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General Derivation of Two-Fluid Model

Hee Cheon No
Korea Advanced Institute of Science and Technology
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2상 유동 모델의 일반적인 유도

노 희 천
한국과학기술원
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Abstract

General time-volume averaged conservation equations and jump conditions for two-phase flows are derived here. The time-averaged equations for a single phase region in two-phase flow are obtained from local instant balance equations by a technique often used for single phase turbulent flow equations. The results obtained by integrating the time averaged equations over a flow volume are spatially averaged twice; first, they are averaged over a single phase region of the k-th phase and then averaged over the total volume of the k-th phase, in a flow volume. The mass, momentum, and energy conservation equations are obtained from the general time-volume averaged equations. The advantages of the present model are explained by comparing it with Ishii's model (1) and Banerjee's model (2). Finally, the assumptions and approximate terms of the equations of the THERMIT-6S are clarified.

요 약

2상 유동에 대한 일반적인 시간과 공간에 대해 평균된 보존방정식과 jump condition을 유도했다. 단상난류 유동 방정식에 사용되는 방법을 써서, 한부분에서 순간적으로 이루어지는 평형방정식(local instant balance eq.)으로부터 2상유동내 단상영역에 관한 시간에 대해 평균된 방정식을 유체체적에 대해 적분하여 얻어진 결과는 공간적으로 두차례에 걸쳐 평균된다. 즉, 한 유체체적내에서 일차적으로 k번째 상의 단상영역에 대해 평균하고 다음에 k번째 상 전체체적에 대해 평균한다. 질량, 운동량 그리고 에너지 보존 방정식은 일반적인 시간과 공간에 대해 평균된 방정식으로부터 얻어진다. 이 모델의 장점은 Ishii모델, 그리고 Banerjee의 model과 비교하여 설명된다. 마지막으로, THERMIT-6S의 방정식에 포함된 가정과 근사항들에 대해 밝혀둔다.

1. Introduction

The most important characteristic of two phase flows is the presence of moving internal interfaces separating phases; a two-phase system consists of a number of single phase regions bounded by moving interfaces. Therefore, a formulation based on local instant variables and moving interfaces results in a multi-boundary problem which are not known a priori. In order to overcome this difficulty, we need the formula-

tion of two-phase flows to integrate the microscopic scales of phenomena but to preserve their effect on macroscopic phenomena.

Three approaches have been used widely to develop a two-phase flow model:

1) Mixture Model Approach
2) Control Volume Approach
3) Averaging Approach

Ishii (1, 3) provides an excellent description of these three approaches. The first and second approaches are mainly based on hypothesis, physical intuition, and assumed similarity with a single phase flow system. In these two approaches, even though easy formulation and practicality in simple cases are main advantages, we encounter the difficulty of identifying the effect of the microscopic scales of phenomena on the macroscopic behavior. On the other hand, the averaging approch enables us to set up mathematically rigorous equations.

The averaging approach can be classified into three main groups: Eulerian, Lagrangian, and Boltzmann statistical averages. In an Eulerian description, time and space coordinates are taken as independent variables and various dependent variables express their changes with respect to these coordinates. As the particle coordinate in a Lagrangian description displaces the spatial variable of the Eulerian description, the Lagran- gian average is taken by following a certain particle and observing it in a time interval. The Boltzmann statistical averaging with a concept of the particle number density is important when the collective mechanics of large number of particles are in question. The Eulerian average has been considered as the most widely used method, due to its close relation to experimental observations and instrumentations. Therefore, here the time-volume averaged conservation equations of two phase flows are derived on a basis of the Eulerian averaging approach.

2. Local Instant Balance Equation

A two-phase flow is considered to subdivided into several single phase regions separated by moving interfaces. Each separate phase can be connected through jump conditions. Jump conditions constitute a characteristic feature of moving boundaries and provide relations between the phase interaction terms. Therefore, the ordinary method used in continuum mechanics will be first used and then jump conditions will be derived.

Let us start from a general balance equation. For a material of the k-th phase with the volume and surface, V_{mk} and A_{mk}, we can set up the general balance equation.

$$\frac{d}{dt}\oint_{V_{mk}}\rho_k\phi_k dV=-\oint_{A_{mk}}\vec{n}_k\cdot\vec{J}_k dA+\int_{V_{mk}}\rho_k S_k dV \tag{1}$$

where the subscript, m, indicates a control mass, ρ_k is the density of k-th phase, ϕ_k is any quantity being conserved in the *k-th* phase, $\vec{J}_k$ and S_k are the efflux and source of ϕ_k, and $\vec{n}_k$ is an outward unit normal vector for the k-th phase. Using the Reynolds transport theorem (1), we have

$$\frac{d}{dt}\int_{V_{mk}}\rho_k\phi_k dV=\int_{V_{mk}}\frac{\partial\rho_k\phi_k}{\partial t}dV+\oint_{A_{mk}}\rho_k\phi_k\vec{v}_k\cdot\vec{n}_k dA \tag{2}$$

where $\vec{v}_k$ denotes the velocity of a material of the k-th phase. Note that, while the Reynolds transport theorem is derived for a material volume, the Leibnitz rule (4) (given later by Eq. (24)) is a purely geometric theorem. The Gauss's theorem (5) gives a transformation between a volume and surface integral.

$$\oint_{A_{mk}}(\rho_k\phi_k\vec{v}_k+\vec{J}_k)\cdot\vec{n}_k dA=\int_{V_{mk}}\nabla\cdot(\rho_k\phi_k\vec{v}_k+\vec{J}_k)dV \tag{3}$$

Substituing Eqs. (2) and (3) into Eq. (1), we have

$$\int_{V_{mk}} \left[\frac{\partial \rho_k \psi_k}{\partial t} + \nabla \cdot (\rho_k \psi_k \vec{v}_k + \vec{J}_k) - \rho_k S_k \right] dV = 0. \quad (4)$$

By the axiom of continuum, we get the local instant differential balance equation

$$\frac{\partial \rho_k \psi_k}{\partial t} + \nabla \cdot (\rho_k \psi_k \vec{v}_k) + \nabla \cdot \vec{J}_k - \rho_k S_k = 0. \quad (5)$$

The local instant differential balance equation, Eq. (5) can be applied in each phase up to an interface. Let us derive an interfacial balance equation based on the control volumes, V_1 and V_2 are surrounded by the areas of each phase, A_1 and A_2, with an interfacial area A_i. As the control volumes, V_1 and V_2, are very thin, we put $\vec{n}_1 = -\vec{n}_2$,

where $\vec{n}_1$ and $\vec{n}_2$ are outward unit normal vectors from surfaces, A_1 and A_2, respectively. Now we have the general balance equation for the control volumes, V_1 and V_2 by summing the integrated result of Eq (5) over each result.

$$\sum_{k=1}^{2} \frac{d}{dt} \int_{V_k} \rho_k \psi_k dV = \sum_{k=1}^{2} \left[\int_{A_k} \vec{n}_k \cdot [\rho_k \psi_k (v_k - \vec{v}_i) + \vec{J}_k] dA + \int_{V_k} \rho_k S_k dV \right] \quad (6)$$

We let the volumes, V_1 and V_2 shrink down to the interface so that the volumes, V_1 and V_2 vanish, while the interfacial area A_i remains finite in the limit. The volume integrals vanish and A_k is equal to A_i, in the limit. Then we have the general interfacial balance equation.

$$\sum_{k=1}^{2} (\dot{m}_k \psi_k + \vec{n}_k \cdot \vec{J}_k) = 0 \quad (7)$$

where $\dot{m}_k = \rho_k (\vec{v}_k - \vec{v}_i) \cdot \vec{n}_k$ (8)

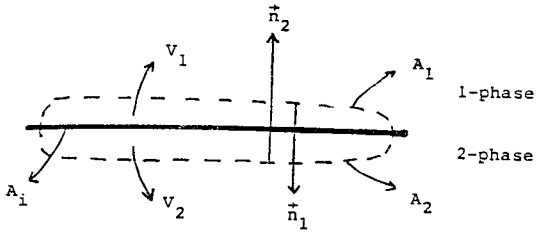

Fig. 1. Configuration of interface.

$\dot{m}_k$ is the local instant mass flux from V_k through the interface.

3. General Time Averaged Balance Equation

In a single phase turbulent flow, the Eulerian time averaging is essential and a well-known technique. As a two-phase flow consits of several single phase regions, the technique used in a single phase turbulent flow is taken to obtain the general time-averagted balance equation for one of single phase regions.

Let us integrate the local instant balance equation, Eq. (5), over a time interval Δt. The time interval Δt should be smaller than the transport time of one single phase to pass through a reference position and than the macroscopic time constant of the unsteadiness of the bulk fluid. Also Δt should be large enough to smooth out the local variations of properties. Ishii (1) and Delhaye (6), used Δt which is a time inerval sufficient for several single phase regions to pass through a local position. However, as turbulence originated in a single phase region will be confined in that phase by the interface and the effect of turbulence in a phase can be transmitted through the other phase, Δt should be smaller than a time interval for a single phase region to pass through a reference point.

As a result, we get a time-averaged equation which is identical to that made in analyzing a single phase turbulent flow.

$$\frac{\overline{\partial \rho_k \psi_k}}{\partial t} + \nabla \cdot \overline{(\rho \psi_k \vec{v}_k)} + \nabla \cdot \vec{J}_k - \overline{\rho_k S_k} = 0, \quad (9)$$

where $\bar{F} = \frac{1}{\Delta t} \int_{\Delta t} F dt.$ (10)

In order to obtain Eq. (9), we used the following relationships which come from the Leibnitz rule.

$$\frac{\overline{\partial F}}{\partial t} = \frac{\partial \bar{F}}{\partial t} \text{ and } \overline{\nabla \cdot F} = \nabla \cdot \bar{F}. \quad (11)$$

By taking a non-zero scalar weighted function w, we define the general weighted mean value of a function F as:

$$\overleftrightarrow{F}=\frac{\overline{wF}}{\overline{w}}. \tag{12}$$

In general, the volume, momentum, energy, and entropy are considered to be extensive variables. If F is taken as a quantity per unit volume of the extensive characteristic, it can be expressed in terms of the variable per unit mass ϕ as:

$$F=\rho\phi. \tag{13}$$

The appropriate mean value for ϕ should be weighted by the density as:

$$\overleftrightarrow{\phi}=\frac{\overline{\rho\phi}}{\overline{\rho}}. \tag{14}$$

Now let us introduce fluctuating components of variables caused by turbulence. In general, as they are defined as a difference between a local instant variable and its weighted mean value, we have

$$F''=F-\overleftrightarrow{F}. \tag{15}$$

Since the mean values of ϕ and $\vec{v}$ are weighted by mass, the fluctuating components are given as follows:

$$\phi_k=\overleftrightarrow{\phi}_k+\phi_k''\text{and }\vec{v}_k=\overleftrightarrow{\vec{v}}_k+\vec{v}_k''. \tag{16}$$

Using Eqs. (14) and (15), we have the additional relationships:

$$\overline{\rho_k''}=0,\ \overleftrightarrow{\phi_k}''=0,\ \overleftrightarrow{\vec{v}_k''}=0,$$
$$\overline{\rho_k\phi_k''}=0,\ \overline{\rho_k\vec{v}_k''}=0. \tag{17}$$

From Eqs (16) and (17), the convective flux term

$$\overline{\rho_k\phi_k\vec{v}_k}=\bar{\rho}_k\overleftrightarrow{\phi}_k\overleftrightarrow{\vec{v}}_k+\overline{\rho_k\phi_k''\vec{v}_k''}. \tag{18}$$

Now we obtain the time-averaged balance equation

$$\frac{\partial\bar{\rho}_k\overleftrightarrow{\phi}_k}{\partial t}+\nabla\cdot(\bar{\rho}_k\overleftrightarrow{\phi}_k\overleftrightarrow{\vec{v}}_k)+\nabla\cdot(\bar{\vec{J}}_k+\bar{\vec{J}}_k^t)-\bar{\rho}_k\overleftrightarrow{S}_k=0, \tag{19}$$

where

$$\bar{\vec{J}}_k^t=\overline{\rho_k\phi_k''\vec{v}_k''}. \tag{20}$$

In the same manner, we can obtain the time-averaged interfacial balance equation by integrating Eq. (7) over the time interval, Δt.

$$\sum_{k=1}^{2}\left(\overleftrightarrow{\dot{m}_k\phi_k}+\vec{n}_k\cdot\bar{\vec{J}}_k\right)=0, \tag{21}$$

where

$$\overleftrightarrow{\phi}=\frac{\overline{\dot{m}_k\phi_k}}{\overline{\dot{m}_k}}. \tag{22}$$

$\overleftrightarrow{\phi}$ is weighted by the time averaged mass flux through the interface.

4. General Time-Volume Averaged Balance Equation

Many practical problems of two-phase flows are dealt with using time-volume averaged equations. A time-volume averaged equation can be obtained by integrating the local time averaged equation, Eq. (19), over a flow volume V_f, which consists of several single phase regions.

$$\sum_{i=1}^{I}\frac{1}{V_f}\int_{V_{ki}}\left[\frac{\partial\bar{\rho}_k\overleftrightarrow{\phi}_k}{\partial t}+\nabla\cdot(\rho_k\overleftrightarrow{\phi}_k\overleftrightarrow{\vec{v}}_k)\right.$$
$$\left.+\nabla\cdot(\bar{\vec{J}}_k+\bar{\vec{J}}_k^t)-\bar{\rho}_k\overleftrightarrow{S}_k\right]dV=0, \tag{23}$$

where I is the maximum number of the k-th phase regions in the volume V_f and V_{ki} is the volume of the i-th single phase region of the k-th phase. The theorems we will use are given below:

Leibnitz rule

$$\frac{\partial}{\partial t}\int_{V_{ki}}F dV=\int_{V_{ki}}\frac{\partial F}{\partial t}dV+\int_{A_{ki}}F\vec{v}_i\cdot\vec{n}_k dA, \tag{24}$$

Gauss' theorem

$$\int_{V_{ki}}\nabla\cdot\vec{F}dV=\nabla\cdot\int_{V_{ki}}\vec{F}dV+\int_{A_{ki}}\vec{n}_k\cdot\vec{F}dA, \tag{25}$$

where A_{ki} is the surface area of the i-th single phase region of the k-th phase in the volume V_f and $\vec{v}_i\cdot\vec{n}_k$ is the surface displacement velocity of A_{ki}. Using these theorems, we obtain

the equation

$$\sum_i \frac{1}{V_f}\Big[\frac{\partial}{\partial t}\int_{V_{ki}} \bar{\rho}_k\overleftrightarrow{\phi}_k\, dV+\nabla\cdot\int_{V_{ki}}\bar{\rho}_k\overleftrightarrow{\phi}_k\overleftrightarrow{\vec{v}}_k dV$$
$$+\nabla\cdot\int_{V_{ki}}(\bar{\vec{J}}_k+\bar{\vec{J}}^{t}_k)\, dV-\int_{V_{ki}}\bar{\rho}_k\overleftrightarrow{S}_k\, dV$$
$$+\int_{Aki}\vec{n}_k\cdot[\bar{\rho}_k\overleftrightarrow{\phi}_k(\overleftrightarrow{\vec{v}}_k-\vec{v}_i)+\bar{\vec{J}}^{t}_k]\, dA$$
$$+\int_{A_{ki}+A_{kwj}}\vec{n}_k\cdot\bar{\vec{J}}_k\, dA\Big]=0, \qquad (26)$$

where A_{kwj} denotes the area of the j-th single region of the k-th phase contacting the solid surface in the volume V_f. The fifth term in Eq. (26) can be simplified by using the definition of the turbulent flux, Eq. (20).

$$B=[\bar{\rho}_k\overleftrightarrow{\phi}_k(\overleftrightarrow{\vec{v}}_k-\vec{v}_i)+\bar{\vec{J}}^{t}_k\cdot\vec{n}_k$$
$$=(\bar{\rho}_k\overleftrightarrow{\phi}_k\overleftrightarrow{\vec{v}}_k+\overline{\rho\phi'_k\vec{v}_k{}''}-\bar{\rho}_k\overleftrightarrow{\phi}_k\vec{v}_i)\cdot\vec{n}_k. \qquad (27)$$

From Eqs. (14) and (18), Eq. (27) becomes

$$B=(\overline{\rho_k\phi_k\vec{v}_k}-\overline{\rho_k\phi_k}\vec{v}_i)\cdot\vec{n}_k. \qquad (28)$$

Then, we have the following equation which is consistent with the jump condition, Eq. (7).

$$B=\bar{\dot{m}}_k\overleftrightarrow{\phi}_k. \qquad (29)$$

Let us introduce spatial averaged relationships for a single phase region.

$$\langle F\rangle=\int_{V_{ki}}F\, dV\Big/\int_{V_{ki}}dV \qquad (30)$$

$$\langle F\rangle_{ai}=\int_{A_{ki}}F\, dV\Big/\int_{A_{ki}}dV \qquad (31)$$

$$\langle F\rangle_{wj}=\int_{A_{kwj}}F\, dA\Big/\int_{A_{kwj}}dA. \qquad (32)$$

Substituting Eqs. (30), (31) and (32) into Eq. (26), we obtain the equation averaged for a single phase region.

$$\sum_i\Big[\frac{\partial}{\partial t}\alpha_{ki}\,\langle\bar{\rho}_k\overleftrightarrow{\phi}_k\rangle+\nabla\cdot\alpha_{ki}\,\langle\overleftrightarrow{\rho_k\phi\vec{v}_k}\rangle$$
$$+\nabla\cdot\alpha_{ki}\,\langle\bar{\vec{J}}_k+\bar{\vec{J}}^{t}_k\rangle$$
$$-\alpha_{ki}\,\langle\bar{\rho}_k\overleftrightarrow{S}_k\rangle+\frac{1}{L_{ai}}\,(\langle\bar{\dot{m}}\overleftrightarrow{\phi}_k\rangle_{ai}$$
$$+\langle\vec{n}_k\cdot\bar{\vec{J}}_k\rangle_{ai})+\frac{1}{L_{wj}}\langle\vec{n}_k\bar{\vec{J}}_k\rangle_{wj}\Big]=0, \qquad (33)$$

where

$$\alpha_{ki}=\frac{V_{ki}}{V_f},\quad \frac{1}{L_{ai}}=\frac{A_{ki}}{V_f},\quad \frac{1}{L_{wj}}=\frac{A_{kwj}}{V_f}. \qquad (34)$$

Then Eq. (33) is averaged over the total volume, or over the total surface, of k-th phase in the volume V_f by using the following relationships:

$$\langle\!\langle F\rangle\!\rangle=\frac{\sum_i\alpha_{ki}\langle F\rangle}{\sum_i\alpha_{ki}}=\frac{\sum_i\alpha_{ki}\langle F\rangle}{\alpha_k}, \qquad (35)$$

$$\langle\!\langle F\rangle\!\rangle_a=\frac{\sum_i\frac{1}{L_{ai}}\langle F\rangle_{ai}}{\sum_i\frac{1}{L_{ai}}}=\frac{\sum_i\frac{1}{L_{ai}}\langle F\rangle_{ai}}{\frac{1}{L_{ak}}}, \qquad (36)$$

$$\langle\!\langle F\rangle\!\rangle_w=\frac{\sum_j\frac{1}{L_{wj}}\langle F\rangle_{wj}}{\sum_j\frac{1}{L_{wj}}}=\frac{\sum_j\frac{1}{L_{wj}}\langle F\rangle_{wj}}{\frac{1}{L_{wk}}}, \qquad (37)$$

where

$$\alpha_k=\sum_i\alpha_{ki},\quad \frac{1}{L_{ak}}=\frac{\sum_i A_{ki}}{V_f},\quad \frac{1}{L_{wk}}=\frac{\sum_j A_{kj}}{V_f}. \qquad (38)$$

Substituting Eqs. (35), (36), and (37) into Eq. (33), we have the general time-volume averaged equation.

$$\frac{\partial}{\partial t}\alpha_k\langle\!\langle\bar{\rho}_k\overleftrightarrow{\phi}_k\rangle\!\rangle+\nabla\cdot\alpha_k\langle\!\langle\bar{\rho}_k\overleftrightarrow{\phi}_k\overleftrightarrow{\vec{v}}_k\rangle\!\rangle$$
$$+\nabla\cdot\alpha_k\langle\!\langle\bar{\vec{J}}_k+\bar{\vec{J}}^{t}_k\rangle\!\rangle-\alpha_k\langle\!\langle\bar{\rho}_k\overleftrightarrow{S}_k\rangle\!\rangle$$
$$+\frac{1}{L_{ak}}\langle\!\langle\bar{\dot{m}}_k\overleftrightarrow{\phi}_k\rangle\!\rangle_a+\frac{1}{L_{ak}}\langle\!\langle n_k\cdot\bar{\vec{J}}_k\rangle\!\rangle_a$$
$$+\frac{1}{L_{wk}}\langle\!\langle n_k\cdot\bar{\vec{J}}_k\rangle\!\rangle_w=0. \qquad (39)$$

Let us obtain the time-volume averaged interfacial jump condition from Eq. (21). Integrating Eq. (21) over the interfacial surface area A_{ki}, we get the jump condition.

$$\sum_{k=1}^{2}\sum_i\int_{A_{ki}}(\bar{\dot{m}}_k\overleftrightarrow{\phi}_k+\vec{n}_k\cdot\bar{\vec{J}}_k)\,dA$$
$$=\sum_{k=1}^{2}\frac{1}{L_{ak}}\langle\!\langle\bar{\dot{m}}_k\overleftrightarrow{\phi}_k+\vec{n}_k\cdot\bar{\vec{J}}_k\rangle\!\rangle_a=0. \qquad (40)$$

5. Time-Volume Averaged Conservation Equations

The forms of the time-volume averaged conservation equations for mass, momentum, and energy of the k-th phase can be written by specifying Eq. (39) as follows:

Mass

In this case,

$$\phi_k=1,\ \vec{J}_k=0,\ S_k=0. \tag{41}$$

We have

$$\frac{\partial}{\partial t}\alpha_k\langle\!\langle\bar{\rho}_k\rangle\!\rangle+\nabla\cdot\alpha_k\langle\!\langle\bar{\rho}_k\overleftrightarrow{\vec{v}}_k\rangle\!\rangle=\Gamma_k' \tag{42}$$

where

$$\bar{\vec{J}}_k'=\overline{\rho_k\vec{v}_k'}'=0$$

and $$\Gamma_k=-\frac{1}{L_{ak}}\langle\!\langle\bar{\dot{m}}_k\rangle\!\rangle_a. \tag{43}$$

Γ_k is the time-volume averaged interfacial mass flow rate per unit volume of k-th phase.

Momentum

In this case,

$$\phi_k=\vec{v}_k,\ \bar{\bar{J}}_k=p_k\,\bar{\bar{I}}-\bar{\bar{\tau}}_k',\ S_k=\vec{F}_k. \tag{44}$$

We have

$$\begin{aligned}&\frac{\partial}{\partial t}\alpha_k\langle\!\langle\bar{\rho}_k\overleftrightarrow{\vec{v}}_k\rangle\!\rangle+\nabla\cdot\alpha_k\langle\!\langle\bar{\rho}_k\overleftrightarrow{\vec{v}}_k\overleftrightarrow{\vec{v}}_k\rangle\!\rangle+\nabla\alpha_k\langle\!\langle\bar{p}_k\rangle\!\rangle\\&-\nabla\cdot\alpha_k\langle\!\langle\bar{\bar{\tau}}_k\cdot\vec{n}_k\rangle\!\rangle-\alpha_k\langle\!\langle\bar{\rho}_k\overleftrightarrow{\vec{F}}_k\rangle\!\rangle-\nabla\cdot\alpha_k\langle\!\langle\bar{\bar{\tau}}_k^t\cdot\vec{n}_k\rangle\!\rangle\\&+\frac{1}{L_{ak}}\langle\!\langle\bar{\dot{m}}_k\overleftrightarrow{\vec{v}}_k\rangle\!\rangle_a+\frac{1}{L_{ak}}\langle\!\langle\bar{p}_{ki}\rangle\!\rangle_a\\&-\frac{1}{L_{ak}}\langle\!\langle\bar{\bar{\tau}}_k\cdot\vec{n}_k\rangle\!\rangle_a-\frac{1}{L_{wk}}\langle\!\langle\bar{\bar{\tau}}_k\cdot\vec{n}_k\rangle\!\rangle_w=0\end{aligned} \tag{45}$$

where

$$\bar{\bar{\tau}}_k^t=-\overline{\rho_k\vec{v}_k'\vec{v}_k'}. \tag{46}$$

and p_{ki} is the local interfacial pressure. The time-averaged local interfacial pressure $\bar{p}_{ki}$ can be divided into three terms as follows:

$$\bar{p}_{ki}=\langle\!\langle\bar{p}_k\rangle\!\rangle+\Delta p_{ki}+\Delta p'_{ki}, \tag{47}$$

where

$$\Delta p_{ki}=\langle\!\langle\bar{p}_{ki}\rangle\!\rangle_a-\langle\!\langle\bar{p}_k\rangle\!\rangle, \tag{48}$$

$$\Delta p'_{ki}=\bar{p}_{ki}-\langle\!\langle\bar{p}_{ki}\rangle\!\rangle_a. \tag{49}$$

and Δp_{ki} is the difference between the average interfacial and average phase pressures and $\Delta p'_{ki}$ is the difference between the local and average interfacial pressures.

Using Gauss' theorem, we obtain the following relationship:

$$\begin{aligned}\frac{1}{L_{ak}}\langle\!\langle\bar{p}_k\rangle\!\rangle_a=&-(\langle\!\langle\bar{p}_k\rangle\!\rangle+\Delta p_{ki})\nabla\cdot\alpha_k\\&+\frac{1}{L_{ak}}\langle\!\langle\Delta p'_{ki}\rangle\!\rangle_a.\end{aligned} \tag{50}$$

The second term on the right-hand side in Eq. (50) is the term which leads to the virtual mass for inviscid flows. Substituting Eq. (50) into Eq. (45), we obtain the momentum conserve rvation equation.

$$\begin{aligned}&\frac{\partial}{\partial t}\alpha_k\langle\!\langle\bar{\rho}_k\overleftrightarrow{\vec{v}}_k\rangle\!\rangle+\nabla\cdot\alpha_k\langle\!\langle\bar{\rho}_k\overleftrightarrow{\vec{v}}_k\vec{v}_k\rangle\!\rangle+\alpha_k\nabla\langle\!\langle\bar{p}_k\rangle\!\rangle\\&-\alpha_k\langle\!\langle\bar{\rho}_k\vec{F}_k\rangle\!\rangle-\nabla\cdot\alpha_k\langle\!\langle\bar{\bar{\tau}}_k\cdot\vec{n}_k\rangle\!\rangle-\nabla\cdot\alpha_k\langle\!\langle\bar{\bar{\tau}}_t^t\cdot\vec{n}_k\rangle\!\rangle\\&-\Delta p_{ki}\nabla\cdot\alpha_k\\&=-\frac{1}{L_{ak}}\langle\!\langle\bar{\dot{m}}_k\overleftrightarrow{\vec{v}}_k+\Delta p'_{ki}-\bar{\bar{\tau}}_k\cdot\vec{n}_k\rangle\!\rangle_a\\&+\frac{1}{L_{wk}}\langle\!\langle\bar{\bar{\tau}}_k\cdot\vec{n}_k\rangle\!\rangle_w.\end{aligned} \tag{51}$$

Energy

In this case,

$$\phi_k=E_k=e_k+\frac{v_k^2}{2},$$

$$\vec{J}_k=\vec{q}_k-(p_k\bar{\bar{I}}-\bar{\bar{\tau}})\cdot\vec{v}_k'$$

$$S_k=\vec{F}_k\cdot\vec{v}_k+Q_k, \tag{52}$$

where e_k and Q_k are the specific internal energy and volumetric internal energy generation rate.

In solving two-phase problems, it is often useful to separate the mechanical and thermal effects in the total energy equation. From dotting the local instant momentum equation derived by combining Eqs. (5) and (44) by the velocity of k-th phase $\vec{v}_k$, we have the mechanical energy balance equation.

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_k v_k^2\right)+\nabla\cdot\left(\frac{1}{2}p_k v_k^2\vec{v}_k+\nabla\cdot p_k\vec{v}_k\right.$$

$$-p_k\nabla\cdot\vec{v}_k+\nabla\cdot(\overline{\overline{\tau}}_k\cdot\vec{v}_k)-\overline{\overline{\tau}}_k:\nabla\vec{v}_k-\rho_k\vec{F}_k\cdot\vec{v}_k=0. \tag{53}$$

Using Eqs. (5) and (52), we have the local instant energy balance equation.

$$\frac{\partial}{\partial t}\rho_k\left(e_k+\frac{1}{2}v_k^2\right)+\nabla\cdot\rho_k\vec{v}_k\left(e_k+\frac{1}{2}v_k^2\right)$$
$$+\nabla\cdot\vec{q}_k+\nabla\cdot(p_k\vec{v}_k)+\nabla\cdot(\bar{\tau}_k\cdot\bar{v}_k)$$
$$-\rho_k(\vec{F}_k\cdot\vec{v}_k+Q_k)=0. \tag{54}$$

Subtracting Eq. (53) from Eq. (54), we can obtain the internal energy equation.

$$\frac{\partial}{\partial t}\rho_k e_k+\nabla\cdot\rho_k e_k\vec{v}_k+\nabla\cdot\vec{q}_k+p_k\nabla\cdot\vec{v}_k$$
$$+\overline{\overline{\tau}}_k:\nabla\vec{v}_k-\rho_k Q_k=0. \tag{55}$$

If we compare the equations of mechanical and thermal energy, Eqs. (53) and (55), we find out that $(p_k\nabla\cdot\vec{v}_k)$ and $(\bar{\tau}_k:\nabla\vec{v}_k)$ are common to both equations and that they appear with opposite signs in the two equations. Therefore, these terms describe the interconversion of mechanical and thermal energy. The term can be either positive or negative, depending on whether the fluid is expanding and contracting. As a result, it represents a reversible mode of interchange. Using the continuity equation, we have

$$p_k\Delta\cdot\vec{v}_k=-p_k\frac{1}{\rho_k}\left[\frac{\partial\rho_k}{\partial t}+\vec{v}_k\cdot\nabla\rho_k\right]$$
$$=-p_k\frac{1}{\rho_k}\frac{D\rho_k}{Dt}. \tag{56}$$

For a fluid of constant density, the term $(p_k\nabla\cdot\vec{v}_k)$ becomes zero. On the othere hand, the term $(-\overline{\overline{\tau}}_k:\nabla\vec{v}_k)$ is always positive and therefore represents an irreversible degradation of mechanical to thermal.

Let us get the time averaged internal energy from Eq. (55).

$$\frac{\partial}{\partial t}\overline{\rho_k e_k}+\nabla\cdot\overline{\rho_k e_k\vec{v}_k}+\nabla\cdot\overline{\vec{q}}_k+\overline{p_k\nabla\cdot\vec{v}}_k+\overline{\overline{\tau}_k:\nabla\vec{v}}_k$$
$$-\overline{\rho_k Q_k}=0. \tag{57}$$

Using Eqs. (16), (17), and (18), we can obtain the time averaged internal energy equation.

$$\frac{\partial}{\partial t}\overline{\rho}\overline{\overleftrightarrow{e_k}}+\nabla\cdot(\overline{\rho}\overline{e_k}\overline{\overleftrightarrow{v}_k})+\nabla\cdot(\overline{\vec{q}}_k+\overline{\vec{q}}_k^{\,t})$$
$$+\overline{p_k\nabla\cdot\vec{v}}_k+\overline{\overline{\tau}:\nabla\vec{v}}_k=0,$$

where

$$\vec{q}_k^{\,t}=\overline{\rho_k e_k''\vec{v}_k''}. \tag{58}$$

The time-volume averaged internal energy equation can be obtained by integrating Eq. (58) over the flow vlume V_f. In order to do it, especially, the term $(\overline{p_k\nabla\cdot\vec{v}_k})$ requires a special manipulation as follows. The local instant pres sure can be divided into two components: time-averaged local pressure and local time fluctuation of pressure.

$$p_k=\bar{p}_k+p_k''. \tag{59}$$

Then $(\overline{p_k\nabla\cdot\vec{v}_k})$ becomes

$$\overline{p_k\nabla\cdot\vec{v}_k}=\overline{\bar{p}_k}\nabla\cdot\bar{\vec{v}}_k+\overline{p_k''\nabla\cdot\vec{v}_k}. \tag{60}$$

Let us integrate Eq. (60) over the flow volume V_f. Then, we have

$$\sum_{i=1}^{I}\frac{1}{V_f}\int_{V_{ki}}\overline{p_k\nabla\cdot\vec{v}_k}dV$$
$$=\sum_i\frac{1}{V_f}\int_{V_{ki}}(\bar{p}_k\nabla\cdot\bar{\vec{v}}_k+\overline{p_k''\nabla\cdot\vec{v}_k})dV. \tag{61}$$

Also the time averaged local pressure can be divided into two components: time-space averaged pressure and time averaged space fluctuation of pressure.

$$\bar{p}_k=\langle\!\langle\bar{p}_k\rangle\!\rangle+p_k''. \tag{62}$$

Substituting Eq. (62) into Eq. (61), Eq. (61) becomes

$$\sum_i\frac{1}{V_f}\left[\langle\!\langle\bar{p}_k\rangle\!\rangle\int_{V_{ki}}\Delta\cdot\bar{\vec{v}}_k dV\right.$$
$$\left.+\int_{V_{ki}}(p_k''\nabla\cdot\bar{\vec{v}}_k+\overline{p_k''\nabla\cdot\vec{v}})\,dV.\right] \tag{63}$$

Using Eq. (25), the first term in Eq. (63) is expressed as follows:

$$\frac{1}{V_f}\langle\!\langle\bar{p}_k\rangle\!\rangle\int_{V_{ki}}\nabla\cdot\bar{\vec{v}}_k\,dV$$
$$=\langle\!\langle\bar{p}_k\rangle\!\rangle\nabla\cdot\frac{1}{V_f}\int_{V_{ik}}\bar{\vec{v}}_k dV$$
$$+\langle\!\langle\bar{p}_k\rangle\!\rangle\frac{1}{V_f}\int_{A_{ki}}\vec{n}_k\cdot\bar{\vec{v}}_k\,dA. \tag{64}$$

Then, taking the procedure described in Section 4 and assuming that $\bar{\vec{v}}_k$ is equal to $\overleftrightarrow{\vec{v}}_k$, we have

$$\sum_i \frac{1}{V_f}\int_{V_{ki}} \overline{p_k \nabla\cdot\vec{v}_k}\,dV = \langle\!\langle \bar{p}_k\rangle\!\rangle \nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\vec{v}}_k\rangle\!\rangle + \langle\!\langle \bar{p}_k\rangle\!\rangle \frac{1}{L_{ak}}\langle\!\langle \vec{n}_k\cdot\overleftrightarrow{\vec{v}}_k\rangle\!\rangle_a + \alpha_k\langle\!\langle \overline{p_k'^s\nabla\cdot\vec{v}_k} + \overline{p_k'^t\nabla\cdot\vec{v}_k}\rangle\!\rangle. \quad (65)$$

Applying the same procedures for other terms in Eq. (58), we have the time-volume averaged internal energy equation.

$$\frac{\partial}{\partial t}\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k e_k}\rangle\!\rangle + \nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k e_k}\overleftrightarrow{\vec{v}}_k\rangle\!\rangle + \nabla\cdot\alpha_k\langle\!\langle\overline{\vec{q}} + \overline{\vec{q}'}_k\rangle\!\rangle + \langle\!\langle \bar{p}_k\rangle\!\rangle\nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\vec{v}}_k\rangle\!\rangle + \langle\!\langle \bar{p}_k\rangle\!\rangle\frac{1}{L_{ak}}\langle\!\langle \vec{n}_k\cdot\overleftrightarrow{\vec{v}}_k\rangle\!\rangle + \alpha_k\langle\!\langle\overline{\bar{\tau}_k:\nabla_k\vec{v}}\rangle\!\rangle + \alpha_k\langle\!\langle \overline{p_k'^s\nabla\cdot\overleftrightarrow{\vec{v}}_k} + \overline{p_k'^t\nabla\cdot\vec{v}_k}\rangle\!\rangle = -\frac{1}{L_{ak}}\langle\!\langle\overleftrightarrow{\dot{\bar{m}}_k\bar{e}}_k + \vec{n}_k\cdot\overline{\vec{q}}_k\rangle\!\rangle_a - \frac{1}{L_{wk}}\langle\!\langle \vec{n}_k\cdot\overline{\vec{q}}_k\rangle\!\rangle_w. \quad (66)$$

The term $(\dot{m}_k e_k)$ can be expressed in terms of enthalpy, h. The enthalpy is defined as followos:

$$h = e + \frac{p}{\rho}. \quad (67)$$

Using the definition $\dot{m}_k = \rho_k(\vec{v}_k - \vec{v}_i)\cdot\vec{n}_k$, we have

$$\dot{m}_k\frac{p_k}{\rho_k} = p_k(\vec{v}_k - \vec{v}_i)\cdot\vec{n}_k. \quad (68)$$

By Eq. (68), we have

$$\langle\!\langle\overleftrightarrow{\dot{\bar{m}}_k e_k}\rangle\!\rangle = \langle\!\langle\overleftrightarrow{\dot{\bar{m}}_k h_k}\rangle\!\rangle - \langle\!\langle \bar{p}_k(\overleftrightarrow{\vec{v}}_k - \vec{v}_i)\cdot\vec{n}_k\rangle\!\rangle. \quad (69)$$

Using the definition Eq. (62), we have

$$\langle\!\langle \bar{p}_k(\overleftrightarrow{\vec{v}}_k - \vec{v}_i)\cdot\vec{n}_k\rangle\!\rangle = \langle\!\langle \bar{p}_k\rangle\!\rangle\langle\!\langle(\overleftrightarrow{\vec{v}}_k - \vec{v}_i)\cdot\vec{n}_k\rangle\!\rangle + \langle\!\langle p_k'^s(\overleftrightarrow{\vec{v}}_k - \vec{v}_i)\cdot\vec{n}_k\rangle\!\rangle. \quad (70)$$

Combining the second term in Eq. (65) and the first term in Eq. (70), then we obtain

$$\frac{1}{L_{ak}}[\langle\!\langle \bar{p}_k\rangle\!\rangle\langle\!\langle \vec{n}_k\cdot\overleftrightarrow{\vec{v}}_k\rangle\!\rangle - \langle\!\langle \bar{p}_k\rangle\!\rangle\langle\!\langle \vec{n}_k\cdot(\overleftrightarrow{\vec{v}}_k - \vec{v}_i)\rangle\!\rangle] = \frac{1}{L_{ak}}\langle\!\langle \bar{p}_k\rangle\!\rangle\langle\!\langle \vec{v}_i\cdot\vec{n}_k\rangle\!\rangle. \quad (71)$$

Applying Leibnitz rule to Eq. (71), Eq. (71) becomes

$$\frac{1}{L_{ak}}\langle\!\langle \bar{p}_k\rangle\!\rangle\langle\!\langle \vec{v}_i\cdot\vec{n}_k\rangle\!\rangle = \langle\!\langle \bar{p}_k\rangle\!\rangle\frac{\partial\alpha_k}{\partial t} \quad (72)$$

Then we obtain the final internal energy equation:

$$\frac{\partial}{\partial t}\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k e_k}\rangle\!\rangle + \nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k e_k}\overleftrightarrow{\vec{v}}_k\rangle\!\rangle + \nabla\cdot\alpha_k\langle\!\langle\overline{\vec{q}}_k + \overline{\vec{q}'}_k\rangle\!\rangle + \langle\!\langle \bar{p}_k\rangle\!\rangle\cdot\nabla\alpha_k\langle\!\langle\overleftrightarrow{\vec{v}}_k\rangle\!\rangle + \langle\!\langle \bar{p}_k\rangle\!\rangle\frac{\partial\alpha_k}{\partial t} + \alpha_k\langle\!\langle\overline{\bar{\tau}_k:\nabla\vec{v}_k}\rangle\!\rangle + \alpha_k\langle\!\langle \overline{p_k'^s\nabla\cdot\vec{v}_k} + \overline{p_k'^t\nabla\cdot\vec{v}_k}\rangle\!\rangle = -\frac{1}{L_{ak}}\langle\!\langle\overleftrightarrow{\dot{\bar{m}}_k\bar{h}}_k + \vec{n}_k\cdot\overline{\vec{q}}_k\rangle\!\rangle_a - \frac{1}{L_{wk}}\langle\!\langle \vec{n}\cdot\overline{\vec{q}}_k\rangle\!\rangle_w. \quad (73)$$

Using the definition of enthalpy, we can obtain the enthalpy energy equation from Eq. (73). The first and second terms on the left-hand side in Eq. (73) become

$$\frac{\partial}{\partial t}\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k e_k}\rangle\!\rangle = \frac{\partial}{\partial t}\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k h_k}\rangle\!\rangle - \frac{\partial}{\partial t}\alpha_k\langle\!\langle \bar{p}_k\rangle\!\rangle \quad (74)$$

and $\nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k e_k}\overleftrightarrow{\vec{v}}_k\rangle\!\rangle$

$$= \nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k h_k}\overleftrightarrow{\vec{v}}_k\rangle\!\rangle - \nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\bar{p}_k\vec{v}}_k\rangle\!\rangle = \nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k h_k}\overleftrightarrow{\vec{v}}_k\rangle\!\rangle - \langle\!\langle \bar{p}_k\rangle\!\rangle\nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\vec{v}}_k\rangle\!\rangle - \langle\!\langle\overleftrightarrow{\vec{v}}_k\rangle\!\rangle\cdot\nabla\alpha_k\langle\!\langle \bar{p}_k\rangle\!\rangle - \nabla\cdot\alpha_k\langle\!\langle p_k'^s\overleftrightarrow{\vec{v}}_k\rangle\!\rangle. \quad (75)$$

Then, we have the final enthalpy energy equation

$$\frac{\partial}{\partial t}\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k h_k}\rangle\!\rangle + \nabla\cdot\alpha_k\langle\!\langle\overleftrightarrow{\bar{\rho}_k h_k}\overleftrightarrow{\vec{v}}_k\rangle\!\rangle + \nabla\cdot\alpha_k\langle\!\langle\overline{\vec{q}}_k + \overline{\vec{q}'}_k\rangle\!\rangle - \langle\!\langle\overleftrightarrow{\vec{v}}_k\rangle\!\rangle\cdot\nabla\alpha_k\langle\!\langle \bar{p}_k\rangle\!\rangle - \alpha_k\frac{\partial}{\partial t}\langle\!\langle \bar{p}_k\rangle\!\rangle + \alpha_k\langle\!\langle\overline{\bar{\tau}_k:\nabla\vec{v}_k}\rangle\!\rangle - \nabla\cdot\alpha_k\langle\!\langle p_k'^s\overleftrightarrow{\vec{v}}_k\rangle\!\rangle + \alpha_k\langle\!\langle p_k'^s\nabla\cdot\overleftrightarrow{\vec{v}}_k + \overline{p_k'^t\nabla\cdot\vec{v}}_k\rangle\!\rangle = -\frac{1}{L_{ak}}\langle\!\langle\overleftrightarrow{\dot{\bar{m}}_k\bar{h}}_k + \vec{n}_k\cdot\overline{\vec{q}}_k\rangle\!\rangle_a - \frac{1}{L_{wk}}\langle\!\langle n_k\cdot\overline{\vec{q}}_k\rangle\!\rangle_w. \quad (76)$$

6. Interfacial Jump Condition

From Eq. (40) we can obtain the interfacial jump conditions.

Mass

In this case.

$\psi_k=1,\ J_k=0,$

we have

$$\sum_k \Gamma_k=0. \tag{77}$$

Momentum

In this case,

$$\psi_k=\vec{v}_k',\quad \overline{\overline{J_k}}=p_k\overline{\overline{I}}-\overline{\overline{\tau_k}},$$

we have

$$\sum_{k=1}^{2}\frac{1}{L_{ak}}\langle\!\langle \overline{\dot{m}_k\overleftrightarrow{\vec{v}}_k}+\vec{n}_k(\bar{p}_k\overline{\overline{I}}-\overline{\overline{\tau_k}})\rangle\!\rangle_a=0. \tag{78}$$

Energy

In this case,

$$\psi_k=E_k=e_k+\frac{v_k^2}{2},\quad \vec{J}_k=\vec{q}_k-(p_k\overline{\overline{I}}-\overline{\overline{\tau_k}})\cdot\vec{v}_k,$$

We have

$$\sum_{k=1}^{2}\frac{1}{L_{ak}}\langle\!\langle \overline{\dot{m}_k\overleftrightarrow{E}_k}+\vec{n}_k\cdot[\overleftrightarrow{q_k}-(\overline{p_k\overline{\overline{I}}-\overline{\overline{\tau_k}})\cdot\vec{v}_k}]\rangle\!\rangle_a$$

$$=0. \tag{79}$$

The equation can be written in enthalpy form

$$\sum_{k=1}^{2}\frac{1}{L_{ak}}\langle\!\langle \overline{\dot{m}_k\overleftrightarrow{h}_k}+\vec{n}_k\cdot(\overline{\vec{q}_k}-\bar{p}_k\cdot\vec{v}_i+\overline{\overline{\tau_k}\cdot\vec{v}_k})\rangle\!\rangle_a$$

$$=0. \tag{80}$$

7. Comparison

Ishii (3) derived time average equations by averaging the local equations over a time interva Δt in which several single phase regions with singular interfaces pass through a reference point. Therefore, he was forced to use limiting forms about the Leibniz rule derived by Delhaye and Achard (6), which lead to a question about the validity about their derivation and an ambiguous physical interpretation of terms describing interfacial transfer phenomena.

These difficulties mainly come from their averaging the local equations over a time interval in which several single phase with singular interface are allowed to pass through a reference point.

Banerjee and Chan (2) derived one dimensional volume averaged equations by averaging the local equations over a fluid volume V_f which consists of several single phase regions. As their equations are not averaged over a time interval, turbulent contribution terms do not appear in the conservation equations. Except for these turbulent terms and the use of instantaneous properties, their Equations are the same as the equations derived here.

Let us compare the time-averaged conservation equations, Eqs. (42), (51), and (73) with the conservation equations of the THERMIT-6S(7). The following equations are used in the THERMIT-6S.

Mass

$$\frac{\partial}{\partial t}(\alpha_k\rho_k)+\nabla\cdot(\alpha_k\rho_k\vec{v}_k)=\Gamma_k \tag{88}$$

Momentum

$$\frac{\partial}{\partial t}\alpha_k\rho_k\vec{v}_k+\nabla\cdot(\alpha_k\rho_k\vec{v}_k\vec{v}_k)+\alpha_k\nabla p_k$$

$$=\Gamma_k\vec{v}_i-\vec{F}_{sk}-\vec{F}_{vk}-\vec{F}_{wk}+\alpha_k\rho_k\vec{g} \tag{89}$$

Energy

$$\frac{\partial}{\partial t}\alpha_k\rho_k e_k+\nabla\cdot(\alpha_k\rho_k e_k\vec{v}_k)+p\nabla\cdot(\alpha_k\vec{v}_k)$$

$$+p\frac{\partial\alpha_k}{\partial t}=Q_{ik}+Q_{wk}+Q_{kk}, \tag{90}$$

where

$\vec{F}_{sk}$, $\vec{F}_{vk}$, and Q_{ik}: interfacial standard drag force and virtual mass force, energy exchange rate, per unit volume,respectively.

F_{wk} and Q_{wk}: wall friction force and wall heat transfer rate, per unit volume, respectively,

Q_{ll}: liquid energy conduction rate per unit volume,

$\vec{g}$: gravitational force.

Note that Q_{vv} is put to be zero for the vapor energy equation of the THERMIT-6S. The relationships between Eq. (89) and Eq. (51), and between Eq. (90) and Eq. (73) are as follows;

$$v_i=\langle\!\langle\overleftrightarrow{v}\rangle\!\rangle_a',\quad \vec{F}_k=\rho_k\vec{g}_k$$

$$\vec{F}_{sk}=-\frac{1}{L_{ak}}\langle\langle\vec{n}_k\cdot\overline{\overline{\tau}}_k\rangle\rangle_a$$

$$\vec{F}_{vk}=\frac{1}{L_{ak}}-\langle\langle\Delta p'_{ki}\rangle\rangle_a$$

$$F_{wk}=-\frac{1}{L_{wk}}\langle\langle\vec{n}_k\cdot\overline{\overline{\tau}}_k\rangle\rangle_w$$

$$Q_{ik}=-\frac{1}{L_{ak}}\langle\langle\overleftrightarrow{\dot{m}_k}\overleftrightarrow{h}_k+\vec{n}_k\cdot\overline{\vec{q}}_k\rangle\rangle_a$$

$$Q_{wk}=-\frac{1}{L_{wk}}\langle\langle n_k\cdot\overline{\vec{q}}_k\rangle\rangle_w$$

$$Q_{ll}=-\nabla\cdot\alpha_l(\vec{q}_l+\vec{q}_l{}^t).$$

Implicitly each property in the equations of the THERMIT-6S can be considered to be averaged in time and volume. Therefore, the THERMIT-6S neglects the fluctuating terms from the nonuniform spatial distribution of properties. Properties can be expressed as the um of a spatial average and spatial fluctuating component:

$$\bar{\rho}_k=\langle\langle\bar{\rho}_k\rangle\rangle+\rho_k{}'^s \text{ and } \overleftrightarrow{\psi}_k=\langle\langle\overleftrightarrow{\psi}_k\rangle\rangle+\psi_k{}'^s.$$

Then, $\langle\langle\rho_k\overleftrightarrow{\psi}_k\rangle\rangle$ and $\langle\langle\bar{\rho}_k\overleftrightarrow{\psi}_k v_k\rangle\rangle$ become

$$\langle\langle\rho_k\overleftrightarrow{\psi}_k\rangle\rangle=\langle\langle\rho_k\rangle\rangle\langle\langle\overleftrightarrow{\psi}_k\rangle\rangle+\langle\langle\rho_k{}'^s\psi_k{}'^s\rangle\rangle$$

$$\langle\langle\rho_k\overleftrightarrow{\psi}_k\overleftrightarrow{v}_k\rangle\rangle=\langle\langle\bar{\rho}_k\rangle\rangle\langle\langle\overleftrightarrow{\psi}_k\rangle\rangle\langle\langle\overleftrightarrow{v}_k\rangle\rangle+\langle\langle\bar{\rho}_k\rangle\rangle\langle\langle\psi_k{}'^s v_k{}'^s\rangle\rangle$$
$$+\langle\langle\overleftrightarrow{\psi}_k\rangle\rangle\langle\langle\rho_k{}'^s v_k{}'^s\rangle\rangle+\langle\langle\overleftrightarrow{v}_k\rangle\rangle\langle\langle\rho_k{}'^s\psi_k{}'^s\rangle\rangle$$
$$+\langle\langle\rho_k{}'^s\psi_k{}'^s v_k{}'^s\rangle\rangle.$$

An adequate assessment should be given for all terms in the equations by assessing their comparative magnitudes and thereby justify the neglect of these spatial fluctuating terms.

In addition to neglecting the distribution terms, in the momentum equation of the THERMIT-6S, several assumptions are made:

1) Negligible turbulent contribution term, $\nabla\cdot\alpha_k\langle\langle\overline{\overline{\tau}}_k{}^t\cdot\vec{n}_k\rangle\rangle$

2) Negligible phase shear force, $\nabla\cdot\alpha_k\langle\langle\overline{\overline{\tau}}_k\cdot\vec{n}_k\rangle\rangle$

3) The average interfacial pressure $\langle\langle\bar{p}_{ki}\rangle\rangle_a$ equals the phase pressure $\langle\langle\bar{p}_k\rangle\rangle$. As a result,we have

$$\Delta p_{ki}=0.$$

Similarily, beside the omission of the distribution terms the following assumptions are made in the internal energy equation of the THERMIT-6S:

1) Negligible irreversible conversion to internal energy, $\langle\langle\overline{\overline{\tau}_k}:\Delta\overline{\vec{v}}_k\rangle\rangle$

2) Negligible vapor energy conduction rate, $\nabla\cdot\alpha\langle\langle\overline{\vec{q}}_v+\overline{\vec{q}}_v{}^t\rangle\rangle$.

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