韓國林學會誌 67 : 52 – 59. 1984
Jour. Korean For. Soc. 67 : 52 – 59. 1984

The Gentan Probability, A Model for the Improvement of the Normal Wood Concept and for the Forest Planning

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1. The normal wood

The practice of forestry consists in the alternative iteration of felling and planting in a forest. During these practices the forest product is yielded. The amount of the yield must annually keep a fixed level for many reasons. On the one hand the forest product as material is indispensable for the human living, on the other hand forest in itself is important for the human society. Hence the forest must be maintained lastingly. As a standard method to realize both purposes simultaneously, it has been assumed hitherto, that the total forest F ha must be divided into age-classes having equal area F/u ha, that the same age class area must get to the felling age every year, and that the forest must supply a constant produce annually. The forest, which will endure lastingly and will be able to supply a constant wood produce as stated above, is called "normal wood" and has been accepted as a foundamental concept in the forest management.

2. Criticism to the normal wood concept

If we consider the sustention of the wood production, the normal wood concept is always inevitable. The principle of the sustention and the normal wood concept can be considered as synonym in this case. Similar principles or concepts were adapted by a lot of foresters and were practically realized. Therefore it is somewhat questionable, whether we can consider this pinciple as "Hundeshagen thought". All the thoughts are always ambiguously felt by many peoples. But an ordinary person cannot express it clearly. A man appears and penetrates the essence of the matter and declares his finding distinctly. He is a genius and his finding is competent to be called his "thought".

In this sense the "Hundeshagen's normal wood" was obviously a splendid "his thought" and it remains even now as the one and only principle of the forest management. It is believed by every forester, that the sustention of wood production can be guaranteed and confirmed only by the normal wood strategy, and that the main problem of the forest management, therefore, is to direct the real wood into its "normal state".

The forest management became entangled in the process of transfering a real wood into normal. Though the theoreticians clinged to the "normal wood" as a fixed notion, realities of the management often contradicted this principle. Usually forest owners neglected the normal wood principle at the decision of their forest felling. In order to form the normal state they must now and then either cut their younger forest earlier or leave their older forest later than the circulation period. On both occasions they always rejected the principle and did not act against their own interests.

Ch. Wagner (Prof. of forest management, Freiburg University) once made an expression of his criticism on the "strictly normal wood concept".

"Wir dürfen daher diese Form nicht, wie es in der Forsteinrichtung und in der ganzen Forstwissenschaft und Forstwirstchaft leider üblich geworden ist, als "Normalzustand" bezeichnen.

Es ist vielmehr der "Idealzustand". Dagegen wollen wir hier als "Normalzustand" denjenigen bezeichnen, der uns alles erreichen lößt, was wir wirtschaftlich brauchen und daher anstreben...."

In the above Wagner points out that "normal wood" is not so much ideal as illusory, and that there is no possibility of its realization in the least. From his point of view it is able to compare normal wood to a reversible engine with 100% efficiency, which contradicts the second law of thermodynamics and therefore has no possibility of its realization.

The Wagner's criticisim on the "normal wood concept" was circulated by his famous textbook "Lehrbuch der theoretischen Forsteinrichtung" (1928). For all that Wagner, who himself denied Hundeshagen, could not propose any counterpart nor set it against the normal wood. In place of "normal wood" there has been no idea till today, which has played the part of an indicator in the forest management.

Baader wrote on this theme in 1933. "Nachhalt und Normalwald sind zwei Begriffe, die sich gegenseitig bedingen. Ohne die Nachbalts idee ist die Normalwaldvorstellung underkbar, und ohne die letztere ist die Nachhalts Forderung vergleichbar einem Schiffe, das ohne Kompaß und Steuer auf der See tribt...."

In this famous article Baader wanted to point out an unsolvable contradiction. The sustention of wood and wood production is indispensable for forestry. On the other hand a forest cannot be sustained without the normal wood. However Wagner denied the normal wood as unrealizable. Therefore the classical forest management must be dead locked on the contradication. Conversely that accounts for the fact that Hundeshagen's normal wood is the one and only conceptual model in the forest management.

3. The Gentan-probability

At first direct our attention to a fixed stand with finite life span. Forests are changing their appearence ceaselessly in the course of years. Younger forests grow older while older forests are felled and replaced by new generations. Therefore when we observe the varied state of forests, we must be concerned with repeated trials. When it is cut, it is replaced almost immediately by a new stand, which in due time is replaced by a third stand, and so on. We assume that the stand life span is a discrete random variable which ranges only over multiples of an unit time, for example a year or a decade. In the following we regard the time unit as a decade and call it period. The corresponding age-class contains a decade too. We assume that it occurs a trial with a possible outcome "felling" or "survival". The renewal process in an individual stand causes the transformation of the forest age distribution and consequently gives variety to the appearence of the forest. These repetative processes may be treated as a recurrent event as follows:

Now define a probability $g(j)$ that a newly planted stand will survive till j years age-class and be cut in the same years age-class. We call $g(j)$ "Gentan probability". "Gentan" in Japanese means the diminution of a planted area. From the definition of $g(j)$ is derived a probability $r(j)$, that the newly planted stand will survive beyond j years age class, as

$$
\begin{aligned}
r(j) &= 1-q(1)-q(2) - \dots - q(j-1) \\
&= q(j) + q(j+1) + q(j+2) + \dots
\end{aligned}
$$

Then define a probability $q(j,k)$, that a stand, which is already j years age-class at the referring time, will survive k years further and will be cut at the j+k years age. From the definitions of the above probabilities follows:

$$q(j,k) = q(j+k)/r(j)$$

Let us suppose an initial forest age distribution a_1, a_2, ..., where a_k is a forest area at the k age-class. By the very definition of the probabilities $q(j)$ and $q(j,k)$, the estimation of the felling area tabulates as follows:

			u_3
		u_2	$u_2 q(1)$
	u_1	$u_1 q(1)$	$u_1 q(2)$
a_1	$a_1 q(1,1)$	$a_1 q(1,2)$	$a_1 q(1,3)$
a_2	$a_2 q(2,1)$	$a_2 q(2,2)$	$a_2 q(2,3)$
...	...	...	...
a_m	$a_m q(m,1)$	$a_m q(m,2)$	$a_m q(m,3)$

The first column represents the initial forest age distribution, the second column the estimated felling area from each age-class of the first period, and the third that of the second period and so on. The term u_n, which perches on the n+1th column and figures out a sum of all the terms of the column, is no more than the regenerated area at the period n.

From the above table the estimation of the forest age distributions are obtained as follows:

		u_2
	u_1	$u_1(1\text{-}q(1))$
a_1	$a_1(1\text{-}q(1,1))$	$a_1(1\text{-}q(1,1)\text{-}q(1,2))$
a_2	$a_2(1\text{-}q(2,1))$	$a_2(1\text{-}q(2,1)\text{-}q(2,2))$
...	...	...
a_n	$a_n(1\text{-}q(2,1))$	$a_n(1\text{-}q(n,1)\text{-}q(n,2))$

It is easy to know how to tabulate this table. And it is also self-evident, that each column represents a forest age destribution at the corresponding period.

If we multiply each growing stock per hectar by a corresponding forest area in the table, we obtain instantly the total harvest and the total stock of the respective period. Of course, the yield table is needed for this purpose.

4. The age class vector

Let us suppose for example a 30 years old stand covering 10 ha. Usually it will be 40 years old after a lapse of 10 years, but a part of the stand may be cut according to circumstances. For the sake of simplicity let us assume, that 3 ha of the stand are cut during the decade and are replanted immediately in the same period. On the same way let us assume again, that 5 ha of the survived 40 years age stand will be cut for the next period. Thus the stand will have an entirely different age distribution from that of the beginning.

Therefore it is possible to discribe the state corresponding to each of the above mentioned stand by the following arrangements of ordered tuples (0, 0, 10, 0, 0,...), (3, 0, 0, 7, 0,...) and (5, 3, 0, 0, 2, 0,...). On the other hand an ordered tuple of numbers is mathematically a vector itself. Therefore the phenomena of the forest age transition can be described by means of vectors, whose components are the area of each age-class.

Assuming a sufficiently large n, the upper bound of the age-class can be restricted practically lower than the prescribed n. And if the number of the components is finite, the vector is n-dimentional, that is n-vector. In the following we call such a n-vector $(a_1, a_2, \ldots, a_n)$ an age-class vector. Owing to the lapse of time the age-class vector $a = (a_1, a_2, \ldots, a_n)$ will be transformed into another age-class vector $a' = (a_1', a_2', \ldots, a_n')$ in the next period.

In ordinary circumstances an j years age-class stand at present will pass over into the j+1 years age-class in one period. Some of its portion will be felled as the case may be. As the felling area is in practice replanted immediately after felling, it is treated as to transit from the j years to the 1 years age-class. Let us in general assign a symbol p(j,k) to the probability, that an j years age-class stand passes over into an k years age-class, and call it age-class transition probability from j to k.

Since the probability p(j,k) is interpreted as an areal ratio of the transferred stand to the original, we immediately obtain the area a_k of the k years age-class in the next period as,

$$a_k = a_1 p(1,k) + a_2 p(2,k) + \ldots + a_n p(n,k) \qquad k = 1, 2, \ldots, n \qquad (1)$$

Accordingly it will be more appropriate to introduce a n x n matrix P, whose components are the probabilities p(j,k)

$$P = (p(j,k)), \qquad j, k = 1, 2, \ldots, n$$

By using the definition of the matrix multiplication we may now write the linear transformation (1) in the simpler form

$$a' = aP, \tag{2}$$

where $a = (a_j)$ and $a' = (a'_j)$ are age-class row vectors at the present and in the following period, respectively. Let us call P an "age-class-transition matrix". The matrix P is a kind of stochastic matrix and controls the process of the forest age transition. In other words the forest age transition can be regarded as a simple Markov chains.

5. Age-class space

Let $e_1 = (1, 0,...,0)$, $e_2 = (0, 1,...,0)$,...,$e_n = (0, 0,..,1)$ be a basis of an n-dimensional vector space R. Then an arbitrary vector $a = (a_1, a_2,...,a_n)$ in the space has an unique expression

$$a = a_1 e_1 + a_2 e_2 +....+ a_n e_n \tag{3}$$

For the age-class vectors the sum of its all components remains constant, that is

$$a_1 + a_2 +....+ a_n = a \tag{4}$$

where a is the total area of the forest. Being all components $a_1, a_2,...,a_n$ non-negative and smaller than a, the age-class vector a is restricted to flow only in an n-1 dimensional simplex. From now on let us call this simplex "age-class space A".

The transformation defined by the age-class-transition matrix P is continuous. If we introduce a kind of norm "a" for the vector a such that

$$\|a\| = |a_1| + |a_2| + ... + |a_n| \tag{5}$$

then there holds an equality

$$\| a' \| = \| a \| \tag{6}$$

This equality represents the fact that the total area of the forest is invariable throughout all the periods. Therefore sufficiently close two age-class vectors a_1 and a_2 correspond to age-class vectors a_1' and a_2' apart as close as before.

As is well known the Brouwer's fixed point theorem is one of the most important in topology. The theorem states; "The continuous mapping of a closed simplex into itself has at least a fixed point". Applying this to our case, it follows that there exists at least a fixed vector a_0 in the age-class space, such that

$$a_0 P = a_0 \tag{7}$$

The vector a_0 is an eigenvector of the matrix P corresponding to an eigenvalue one. The equation (7) shows that the vector a_0 will remain unchanged everlastingly, so we will be able to regard this as a normal age-class distribution. It is easily shown that the vector a_0 satisfying (7) is unique for each transition matrix P.

6. The generalized normal wood

Through a lapse of time the age-class-transition matrix P in reality may undergo a change in greater or less, but for simplicity we assume its uniformity in the following.

Under this assumption an initial age-class-vector a will be transformed into another vector a_l after l periods, which is written as

$$a_l = aP^l \tag{8}$$

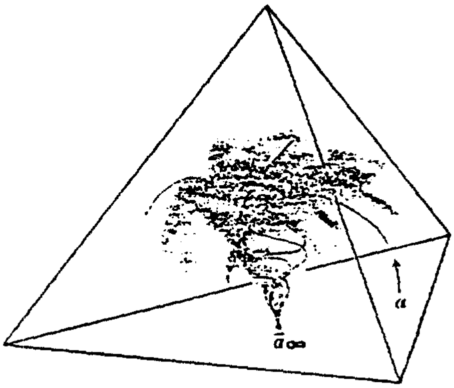
where p^l is a l^{th} power of the matrix P. If a sequence of the matrices P^l converges to a fixed matrix as l tends to an infinity, we call such a matrix P stable. By taking limit both the sides of (8) it is immediately obtained that for such a stable matrix P there exists a vector a^∞ such that

$$a_\infty = \lim_{l \to \infty} a_l = a \lim_{l \to \infty} P^l \tag{9}$$

A sufficient condition for the stability of an age-class-transition matrix P is that the set of numbers k_i such that $P^{(k_i)}(1,1) = 0$ is mutually prime, where $P^{(k_i)}(1,1)$ is the probability that the first age-class stand will recur to the original class after k_i periods. Since the limiting vector a_∞ is a fixed point of the matrix P and there is no fixed point other than a_0 as above mentioned, we conclude that

$$a_\infty = a_0 \tag{10}$$

The ultimate forest state thus obtained has its own fixed age distribution and constancy of the wood productivity. Consequently the newly obtained state deserves to be called as a generalized normal state, though it is seemingly different from the old one. While the latter was criticized as unrealizable, the former will be realized spontaneously with the lapse of time. To each matrix there corresponds one normal wood, and to infinitely many matrices there exist infinitely many normal woods.



An age-class vector a moves along the flow line in the n-1 dimensional age-class space and sinks into the limiting normal vector a_∞.

The limiting vector a_∞ can be always accessible from an arbitrary vector in the age-class space. Therefore we can imagine a flow of the age-class vector, which pours into the vector a_∞ as a sink. But the conclusion may seem contradictory. It seems illogical that the vector $a_\infty = a \lim P^1$ remains fixed for any vector a, while the limiting matrix $\lim P^1$ is constant. This pretended contradiction is explained by means of the Gnedenko's Theorem. This theorem states that an infinite power of an arbitrary age-class-transition matrix P is convergent, such that

$$P^\infty = \lim_{l\to\infty} P^l = \begin{pmatrix} p_1 & p_2 & \cdots & p_n \\ p_1 & p_2 & \cdots & p_n \\ p_1 & p_2 & \cdots & p_n \end{pmatrix}, \quad p_1 + p_2 + \ldots + p_n = 1 \tag{11}$$

Accordingly for an arbitrary initial age-class vector a holds

$$a_\infty = a \lim_{l\to\infty} P^l = (a_1, a_2, \ldots, a_n) \begin{pmatrix} p_1 & p_2 & \cdots & p_n \\ p_1 & p_2 & \cdots & p_n \\ p_1 & p_2 & \cdots & p_n \end{pmatrix}$$
$$= (ap_1, ap_2, \ldots, ap_n) \tag{12}$$

where $a_1 + a_2 + \ldots + a_n = a$ is the total forest area. This shows that the above conclusion is not contradictory despite of the superficial skepticism.

7. Another proof of the forest transition stability by means of the Gentan-probability q(j)

By using the Gentan-probability the similar conclusion can be deduced. This method is founded upon the fact that a replanted area in the k^{th} period u_k is represented by a difference equation

$$u_k = u_{k-1}q(1) + u_{k-2}q(2) + \ldots + u_1 q(k-1) + b_k \tag{13}$$

where

$$b_k = a_1 q(1,k) + a_2 q(2,k) + \ldots + a_n q(n,k) \tag{14}$$

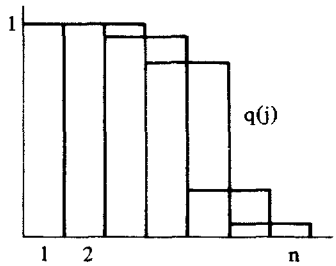
The equation (13) derived directly from the tables aforementioned is a discretization of an integral equation of the convolution type, which is the same equation obtained and defined as "renewal equation" by Feller.

By considering the generating functions of the series u_k, $q(k)$ and b_k, it is concluded that

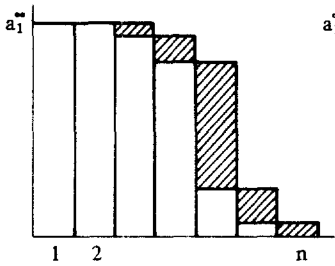
$$u_k \to a/u \tag{15}$$

as k tends to the infinity, where a is the total forest area and $u = \Sigma j.q(j)$ is its average felling age. The relation (15) shows that the felling area in each period becomes constant after a long lapse of time.

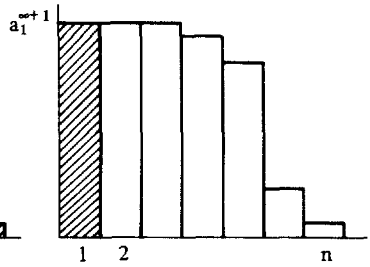
As the result of this the age-class distribution of the forest becomes similar to a curve, which declines with steps 1, $1\text{-}q(1) = p_1$, $1\text{-}q(1) - q(2) = p_2$,.... and is called "Gentan curve". The appended figures show why the limiting age-class distribution resembles the Gentan curve. The shaded area of the second figure indicates the felling in the ∞ period and that of the third the regenerated in the ∞+1 period.



"Gentan curve"



"Forest age distribution at ∞".
"The shaded are harvested."



"Forest age distribution at ∞+1".
"The shaded is replanted."

Both of the age distribution are similar to the Gentan curve on the left and sustain their form unchanged for ever.

From the definitions of the age-class transition probability p(j,k) and the Gentan probability q(j) hold the following relations.

$$q(j) = p(1,2)p(2,3) \ldots\ldots p(j\text{-}1,j)p(j,1) \quad (16)$$

$$p(j,1) = q(j,1) \quad (17)$$

Therefore the theory by the age-class-transition matrix and that by the Gentan probability reduce to the same conclusion.

From the very beginning the quantum mechanics adopted two different approaches, one by the matrix and the other by the wave equation. And the wave equation is equivalent to the corresponding integral equation. Furthermore the core of the quantum theory is reduced to find eigenvalues or corresponding eigenvectors of the matrix or those of the integral equation. On the other hand the age-class transition theory adopts two different approaches also, one by the matrix and the other by the renewal equation. We have seen that the renewal equation is equivalent to a kind of integral equation. That is the reason why we studied the same age-class transition process by the different two methods.

8. Estimation of the Gentan Probability from the felling statistics

For all restrictions each individual forest owner in all countries wishes to treat his forest of his own will. Usually he adopts the most profitable felling age for his forest. Accordingly there is caused a forest life span distribution in each forest district. From the definition the Gentan probability is nothing else but a waiting time of a stand up to its felling. It is therefore possible to determine the probability from the statistics of the forest life span.

Let us assume that a stand is harvested, if its mean diameter is k cm thick, and that the mean diameter grows in average m cm thicker in a period. Under these assumptions the probability f(k, t) that the mean diameter of the stand is k cm thick within t period, is given by the following Poisson's dinsity function.

$$f(k,t) = \exp(-mt)\,(mt)^k/k! \quad (18)$$

The mean diameter becomes k cm thick at the t th period, if and only if it has already been k-1 cm till the t-1th period and becomes 1 cm thicker within the successive period (t-1, t). Assuming the consecutive events independent the probability F(k,t)dt, that the mean diameter is k cm thick within the period (t-1,t), is obtained by multiplying two probabilities f(k-1,t) and mdt.

$$F(k,t)dt = f(k\text{-}1,t)mdt \quad (19)$$

From the derivation it is clear that this formula gives the life span distribution of the forest. Then the probability, that the average life span of the forests lies in the time interval (j,j+1), that is the Gentan probability q(j), is given by

$$q(j) = \int_j^{j+1} F(k,t)dt \quad (20)$$

Letting

$$k = \frac{n}{2}, \qquad mt = \frac{x^2}{2} \tag{21}$$

this integral (20) leads to

$$q(j) = \frac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})} \int_{2mn}^{2m(n+1)} e^{-\frac{x^2}{2}} (x^2)^{\frac{n}{2}-1} dx^2 \tag{22}$$

This integral is easily obtained by the x^2 table.

In order to apply the formula to a concrete case, two parameters m and k must be estimated previously. But there exist two relations

$$E(t) = k/m, \qquad \sigma^2(t) = k/m^2 \tag{23}$$

where $E(t)$ and $\sigma^2(t)$ are the mean and the variance of the felling age, respectively. Thus on the basis of the statistics we can predict the transition and the product in a forest district by means of the Gentan probability.

9. Yield estimation by the linear programming

We have shown above that all the forests will be able to arrive in normal state inevitably in the long run. However the process to the normal state is not necessarily optimal, but only feasible. Now let us assume an initial age-class vector a and a target vector a′ in the age-class space. Each method of the yield estimation corresponds to a path connecting two vectors. Thereore the yield estimation accounts to a problem to find an optimal path according to the purpose of the management.

The problem will be successfully solved by means of the linear programming as follows. We shall explain the method by an example. Without loss of generality we confine outselves to the first three periods in the example. Denoting the initial age-class distribution $a_1, a_2, \ldots, a_n, \ldots$ and the final one $b_1, b_2, \ldots b_n, b_{n+1}, \ldots$, the following two tables are laid out on the same way as the "Gentan method".

			u_3
		u_2	z_1
	u_1	y_1	z_2
a_1	x_1	y_2	z_3
a_2	x_2	y_2	z_2
----	----	----	----
a_n	x_n	y_{n+1}	z_{n+2}

			u_3
		u_2	u_2 $-z_1$
	u_1	u_1 $-y_1$	u_1 $-y_1-z_2$
a_1	a_1-x_1	$a_1-x_1-y_2$	$a_1-x_1-y_2-z_3$
a_2	a_2-x_2	$a_2-x_2-y_2$	$a_2-x_2-y_3-z_4$
----	----	----	----
a_n	a_n-x_n	$a_n-x_n-y_{n+1}$	$a_n-x_n-y_{n+1}-z_{n+2}$

where $x_1, x_2, \ldots, x_n$; $y_1, y_2, \ldots, y_{n+1}$; $z_1, z_2, \ldots, z_{n+2}$ are the undetermined felling areas and u_1, u_2 and u_3 are their totals of the periods, respectively. That is

$$\begin{aligned} u_1 &= x_1 + x_2 + \ldots + x_n \\ u_2 &= y_1 + y_2 + \ldots + y_{n+1} \\ u_3 &= z_1 + z_2 + \ldots + z_{n+2} \end{aligned} \tag{24}$$

The conditions, that the final age-class distribution at the thrid period is $b_1, b_2, \ldots$, are represented by the following equalities.

$$\begin{aligned} u_3 \qquad\quad &= z_1 + z_2 + \ldots + z_{n+2} &&= b_1 \\ u_2 \qquad - z_1 &= y_1 + y_2 + \ldots + y_{n+1} \quad - z_1 &&= b_2 \\ u_1 - y_1 - z_2 &= x_1 + x_2 + \ldots + x_n - y_1 - z_2 &&= b_3 \end{aligned} \tag{25}$$

Besides these conditions we can list up linear equalities and inequalities of these unknowns which express the needs of the forest management. For example, if we adopt the method of periods by area (Flächenfachwerk), there must be conditions such that

$$\begin{aligned} x_1 + x_2 + \ldots + x_n &= y_1 + y_2 + \ldots + y_{n+1} \\ &= z_1 + z_2 + \ldots + z_{n+2} \end{aligned} \tag{26}$$

or the periodic method by volume (Massenfachwerk), conditions such that

$$\begin{aligned} v_1 x_1 + v_2 x_2 + \ldots + v_n x_n &= v_1 y_1 + v_2 y_2 + \ldots + \\ v_{n+1} y_{n+1} &= v_1 z_1 + v_2 z_2 + \ldots + v_{n+2} z_{n+2} \end{aligned} \tag{27}$$

where $v_1, v_2, \ldots$ are the average stock per hectare at each age-class, respectively.

Finally if we select tentatively the total yield W throughout the periods as the object function,

$$W = v_1(x_1 + y_1 + z_1) + v_2(x_2 + y_2 + z_2) + \ldots \rightarrow \max \tag{28}$$

The problem becomes the typical linear programming.

From this point of view all the classical yield estimation of the forest management can be regarded as problems of the linear programming.

10. Supplement

(1) The method of the Gentan-probability has been used for the prediction of the product from

the private forests in Japan for about thirty years. There has occurred a lot of problems with respect to the estimation of the parameters.

(2) Even if there are no available statistical data of the felling age, it is possible to estimate it only from the annual felling area by means of the "forest renewal equation". For this purpose it is conveniently used the sufficient sequences of the aerial or of the satellite photos taken annually. This method will be the most fitted to predict the global forest transition.

(3) Of course, the age-class transition matrix fluctuates from period to period. But if the fluctuations are confined within some small range, an infinite product of such matrices converges stochastically to a fixed matrix. This fact is treated on the other paper presented to this meeting.

Summary

A Gentan probability q(j) is the probability that a newly planted forest will be felled at age-class j. A future change in growing stock and yield of the forests can be predicted by means of this probability. On the other hand a state of the forests is described in terms of an n-vector whose components are the areas of each age-class. This vector, called age-class vector, flows in a n-1 dimensional simplex by means of n x n matrices, whose components are the age-class transition probabilities derived from the Gentan probabilities.

In the simplex there exists a fixed point, into which an arbitrary forest age vector sinks. Theoretically this point means a normal state of the forest. To each age-class-transition matrix there corresponds a single normal state; this means that there are infinitely many normal states of the forests.