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An Effect of Drift Current on Generation Stage of Wind Waves

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風波發生에 있어서 吹送流의 影響

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Abstract: Effect of drift current on the first stage of wave generation by wind is studied theoretically. The viewpoint is similar to the one described by Phillips (1957) except that drift current is considered. It is found that inclusion of the effect of the drift current modifies significantly the results obtained by Phillips, particularly the resonance condition and wave spectrum.

要約: 풍파발생초기 단계에서 바람에 의해 형성된 취송류가 초기단계의 파의특성을 결정하는데 미치는 영향을 이론적으로 다루었다. 본 연구의 관점은 Phillips(1957)의 풍파발생이론의 관점과 유사하다. 취송류를 고려하면 Phillips가 구한 결과를 상당히 변경시키는 것으로 판명되였다. 특히 공명조건에 있어서 취송류가 큰 역활을 하고 있다.

INTRODUCTION

On a calm surface of water at rest, a sudden blow of wind produces strong drift current near the air-water interface and waves on the surface. In wind-wave tunnel where the fetch is limited, the surface is divided generally into three zones with fetch value increasing when statistically stationary state is reached; the first zone where there is only drift current, the second one where waves develope in narrow streaks and the last one where streaks meet one another to form rodinary three dimensional waves (Choi 1977). Since at first only drift current is developed at the lower fetch value and then followed by the formation of waves as fetch increases, it seems that the drift current may have an important effect on the wave formation.

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Until now many authors(e.g., Kawaii 1979, Valenzuela 1976, Stern and Adam 1973) have paid considerable attention to the drift current in explaining the physical mechanism of wind-wave generation, particularly in the viewpoint of the hydrodynamic instability of air and water flows. In this paper, an attempt is made to gain an information on the role of the drift current in the first stage of wave generation by wind from the viewpoint of Phillips' resonance mechanism. The primary objective of this paper is to see the differences between the results derived by Phillips (1957), particularly the resonance condition and the wave spectrum of initial stage of wave generation by wind, and those obtained when the drift current is considered. For mathematical simplicity the drift current is assumed to decrease linearly with depth.

FORMULATION OF THE PROBLEM

Schematic representation of the physical situation is given in Figure 1. Fluid is assumed to beincompressible and is of a layer of finite depth d. Viscous effect will be neglected under the assumption that Reynolds number concerned is sufficiently large. Wave motions are taken to be two dimensional. Positive x is in the direction of wind and positive y is normal to it and upwards direction. Basic governing equations are the continuity and the momentum equations. The physical quantities concerned such as velocity and pressure are decomposed into the mean value and the fluctuating one associated with the wave motion. The basic equations are linearized by traditional scheme.

The continuity equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

where u and v are x and y component of fluctuating velocity, respectively. The momentum equations can be put as

$$\begin{aligned} \frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}+v\frac{\partial U}{\partial y}&=-\frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}&=-\frac{1}{\rho}\frac{\partial p}{\partial y} \end{aligned} \tag{2}$$

where U, p and ρ are the mean drift current in x direction, the fluctuating pressure term and the density of water, respectively. Equation (1) enables us to use the stream function Φ defined by

$$u=-\frac{\partial\Phi}{\partial y} \quad v=\frac{\partial\Phi}{\partial x} \tag{3}$$

Since in this paper the aim is to see the effect of the drift current on the generation of wind waves, surface pressure and tangential stress fluctnations acting on the surface must be considered. In the following analysis the notation employed by Phillips (1977) will be used for convenience. The Fourier component of the surface pressure and the shear stress variations are

$$d\hat{w}=\langle d\hat{w}(k)\rangle+dw(k,t) \tag{4}$$

$$d\hat{\tau}=\langle d\hat{\tau}(k)\rangle+d\tau(k,t) \tag{5}$$

where $\langle d\hat{w}\rangle$ and $\langle d\hat{\tau}\rangle$ represent terms correlated with wave form. dw and $d\tau$ are randomly varying component arising from the turbulent wind, and k and t are wave number and time, respectively. The first terms on the right-hand side of equations (4) and (5) can be put as

$$\langle d\hat{w}\rangle=(\nu_1+i\mu_1)\rho c^2 k dA \tag{6}$$

$$\langle d\hat{\tau}\rangle=(\nu_2+i\mu_2)\rho c^2 k dA \tag{7}$$

where dA is the amplitude of an wave component, ν_1, ν_2, μ_1, and μ_2 are coupling coefficients and c is the phase velocity of waves with wave number k. Thus normal stress can be put equal to the sum of surface pressure and tangential stress advanced in phase by $\pi/2$. The Fourier component of p in turbulent wind acting on the surface is

$$p=(\nu+i\mu)\rho c^2 k dA+dw'(k,t) \tag{8}$$

where $\nu=\nu_1+\nu_2$, $\mu=\mu_1+\mu_2$ and $dw'=dw+id\tau$.

When the normal stress p of equation (8) is considered, the equations governing the surface elevationcan be obtained form equations (1), (2) and (3) together with kinematic boundary condition at the surface

$$\frac{\partial\eta}{\partial t}+U_0\frac{\partial\eta}{\partial x}=v_0 \tag{9}$$

where η is the surface elevation. η is related to dA by Fourier-Stieltjes integral

$$\eta=\int_k dA(k,t)e^{ikx}$$

Following the same procedure described by Choi (1982), one obtains

$$\left(\frac{d}{dt}+ikU_0\right)^2 dA-ik\frac{\langle\frac{dU}{dy}\int\phi dy\rangle}{\langle\phi\rangle}\cdot \cdot\left(\frac{d}{dt}+ikU_0\right)dA+\left\{\frac{\phi_0}{\langle\phi\rangle}(\gamma k^3+gk) +\nu c^2k^2+i\mu c^2k^2\right\}dA=-\frac{k}{\rho}dw' \tag{10}$$

with initial conditions

$$dA=0,\ \ d\dot{A}=0 \text{ at } t=0 \tag{11}$$

where $\langle\ \rangle=\int_{-d}^{0}dy$ and '·' designates $\frac{d}{dt}$ and $\gamma=\frac{T}{p}$ in which T is the surface tension of water and g is the gravitational acceleration. ϕ is related to the stream function Φ in terms of Fourier-Stieltjes integral

$$\Phi=\int_k \phi(k,y)a(k,t)e^{ikx} \tag{12}$$

where a is the time dependent part of the stream function. ϕ is found to satisfy

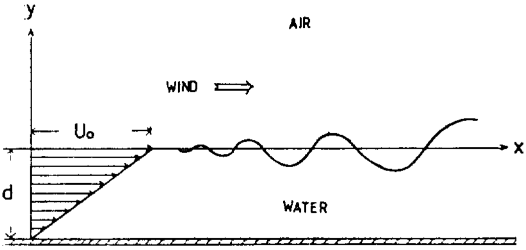
$$(U-c)\ \frac{d^2}{dy^2}-k^2\ \phi-\phi\ \frac{d^2U}{dy^2}=0 \tag{13}$$

where

$$c=-\frac{1}{ik}\ \frac{1}{a}\ \frac{da}{dt}$$

with boundary condition

$$\phi=0 \text{ at } y=-d \tag{14}$$



ANALYSIS

As pointed out in introduction, for mathematical simplicity the drift current will be assumed to be decreasing linearly with depth:

$$U=\frac{U_0}{d}(y+d) \tag{15}$$

Introducing equation (15) into equation (13), one obtains

$$\left(\frac{d^2}{dy^2}-k^2\right)\phi=0 \tag{16}$$

whose solution satisfying boundary condition (14) is

$$\phi=A\ \sinh\ k(y+d)$$

where A is an arbitrary constant. After a little manipulation equation (10) is found to be

$$\frac{d}{dt}+ikU_0\ {}^2dA$$
$$-i\frac{U_0}{d}\ S\ \frac{d}{dt}+ikU_0\ dA$$
$$+S(kg+\gamma k^3)dA+\nu c^2k^2dA$$
$$+i\mu c^2k^2dA=-\frac{k}{\rho}dw' \tag{18}$$

where

$$S=\sinh\ kd/(\cosh\ kd\text{-}1)$$

It should be noted that, when the drift current can be ignored and d goes to infinite, equation (18) becomes exactly the same as one obtained by Phillips.

When $|\nu|$ and $|\mu|$ are much smaller than unity, equation (18) has a solution satisfying the initial condition (11) in the form of

$$dA(k,t)=-\frac{k}{\rho\left(\mu\frac{M^2}{N}-2iN\right)}\cdot$$
$$\cdot\int_0^t dw'(\tau)\left[e^{r(t-\tau)}-e^{r_1(t-\tau)}\right]d\tau \tag{19}$$

where

$$r_1=-ikU_0+i\frac{U_0}{2d}S+iN\left(1+i\frac{1}{2}\ \mu\ \frac{M^2}{N^2}\right)$$
$$r_2=-ikU_0+i\frac{U_0}{2d}S-iN\left(1+i\frac{1}{2}\ \mu\ \frac{M^2}{N^2}\right)$$
$$N^2=\left(\frac{U_0}{2d}S\right)^2+S(kg+\gamma k^3)$$
$$M^2=\frac{1}{4}c^2k^2.$$

Equation (19) gives surface dislacement spectrum ϕ

$$\phi(k,t)=\frac{\overline{dA(k,t)dA^*(k,t)}}{dk}$$
$$=\frac{k^2}{\rho^2}\ \frac{1}{N^2\left(1-\frac{1}{4}\mu^2\frac{M^4}{N^4}\right)}\cdot$$
$$\cdot\int_0^t\int_0^t \Omega'(k,\tau'-\tau'')$$
$$\left[e^{r_2(t-\tau')}-e^{r_1(t-\tau')}\right]$$
$$\left[e^{r_2*(t-\tau'')}-e^{r_1*(t-\tau'')}\right]d\tau''d\tau' \tag{20}$$

where

$$\Omega'(k,t)=\frac{1}{(2\pi)^{-2}}$$
$$\int p(x,t_0)p(x+r,t_0+t)e^{-ikr}dr \tag{21}$$

and '*' means complex conjugate. When $t\gg$Ⓗ(k) and $Nt\gg 1$, where Ⓗ(k) is the integral time scale of the pressure components with wave number K measured in the frame of reference moving with convection velocity $U_c(k)$ of the turbulent pressure pattern, the wave spectrum (20) can be written as

$$\phi(k,t)=\frac{1}{2}\ \frac{k^2}{\rho^2N^2}\frac{\sinh\mu\frac{M^2}{N}t}{\mu\frac{M^2}{N}}\cdot$$

$$\cdot \int_{-\infty}^{\infty} [\Omega'(k,\tau) e^{i(kU_0 - U_0 S/2d + N)\tau} + \Omega'(k,\tau) e^{i(kU_0 - U_0 S/2d - N)\tau}] d\tau. \quad (22)$$

By defining wave number-frequency spectrum $\Pi(k,n)$ as the Fourier transform of $\Omega(k,t)$ with respect to time t:

$$\Pi(k,n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Omega(k,t) e^{int} dt \quad (23)$$

equation (22) can be written as

$$\phi(k,t) = \frac{1}{2} \frac{\pi k^2}{\rho^2 N^2} \frac{\sinh \mu \frac{M^2}{N} t}{\mu \frac{M^2}{N}} \cdot \quad (24)$$

$$\cdot \left[\Pi'(k, kU_0 - \frac{U_0}{2d} S + N) + \Pi' k, kU_0 - \frac{U_0}{2d} S - N) \right].$$

It should be noted that the first term on the right-hand side of equation (24) is the wave spectrum associated with the waves propagating in the direction of the drift current, i.e. in the direction of wind and the second one is the spectrum of the waves travelling in the opposite direction of wind. When t is sufficiently small, equation (24) becomes

$$\phi(k,t) = \frac{\pi k^2}{2\rho^2 N^2} \left[\Pi'(k, kU_0 - \frac{U_0}{2d} S + N) + (\Pi' k, kU_0 - \frac{U_0}{2d} S - N) \right] t \quad (25)$$

The wave number-frequency spectrum defined in equation (23) is maximum for $n = kU_c$, if the turbulent pressure patterns were convected rigidly in the direction of positive x with velocity U_c. Therefore, the wave spectrum $\phi(k,t)$ given in equation (25) has a maximum value when either

$$kU_c = kU_0 - \frac{U_0}{2d} S + N \quad (26)$$

or

$$kU_c = kU_0 - \frac{U_0}{2d} S - N \quad (27)$$

is satisfied. For a wind blowing in the positive x direction, condition (27) can not be satisfied. It should be noted that when the drift current is ignored and depth of fluid d becomes large, the condition (26) becomes exactly the resonance condition obtained by Phillips (1957) as expected.

DISCUSSION

Consideration of the drift current gives wave spectrum composed of two parts, corresponding to the waves travelling in downwind and upwind direction, respectively, which is not present in Phillips' case. Actually the upwind component is expected to give negligible contriution to the total wave spectrum because the resonance condition can not be satisfied. When the drift current is neglected and the depth goes to infinite, the wave spectrum in equation (25) becomes one half of the wave spectrum obtained by Phillips.

The resonance condition is changed in such a way that the air flow as well as the drift current has an effect on determining wave characteristics. Surface veloity U_0 is a certain fraction of the wind velocity ranging from 2% to 4% depending on the measurement. At present, unfortunately no experimental data on wave characteristics and the profile of drift current measured at the same time are available to evaluate the results obtained in this paper. In order to check the results of this this paper, more precise measurement, particularly in the generation stage of wind wave, is desirable.

In this paper the drift current is approximated simply as a linear function of depth. For more realistic resuts it is hopeful to use an analytical form of the drift current fitted to measured values, if possible.

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