部分 等分布 剪斷荷重을 받는 異方性 構造體의 解析

Analysis of Orthotropic Body Under Partial-Uniform Shear Load

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要 旨

本論文은 材料의 性質이 直交하는 方向으로 相異한 異方性 構造體에 部分等分布 剪斷荷重이 境界에 作用할 경우의 垂直應力과 剪斷應力을 나타내는 엄밀解法을 提示하였다.

이 解法은 平衡條件과 適合條件을 동시에 만족하는 彈性論的인 엄밀 解法이다. 따라서 이러한 問題를 解析하기 위하여 Airy 應力函數를 利用하였다.

本解法의 妥當性을 證明하기 위하여 異方性인 경우의 方程式들의 異方性常數들을 等方性인 경우의 常數들로 代置한 경우에 等方性인 경우의 方程式들로 變換되지 않으면 안된다. 이를 檢討하기 위하여 L'hospital의 法則을 이용하였다. 그 結果 異方性인 경우의 모든 方程式들은 等方性인 경우의 方程式들로 정확히 變換되었고 이 식들은 이미 연구된 資料의 값들과 比較된 結果 정확히 一致되었다.

또한 集中荷重의 경우와의 關係에서는 部分等分布荷重의 特別한 경우가 集中荷重임을 고려하고 L'hospital의 法則을 利用하면 部分等分布荷重의 경우의 方程式들은 바로 集中荷重의 경우의 方程式들로 變換됨을 알 수 있다.

본 結果로 미루어 보아 解法의 妥當性이 立證되었다고 할 수 있다.

本解法의 方程式들은 簡單한 形態로 構成되어 있어 數値結果를 정확히 누구나 얻을 수 있는 장점이 있다.

應力의 값을 나타내는 數値結果를 異方性材料인 3단合板과 중첩합판을 예로 들어 나무결을 2가지 方向으로 强軸을 바꾸어 각각의 垂直 및 剪斷應力을 求하여 圖表로 表示하였으며, 그 결과 應力의 分布는 材料의 성질과 强軸의 方向에 따라 현저하게 달라지는 현상을 볼 수 있다.

Abstract

This dissertation presents an exact solution for the shearing and normal stresses of an orthotropic plane body loaded by a partial-uniform shear load. The solution satisfies the equilibrium and compatibility equations concurrently. An Airy stress function is introduced to solve the problem related to an orthotropic half-infinite plane under a partial-uniform shear load.

All the equations for orthotropy must be degenerated into the expressions for isotropy when orthotropic constants are replaced by isotropic ones. The author has evaluated all the equations of orthotropy and succeeded in obtaining exactly identical expressions to the equations of

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isotropy which were derived independently by means of L'hospital's rule.

The analytical results of isotropy are compared with the simple results of other investigator. Since a concentrated shear load is a particular case of partial-uniform shear load, all the equations of partial-uniform shear load case are degenerated into the expressions for concentrated load case of isotropy and orthotropy.

The formal solution is expressed in terms of closed form. The numerical results for orthotropy are evaluated for two kinds and two different orientations of the grain of wood. The type of wood considered are three-layered plywood and laminated delta wood. The distribution of normal and shearing stresses are shown in figures. It is noted that the distribution of stresses of orthotropic materials dependson the type of materials and orientations of the grain.

1. Introduction

The investigation of the problem of a partial uniform shear load applied to the orthotropic half-infinite plane is a great practical interest. Many theoretical paper, in the past, have been written on the subject of orthotropy. An introduction to be elasticity of anisotropic materials was covered in two classical books; one by Love, A.E.H., (16) and the other by Green, A.E., and Zerna, W., (8). Lekhnitskii, S.G. (15) discussed in detail the generalized plane problems related to anisotropy. Silverman, I.K., (19) presented, in 1964, a closed form solution for an orthotropic beam subjected to arbitrary normal and shear loads which can be described by polynomials. More recently, Hashin, Z., (9) developed a general method to solve Silverman's problem relevant to any anisotropic beam since wood is assumed to be orthotropic, in 1967, Hooley, R.F., and Hibbert, P.D., (10) investigated the stress concentrations in the vicinity of external loads applied to timber beam. They assumed the external loads were spread over a finite area and utilized a finite element technique to obtain numerical values for Douglas fir. Since then, many technical paper are written including some relating to the problem of concentrated and a partial uniform load applied to orthotropic materials (20, 24, 25, 26).

This paper presents an exact solution for the stresses of isotropic and orthotropic half-infinite plane loaded by an uniform shear load, with a definite width. Fourier integral is introduced to solve the problem. and the solution satisfies the equilibrium and compatibility equations. The equations of orthotropy are degenerated into the expressions for isotropy when orthotropic constants are replaced by isotropic ones. Also, the equaitions of a partial uniform load case can be degenerated into the expressions for concentrated load case since the sum of uniform shear loads becomes a single concentrated shear load.

The numerical results of isotropic case agree quite closely with simple results of other investigators. The numerical results for orthotropy are evaluated for two kinds of wood and two different orientations of the grain. The type of wood considered are three-layered plywood and laminated delta wood. The formal solutions are expressed in terms of closed form. The numerical results are shown in figures.

2. Analysis of Orthotropic Half-Infinite Plane.

2-1. Formulation of Governing Equations

Considering a unit thickness of a half-infinite isotropic plane as shown in Figure 1 the partial uniform shear load on $y=0$ can be expressed in terms of Fourier cosine integral of frequency β' as

$$f(x)=\frac{2q}{\pi}\int_0^\infty \frac{1}{\beta'}\sin(a\beta')\cos(x\beta')d\beta' \tag{1}$$

Where "*a*" represents a half of the loaded length. Then, the boundary conditions are represented by the Fourier integral as

1) at $y=0$; $\sigma_y=0$ (2a)

2) at $y=0$; $\tau_{xy}=\frac{2q}{\pi}\int_0^\infty \frac{1}{\beta'}\sin(a\beta')\cos(x\beta')d\beta'$ (2b)

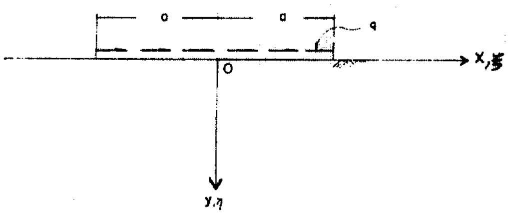

Fig. 1. Partial-Uniform Shear Load on an Orthotropic Half-Infinite Plane

With the axes of coordinates taken along the principal axes of orthotropy, the governing equation for the plane problem of orthotropy, which is equivalent to the biharmonic equation of the plane isotropy can be expressed as

$$\frac{\partial^4\varphi}{\partial x^4}+(D_1^2+D_2^2)\frac{\partial^4\varphi}{\partial x^2\partial y^2}+D_1^2D_2^2\frac{\partial^4\varphi}{\partial y^4}=0 \tag{3}$$

where $D_1^2+D_2^2=\frac{2C_{12}+C_{66}}{C_{66}}$

$$D_1^2D_2^2=\frac{C_{11}}{C_{22}} \tag{3a}$$

the elastic constants can be expressed in terms of the moduli as

$$C_{11}=\frac{1}{E_x},\qquad C_{22}=\frac{1}{E_y} \tag{4a}$$

$$C_{12}=\frac{-\nu_x}{E_y}=\frac{-\nu_y}{E_x},\quad C_{66}=\frac{1}{G} \tag{4b}$$

For the isotropic case, from Eqs. 3a, D_1 and D_2 become unity and Eq. 3 is reduced to the biharmonic equation

$$\frac{\partial^4\varphi}{\partial x^4}+2\frac{\partial^4\varphi}{\partial x^2\partial y^2}+\frac{\partial^4\varphi}{\partial y^4}=0 \tag{5}$$

The substitution of the assumed Airy stress function

$$\varphi=\int_0^\infty f(y)\sin(x\beta')d\beta'$$

into Eq. 3 yields

$$\int_0^\infty\Big[\beta'^4f(y)-\beta'^2f^{II}(y)(D_1^2+D_2^2)+f^{IV}(y)D_1^2D_2^2\Big]\sin(x\beta')d\beta'=0$$

which is reduce to

$$D_1^2D_2^2f^{IV}(y)-\beta'^2(D_1^2+D_2^2)f^{II}(y)+\beta'^4f(y)=0 \tag{6}$$

The solution of Eq. 6 gives

$$f(y)=E_0e^{my}+F_0e^{ny}+G_0e^{py}+H_0e^{qy} \tag{7a}$$

in which

$$m=-\frac{\beta'}{D_2}\ ;\ n=-\frac{\beta'}{D_1}$$

$$p=\frac{\beta'}{D_2}\ ;\ q=\frac{\beta'}{D_1} \tag{7b}$$

and E_0 and F_0 are orthotropic constants to be found from the boundary conditions at $y=0$, while the constants G_0 and H_0 are zero because φ should be bounded as $y\to\infty$ Therefore, φ becomes

$$\varphi=\int_0^\infty(E_0e^{my}+F_0e^{ny})\sin(x\beta')d\beta' \tag{8}$$

From Eq. 8 it follows that

$$\sigma_y=\frac{\partial^2\varphi}{\partial x^2}=-\int_0^\infty\beta'^2(E_0e^{my}+F_0e^{ny})\sin(x\beta')d\beta' \tag{9a}$$

$$\tau_{xy}=-\frac{\partial^2\varphi}{\partial x\partial y}=-\int_0^\infty\beta'(mE_0e^{my}+nF_0e^{ny})\cos(x\beta')d\beta' \tag{9b}$$

substituting Eq. 9a and 9b into the boundary conditions given by Eqs. 2, the following results.

$$\beta'^2(E_0+F_0)=0 \tag{9c}$$

$$\int_0^\infty\beta'(mE_0+nF_0)\cos(x\beta')d\beta'=-\frac{2q}{\pi}\int_0^\infty\frac{\sin(a\beta')}{\beta'}\cos(x\beta')d\beta' \tag{9d}$$

Solving the Eqs. 9c and 9d simultaneously, E_0 and F_0 are found to be

$$E_0=-\frac{2q}{\pi}\frac{\sin(a\beta')}{\beta'^2(m-n)} \tag{10a}$$

$$F_0=\frac{2q}{\pi}\frac{\sin(a\beta')}{\beta'^2(m-n)} \tag{10b}$$

substituting E_0 and F_0 in Eq. 3~8, φ becomes

$$\varphi=-\frac{2q}{\pi}\int_0^\infty \frac{\sin(a\beta')}{\left[-\frac{\beta'}{D_2}+\frac{\beta'}{D_1}\right]\beta'^2}\left[e^{-\frac{\beta'}{D_2}y}-e^{-\frac{\beta'}{D_1}y}\right]\sin(x\beta')\,d\beta' \tag{11}$$

To non-dimensionalize x and y, the following substitutions can be made.

$$\xi=\frac{x}{a}\ ;\ \eta=\frac{y}{a}\ ;\ \beta=a\beta'\ ;\ d\beta=ad\beta' \tag{12}$$

Finally, φ can be expressed in terms of the coordinates ξ and η

$$\varphi=\frac{2qa^2D_1D_2}{\pi(D_1-D_2)}\int_0^\infty\frac{\sin\beta}{\beta^3}\left[e^{-\frac{\beta}{D_2}\eta}-e^{-\frac{\beta}{D_1}\eta}\right]\sin(\xi\beta)\,d\beta \tag{13}$$

2-2. Stresses

It follows from Eq. 13 that

$$\sigma_x=\frac{2q}{\pi(D_1-D_2)}\int_0^\infty\frac{\sin\beta}{\beta}\left[\frac{D_1}{D_2}e^{-\frac{\beta}{D_2}\eta}-\frac{D_2}{D_1}e^{-\frac{\beta}{D_1}\eta}\right]\sin(\xi\beta)\,d\beta \tag{14a}$$

$$\sigma_y=-\frac{2qD_1D_2}{\pi(D_1-D_2)}\int_0^\infty\frac{\sin\beta}{\beta}\left[e^{-\frac{\beta}{D_2}\eta}-e^{-\frac{\beta}{D_1}\eta}\right]\sin(\xi\beta)\,d\beta \tag{14b}$$

$$\tau_{xy}=\frac{2q}{\pi(D_1-D_2)}\int_0^\infty\frac{\sin\beta}{\beta}\left[D_1e^{-\frac{\beta}{D_2}\eta}-D_2e^{-\frac{\beta}{D_1}\eta}\right]\cos(\xi\beta)\,d\beta \tag{14c}$$

Performing integrations, the stresses are given by the closed form as

$$\sigma_x=\frac{q}{2\pi(D_1-D_2)}\left[\frac{D_1}{D_2}\left\{\ln\left[\left(\frac{\eta}{D_2}\right)^2+(\xi+1)^2\right]-\ln\left[\left(\frac{\eta}{D_2}\right)^2+(\xi-1)^2\right]\right\}-\frac{D_2}{D_1}\left\{\ln\left[\left(\frac{\eta}{D_1}\right)^2+(\xi+1)^2\right]-\ln\left[\left(\frac{\eta}{D_1}\right)^2+(\xi-1)^2\right]\right\}\right] \tag{15a}$$

$$\sigma_y=\frac{q}{2\pi(D_1-D_2)}\left[\ln\left[\left(\frac{\eta}{D_2}\right)^2+(\xi-1)^2\right]-\ln\left[\left(\frac{\eta}{D_2}\right)^2+(\xi+1)^2\right]-\ln\left[\left(\frac{\eta}{D_1}\right)^2+(\xi-1)^2\right]+\ln\left[\left(\frac{\eta}{D_1}\right)^2+(\xi+1)^2\right]\right] \tag{15b}$$

$$\tau_{xy}=\frac{q}{\pi(D_1-D_2)}\left[D_1\left\{\tan^{-1}\frac{D_2(\xi+1)}{\eta}-\tan^{-1}\frac{D_2(\xi-1)}{\eta}\right\}-D_2\left\{\tan^{-1}\frac{D_1(\xi+1)}{\eta}-\tan^{-1}\frac{D_1(\xi-1)}{\eta}\right\}\right] \tag{15c}$$

2-3. Particular Case of Isotropy

The general case of orthotropy has been discussed in Section 2. Since isotropy is a particular case of orthotropy, all the equations presented in Section 2 must be degenerated into the expressions for isotropy when orthotropic constants are replaced by isotropic ones. In other words, when D_1 and D_2 become unity. the orthotropic equations should be reduced to isotropic expressions. The anthor has evaluated all the equations of orthotropy presented in this section $D_1=D_2=1$ and succeeded in obtaining exactly identical expressions to the equations of isotropy which were derived independently by means of L'hospital's rule. As shown in Eqs. 15, if D_1 and D_2 given by unity applying L'hospital's rule, the denominator and numerator of the integrand are separately differentiated once with respect to other D_1 or D_2 are substituting the unity into other D_1 or D_2.

when $D_1=1$ is substituted, the Eq. 15a becomes

$$\sigma_x\Big]_{D_1=1}=\frac{q}{2\pi D_2(1-D_2)}\left[\left\{\ln\left[\left(\frac{\eta}{D_2}\right)^2+(\xi+1)^2\right]-\ln\left[\left(\frac{\eta}{D_2}\right)^2+(\xi-1)^2\right]\right\}-D_2^2\{\ln[\eta^2+(\xi+1)^2]-\ln[\eta^2+(\xi-1)^2]\}\right] \tag{16}$$

The first differentiation of the numerator and denominator of the integrand gives a definite limiting value for the integrand. Thus, when L'hospital's rule is applied

$$[\text{Integrand } \sigma_x]_{D_1=1}=\frac{q}{2\pi(1-2D_2)}$$

$$\left[-\frac{2n^2\frac{1}{D_2^3}}{\left(\frac{\eta}{D_2}\right)^2+(\xi+1)^2}\right.$$

$$+\frac{2\eta^2\frac{1}{D_2^3}}{\left(\frac{\eta}{D_2}\right)^2+(\xi-1)^2}-2D_2$$

$$\left.\{\ln[\eta^2+(\xi+1)^2]-\ln[\eta^2+(\xi-1)^2]\}\right] \quad (17)$$

substituting the unity into D_2 in Eq. 17

$$[\text{Integrand }\sigma_x]_{D_1=D_2=1}=\frac{q}{\pi}\left[\left\{\frac{\eta^2}{\eta^2+(\xi+1)^2}\right.\right.$$

$$\left.-\frac{\eta^2}{\eta^2+(\xi-1)^2}\right\}+\{\ln[\eta^2+(\xi+1)^2]$$

$$\left.-\ln[\eta^2+(\xi-1)^2]\}\right] \quad (18)$$

Similary, the following limiting values are found to be

$$[\text{Integrand }\sigma_y]_{D_1=D_2=1}=-\frac{q}{\pi}\left[\frac{\eta^2}{\eta^2+(\xi+1)^2}\right.$$

$$\left.-\frac{\eta^2}{\eta^2+(\xi-1)^2}\right] \quad (19)$$

$$[\text{Integrand }\tau_{xy}]_{D_1=D_2=1}=\frac{q}{\pi}\left[\left\{\tan^{-1}\frac{(\xi+1)}{\eta}\right.\right.$$

$$\left.-\tan^{-1}\frac{(\xi-1)}{\eta}\right\}-\left\{\frac{\eta(\xi+1)}{\eta^2+(\xi+1)^2}\right.$$

$$\left.\left.-\frac{\eta(\xi-1)}{\eta^2+(\xi-1)^2}\right\}\right] \quad (20)$$

2-4. Particular Case of Concentrated Shear Load for Orthotropy.

As shown in Eqs. 15, all the equations of partial-uniform shear load case also must be degenerated into the expressions for concentrated shear load P. If the loaded length *"a"* approaches to zero and $2qa$ are replaced by P, the equations of partial-uniform shear load case should be reduced to equations of concentrated shear load case.

Then, it is necessary to examine each integrand for the abnormal behavior as $a\to 0$

This indeterminate form should be evaluated by L'hospital's rule, Applying L'hospital's rule, the denominator and numerator of the integrand are separately differentiated once with respect to *"a"*

Then,

$$[\text{Integrand }\sigma_x]_{a\to 0}=\frac{P}{\pi c(D_1-D_2)}$$

$$\left[\frac{D_1}{D_2}\frac{\xi}{\left(\frac{\eta}{D_2}\right)^2+\xi^2}-\frac{D_2}{D_1}\frac{\xi}{\left(\frac{\eta}{D_1}\right)^2+\xi^2}\right] \quad (21)$$

$$[\text{Integrand }\sigma_y]_{a\to 0}=-\frac{P}{\pi c(D_1-D_2)}$$

$$\left[\frac{\xi}{\left(\frac{\eta}{D_2}\right)^2+\xi^2}-\frac{\xi}{\left(\frac{\eta}{D_1}\right)^2+\xi^2}\right] \quad (22)$$

$$[\text{Integrand }\tau_{xy}]_{a\to 0}=\frac{PD_1D_2}{\pi c(D_1-D_2)}$$

$$\left[\frac{\eta}{\eta^2+D_2^2\xi^2}-\frac{\eta}{\eta^2+D_1^2\xi^2}\right] \quad (23)$$

where "c" is a unit length of coordinates

2-5. Particular Case of Concentrated shear Load for Isotropy.

In the concentrated shear load case, Since isotropy is a particular case of orthotropy, all the equations of orthotropic case must be degenerated into the expressions for isotropy when orthotropic constants are replaced by isotropic ones. As shown in Eqs. 21, 22 and 23, if D_1 and D_2 become unity, the orthotropic equations should be reduced to isotropic equations. The following equations were obtained by means of L'hospital's rule from orthotropic case

$$[\text{Integrand }\sigma_x]_{D_1=D_2=1}=\frac{P}{\pi c}\;\frac{2\xi^3}{[\eta^3+\xi^2]^2} \quad (24)$$

$$[\text{Integrand }\sigma_y]_{D_1=D_2=1}=\frac{P}{\pi c}\;\frac{2\xi\eta^2}{[\eta^2+\xi^2]^2} \quad (25)$$

Table 1. Elastic and Orthotropic Constants

Type of wood		Elastic Constants				Orthotropic Constants	
		$E_x \times 10^6$psi	$E_y \times 10^6$psi	ν_x	$G \times 10^6$psi	D_1	D_2
Three-layered plywood	x-strong axis	1.71	0.85	0.036	0.1	0.25	3.0
	y-strong axis	0.85	1.71	0.07	0.1	0.34	4.12
Laminated delta wood	x-strong axis	4.3	0.67	0.02	0.31	0.872	1.415
	y-strong axis	0.67	4.3	0.031	0.31	0.71	3.62

$$\left[\text{Integrand}\cdot\ \tau_{xx}\right]_{D_1=D_2=1} = \frac{P}{\pi c}\ \frac{2\xi^2\eta}{[\eta^2+\xi^2]^2} \qquad (26)$$

Eps. 24, 25 and 26 coincide with the simple equations of other investigators 23.

3. Discussion of Numerical Results

The exact solutions of stresses of both isotropic and orthotropic cases have been expressed by the closed forms. Isotropic results are presented first and they are compared with the results of other investigators.

The stresses given by Eqs 15a, 15b and 15c is good for any kind of timber which exhibit physical property of orthotropy. For the sake of comparison, numerical values are calculated

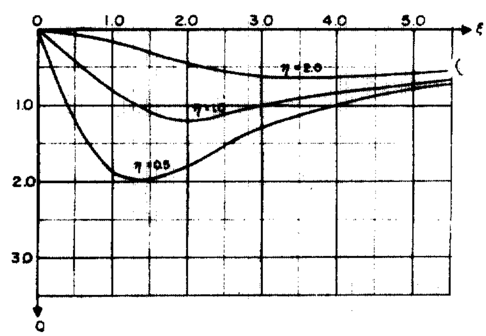

Fig. 2. Distribution of Normal Stress

$\sigma_x = \left(\frac{q}{\pi}\right)Q$

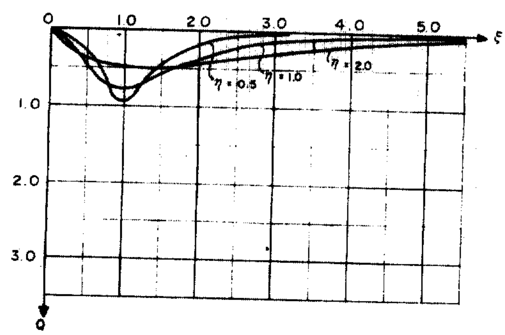

Fig. 3. Distribution of Normal Stress

$\sigma_y = \left(\frac{q}{\pi}\right)Q$

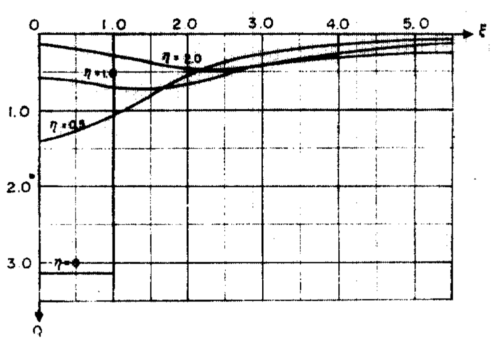

Fig. 4. Distribution of Shearing Stress

$\tau_{xy} = \left(\frac{q}{\pi}\right)Q$

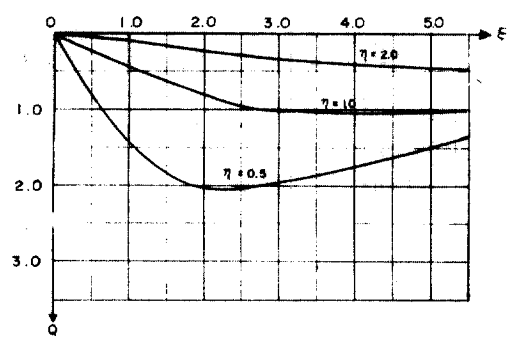

Fig. 5. Distribution of Mormal Stress

$\sigma_x = \left(\frac{q}{\pi}\right)Q$

For $D_1 = 0.25$, $D_2 = 3.00$

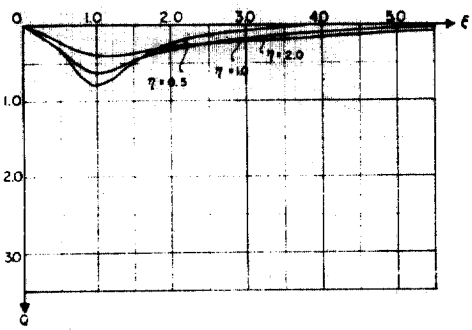

Fig. 6. Distribution of Normal Stress

$$\sigma_y = \left(\frac{q}{\pi}\right)Q$$

For $D_1=0.25$, $D_2=3.00$

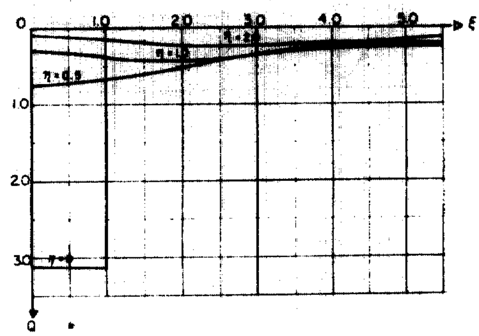

Fig. 7. Distribution of Shearing Stress

$$\tau_{xy} = \left(\frac{q}{\pi}\right)Q$$

For $D_1=0.25$, $D_2=3.00$

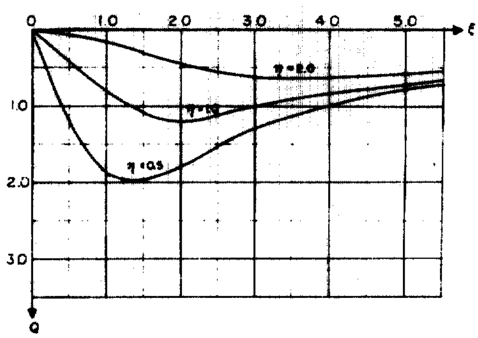

Fig. 8. Distribution of Normal Stress

$$\sigma_x = \left(\frac{q}{\pi}\right)Q$$

For $D_1=0.34$, $D_2=4.12$

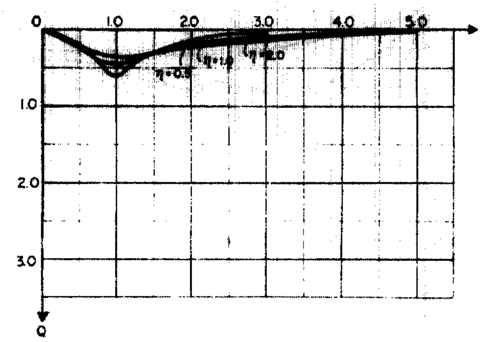

Fig. 9. Distribution of Normal Stress

$$\sigma_y = \left(\frac{q}{\pi}\right)Q$$

For $D_1=0.34$, $D_2=4.12$

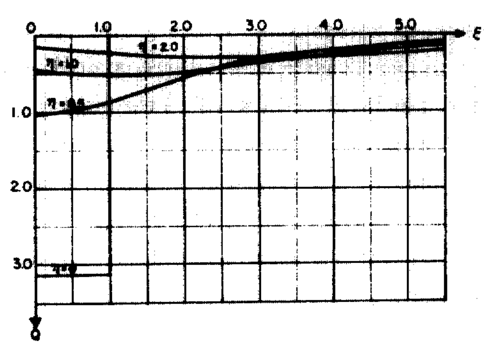

Fig. 10. Distribution of Shearing Stress

$$\tau_{xy} = \left(\frac{q}{\pi}\right)Q$$

For $D_1=0.34$, $D_2=4.12$

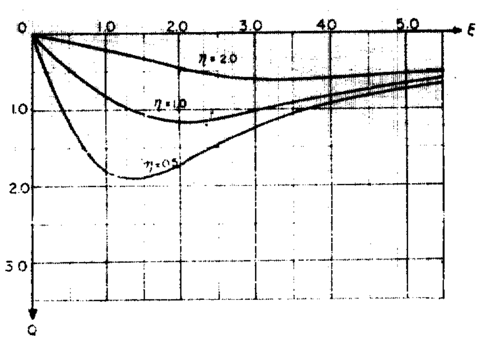

Fig. 11. Distribution of Normal Stress

$$\sigma_x = \left(\frac{q}{\pi}\right)Q$$

For $D_1=0.872$, $D_2=1.415$

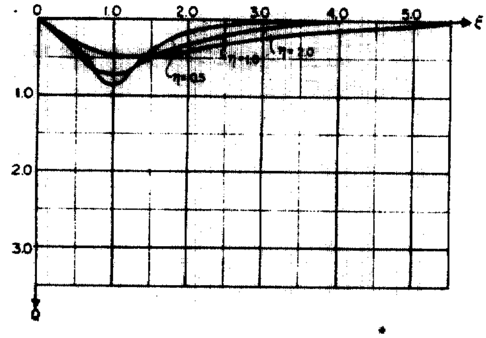

Fig. 12. Distribution of Normal Stress

$\sigma_y=\left(\frac{q}{\pi}\right)Q$

For $D_1=0.872$, $D_2=1.415$

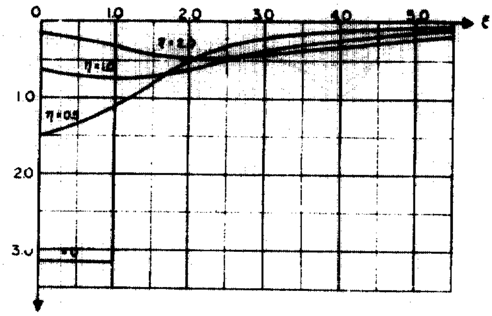

Fig. 13. Distribution of Shearing Stress

$\tau_{xy}=\left(\frac{q}{\pi}\right)Q$

For $D_1=0.872$, $D_2=1.415$

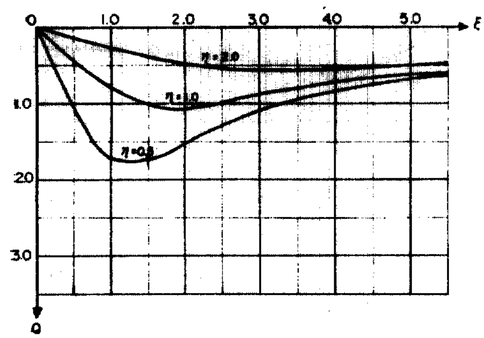

Fig. 14. Distribution of Normal Stress

$\sigma_x=\left(\frac{q}{\pi}\right)Q$

For $D_1=0.71$, $D_2=3.62$

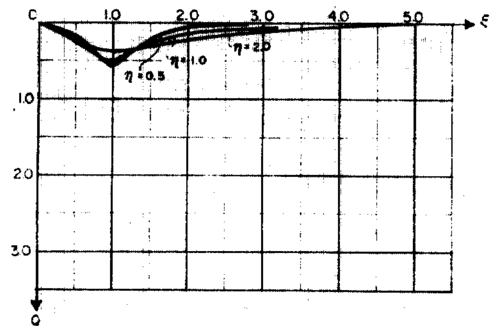

Fig. 15. Distribution of Normal Stress

$\sigma_y=\left(\frac{q}{\pi}\right)Q$

For $D_1=0.71$, $D_2=3.62$

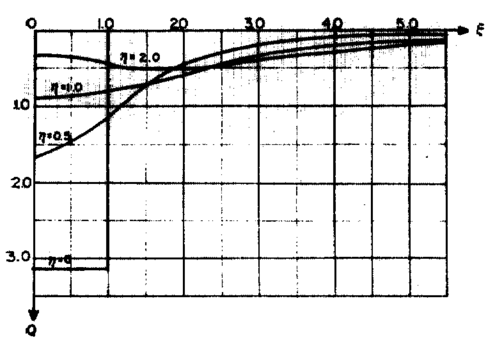

Fig. 16. Distribution of Shearing Stress

$\tau_{xy}=\left(\frac{q}{\pi}\right)Q$

For $D_1=0.71$, $D_2=3.62$

for two different kinds of timber and two different orientations of the grain. The kinds of timber considered are three-layered plywood and laminated delta wood. Two different orientations of the grain are strong axis in the x direction and strong axis in the y direction.

Table 1 indicates the values of elastic constants and the values of D_1 and D_2 for each case studies.

The values for elastic constants, E_x, E_y, G and ν_x are as given by Lekhnitskii. With this constants, the values of D_1 and D_2 are computed using Eq. 3a. 4a, and 4b.

In the table, the grain of the surface layers of three-layered plywood and laminated delta wood are assumed to be paralled to x axis in the case of x-strong axis.

The distribution of two normal stresses and shearing stress are shown in Fig. 2, 3, and 4 for isotropy.

Also, the same stresses are shown in Fig. 5 to Fig. 16 for two orientations (x-strong axis, y-strong axis) of three-layered plywood and laminated delta wood, respectively.

4. Conclusions

An analytical solution for the stresses of isotropic and orthotropic half-infinite plane under a partial uniform shear load is presented using Fourier integral and Airy stress function. The solution of orthotropy is reduced to the solution of isotropy when orthotropic constants are replaced by isotropic ones.

The solutions of a partial uniform shear load case are degenerated into the expressions for concentrated load case since the sum of uniformly distributed shear loads becomes a single concentrated load. Also, the solution of isotropy in equal to the results of other investigators.

Numerical values are computed and reported in figures for the cases of isotropy, three-layered plywood and laminated delta wood.

Two orientations of grain are considered in the timber materials. It is noted that distributions of the stresses of a orthotropic body depend on the type of material and orientation of the grain.

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