$ semistratifiable.*

We now turn our attention to the separation properties of closed sets from compacta. As in the case of Theorem 2.1 vs Theorems 2.3 and 2.4, the conditions with compacta are stronger than those with points.

THEOREM 3.4. *A space is* (1) *k-semistratifiable if and only if it has a COC-map which separates closed sets from compacta,*

(2) *stratifiable if and only if it has a COC-map which separates regularly closed sets from compacta,*

(3) *metrizable if and only if it has a COC-map which separates disjointly* (*or starly*) *closed sets from compacta.*

Proof. (1) It is clear. See [8].

(2) Let X be stratifiable. There exists a COC-map g separating closed sets from points regularly by Theorem 3.2 (3). We claim that g separates regularly closed sets from compacta. Let $F\cap K=\phi$ with F closed and K compact. For each $x\in K$, there exists $n_x\in N$ so that $g(n_x, F)^-\not\ni x$. In other words, $g(n_x, F)^{co}=g(n_x, F)^{-c}$ is an open neighborhood of x. Since K is compact, $\{g(n_x, F)^{-c} : x\in K\}$ has a finite subcover, say $\{g(n_i, F)^{-c} : x_i\in K,\ 1\leqslant i\leqslant \gamma\}$. Here $n_i=n_{x_i}$. Let $k=\max\{n_i : 1\leqslant i\leqslant \gamma\}$. Then $g(k, F)^-\cap K=\phi$ which we desired. The converse is trivial by Theorem 3.2 (3).

(3) If a COC-map separates disjointly (or starly) closed sets from compacta, it does so compacta from closed sets. Now we can apply Theorem 2.4.

4. γ^*-spaces and $k\gamma^*$-spaces

We have defined γ^*-spaces in section 3. Now we study such spaces and shall find that they are very similar to semistratifiable ones.

THEOREM 4.1. *The countable product of γ^*-spaces is a γ^*-space.*

Proof. For each i, let X_i be a γ^*-space with a COC-map g_i separating closed sets from points doubly. (See Theorem 3.2.) Let $X=\prod X_i$ be the product space, and let $\pi_i : X\to X_i$ be the projection. For each i, n and $x\in X$, let $h_i(n, x)=g_i(n, \pi_i(x))$ if $i\leqslant j$, and X_i if $i>j$. Now let $g(n, x)=\prod_{i=1}^{\infty} h_i(n, x)$ for each $(n, x)\in N\times X$. That is,

$$g(n, x)=g_1(n, x_1)\times g_2(n, x_2)\times\cdots g_n(n, x_n)\times X_{n+1}\times X_{n+2}\times\cdots$$

where $x=(x_1, x_2, \ldots)$.

Clearly each $g(n, x)$ is open, $x \in g(n, x)$ and $g(n+1, x) \subset g(n, x)$ for each $(n, x) \in N \times X$. We claim that g separates closed sets from points doubly. Let $U = \bigcap_{i=1}^{k} \prod_i^{-1} U_i$ be a basic open set, where $U_i \subset X_i$ is open. Then

$$F = U^C = \bigcup_{i=1}^{k} \prod_i^{-1}(U_i^c).$$

Therefore, it is enough to prove that: For $x \notin \prod_i^{-1}(U_i^c)$. there exists $n_i \in N$ such that $x \notin g^2(n_i, \prod_i^{-1}(U_i^c))$. Note that $\prod_i^{-1}(U_i^c) = X_1 \times X_2 \times X_3 \times \cdots \times X_{i-1} \times U_i^c \times X_{i+1} \times \cdots$, and $x_i \notin U_i^c$. Since g_i separates U_i^c from x_i doubly by the hypothesis, there exists $n_i \in N$ such that $x_i \notin g_1^2(n_i, U_i^c)$. Now $\prod_i^{-1}(g_i^2(n_i, U_i^c)) = g^2(n_i, \prod_i^{-1}(U_i^c))$ if $i \leqslant n_i$ (we can take n_i as big as we please). Therefore $x \notin g^2(n_i, \prod_i^{-1}(U_i^c))$.

THEOREM 4.2. *A subspace of γ^*-space is a γ^*-space.*

Proof. Let g be a COC-map on X separating doubly closed sets from points. Let Y be a subspace. Then the restriction h of g on $N \times Y$,

$$h(n, x) = g(n, x) \cap Y$$

is a desired COC-map.

THEOREM 4.3. *The union of two closed (in the union) γ^*-spaces is a γ^*-space.*

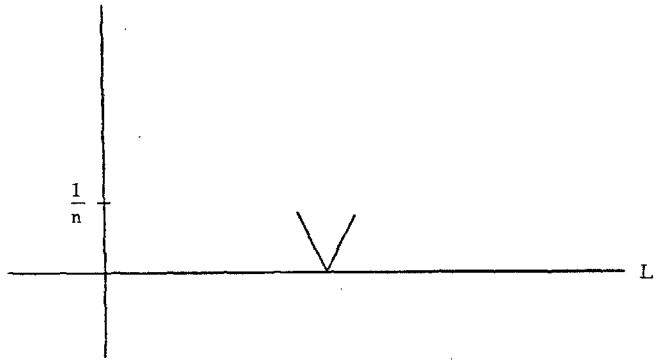
THEOREM 4.4. *A k-semistratifiable space is a γ^*-space.*

Proof. Let g be a COC-map on X separating closed sets from compacta. Suppose that there exists $x \notin F$ with F closed and $x \in g^2(n, F)$ for every n. Then we have sequences $\{x_n\}$, $\{y_n\}$ such that

$$x \in g(n, y_n),\ y_n \in g(n, x_n),\ x_n \in F.$$

Since $\{y_n\}$ converges to x, $K = \{x, y_1, y_2, \ldots\}$ is compact. We may assume that $K \cap F = \phi$. As g separates closed sets from compacta, there exists $k \in N$ such that $g(k, F) \cap k = \phi$. However, $y_k \in g(k, x_k) \cap K$, which is a contradiction. Thus we have shown that g separates doubly closed sets from points.

EXAMPLE 4.5. The converse is not true. There exists a γ^*-space which is not k-semistratifiable. Let X be the upper half plane including the real axis L. We let each point of $X-L$ be open and take as a neighborhood basis of points $x \in L$ a V-vertex at x, sides of slopes ± 1 and height $\frac{1}{n}$.



We define a COC-map by

$$g(n, x) = \begin{cases} \{x\}, & \text{if } x \in X-1 \\ \text{the } V\text{-vertex at } x \text{ of height } \frac{1}{n}, & \text{if } x \in L \end{cases}$$

Clearly $g^2 = g$. Since g separates closed sets from points, so does g^2. Thus X is a γ^*-space. It is known that X is a Moore space (see [5]), and hence first countable. If X is k-semistratifiable, it would be stratifiable by [[10], Theorem 3.2] and hence it would be paracompact. However X is not even normal. Consider the two closed sets, rationals and irrationals in L.

THEOREM 4.6. *If a space is a γ^*-space and a γ-space then it is a semistratifiable γ-space. A semistratifiable γ-space is developable.*

Proof. The first implication is obvious by definition. For the second, see [[6]; Proposition 4.2, and [5]].

DEFINITION 4.7. A space is *$k\gamma^*$-space* if it has a COC-map which separates doubly closed sets from compacta.

THEOREM 4.8. *For a space X, stratifiable $\Rightarrow$ $k\gamma^*$-space $\Rightarrow$ k-semistratifiable. If X is first countable, all these three conditions are equivalent (and X is Nagata.)*

Proof. Note that if a COC-map g separates regularly closed sets from $\mathcal{A}$ (any family), then it separates doubly closed sets from $\mathcal{A}$. (Take $\mathcal{A}$=compacta.) For the second statement, see [[10], Theorem 3.2].

5. Symmetric COC-maps

Since symmetry properties of COC-maps may give rise to stronger results, we take up this subject in this section.

DEFINITION 5.1. A COC-map g on X is said to be *symmetric* if $g=g^*$, where $g^*(n,x)$ is defined by $x\in g^*(n,y)$ if and only if $y\in g(n,x)$. That is, $x\in g(n,y)$ if and only if $y\in g(n,x)$ for every $(n,x)\in N\times X$.

THEOREM 5.2. *A regular space is* (1) *σ-semimetrizable if and only if it has a symmetric COC-map separating points from closed sets,* (2) *metrizable if and only if it has a symmetric COC-map separating doubly (disjointly, or starly) points from closed sets.*

Proof. (1) A COC-map separates points from closed sets if and only if $x_n\in g(n,x)$ for each n implies that the sequence $\{x_n\}$ converges to x. Now, by [[4], Theorem 2.1], we are done.

(2) If X is a metric space, we take

$$g(n,x)=S\left(x;\frac{1}{n}\right)$$

the open $\frac{1}{n}$-ball centered at x. Then g is a symmetric COC-map separating points from closed sets doubly (regularly, disjointly or starly).

For the converse, let g be a symmetric COC-map. If g separates doubly points from closed sets, then g is a γ-function so that $x_n\in g(n,y_n)$ and $y_n\in g(n,x)$ imply $\{x_n\}$ converges to x. Now suppose that $y_n\in g(n,x)\cap g(n,x_n)$. Then, by symmetry, $x_n\in g(n,y_n)$. Therefore $\{x_n\}$ converges to x. However, this is exactly a characterization of a Nagata space. Since a Nagata γ-space is metrizable, we are done.

If g separates disjointly points from closed sets, then g is a Nagata COC-map. That is, if $g(n,x)\cap g(n,x_n)\neq\phi$, then $\{x_n\}$ converges to x. By symmetry, g is a COC-map giving rise to a developable space. Since a developable Nagata space is metrizable, X is metrizable.

If g separates starly points from closed sets, g is a COC–map giving rise to a Nagata function as can be seen by the argument above. Therefore, X is again a developable Nagata space, and hence is metrizable.

QUESTION 5.3. Suppose that X has a symmetric COC–map separating regularly points from closed sets. Is X metrizable? It can be shown that such a space is a developable γ–space.

THEOREM 5.4. *A regular space is metrizable if and only if it has a symmetric COC–map separating doubly (regularly, disjointly, or starly) closed sets from points.*

Proof. Suppose that g separates doubly closed sets from points. Then g separates disjointly closed sets from points since g is symmetric. Thus X is a Nagata γ–space, and hence is metrizable.

Let g separate regularly closed sets from points. Let $x \notin F$ with F closed. Then $x \notin g(k, F)^-$ for some k. Since $\{g(n, x) : n \in N\}$ is a local base at x, $g(m, x) \cap g(m, F) = \phi$ for some m. Thus such a g separates disjointly closed sets from points.

If a symmetric COC–map g separates disjointly (or starly) closed sets from points, it separates disjointly (or starly) points from closed sets. Now by Theorem 5.2 (2), X is metrizable.

REMARK 5.5. Let g be a COC–map. If g is symmetric and separates points from closed sets, then it separates closed sets from points. However, the converse is not true. That is, even though X has a COC–map g_1 separating points from closed sets and a COC–map g_2 separating closed sets from points, it is not true in general that X has a COC–map, separating points from closed sets, which is symmetric. For example, consider a semimetric space which is not o–semimetrizable.

Similarly, a space which is γ and γ^* is not necessarily metrizable. For example, the space described in Example 4.5 is γ and γ^*, because the COC–map defined there separates doubly points from closed sets, and closed sets from points. There does not exist a symmetric COC–map separating doubly points from closed sets.

THEOREM 5.6. *A regular space is metrizable if and only if it has a symmetric COC–map g separating compact sets from closed sets.*

Proof. Such a space is a γ-space by Theorem 2.3. Since g is symmetric, g separates closed sets from compacta, and hence X is k-semistratifiable. By [10], a first countable k-semistratifiable space is stratifiable. Therefore, X is a Nagata γ-space, so it is metrizable.

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