The Journal of the Oceanological Society of Korea
Vol. 19, No. 2, p. 133-140, December 1984

A Note on Oceanographic Applications of Digital Filters

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디지탈필터의 海洋學的 應用

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Abstract: Oceanographic applications of digital filters are studied with special emphasis on the convolution filter with Hamming window and the recursive filters. Convolution filters are simple to understand and easy to design but not efficient for a long data set. Recursive filters, despite of the complexities, have advantages in economy and filter characteristic. By means of digital filtering technique we find that the alongshore wind at Pusan and the sea surface temperature at Gampo in summers during 1973 to 1979 are negatively correlated at low frequencies (periods longer than 5 days) but not so at high frequencies.

要約: 디지탈필터(digital filter)중에서 특히 순환필터(recursive filter)와 Hamming window를 사용한 중합필터(convolution filter)의 해양학적 응용에 대해 논하였다. 중합필터는 이해하기 쉬우며 설계가 간단하지만, 자료수가 많은 경우에는 비경제적이다. 순환필터는 설계가 다소 복잡하지만, 필터의 특성이 좋으며 경제적이다. 여름철(1973~1979)부산 해안선에 평행한 바람과 감포의 표면해수온도 간에 저주파(주기 5일 이상) 성분은 상관관계가 뚜렷하나, 고주파성분은 상관관계가 거의 없음을 디지탈 필터를 사용함으로써 밝혀 내었다.

INTRODUCTION

The word "filter" is derived from electrical engineering, where filters are used to transform electrical signals from one form to another, especially to eliminate (filter out) various undesired frequencies in a signal. Many aspects of the filter theory, such as the design and use of digital filters, originated in the field of analog filters. Digital filters generally have many advantages compared with the analog filter because of their flexibility and economy, particularly at low frequencies (Kanasewich, 1973, p. 204).

In oceanography, running average method is frequently used as a low-pass filter, but its performance is not so good as shown in Fig. 1. The main shortcomings of the running average are that the transition band between pass and stop is too wide, and the amplitude of side lobe is too large. Thus the running average method is not suitable for a precise analysis. Lie (1978) reviewed four filters (mean of 24 hourly heights, mean of 25 hourly heights, Doodson's filter and Demerliac's filter) for the calculation of daily mean sea level, and showed that among these the Doodson's filter and Demerliac's filter eliminate almost completely the tidal effects. However these filters require rather long filter coefficients.

The purpose of this paper is to present effective and accurate digital filters suitable for oceanographic applications. For this purpose convolution and recursive filters are examined

Manuscript received 13 April 1984, revised 4 June 1984.

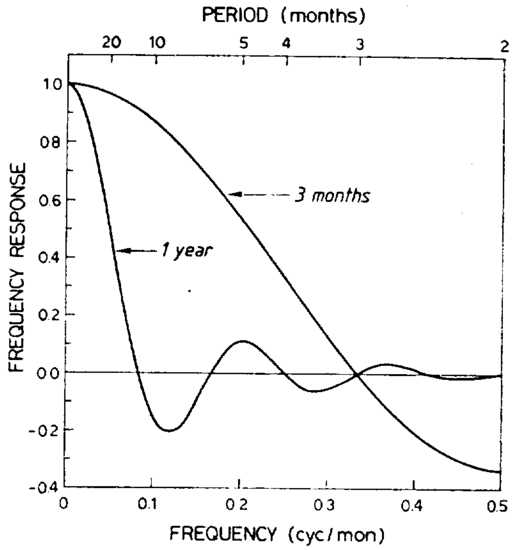

Fig. 1. Frequency responses of the running averages by 1 year (12 months) and by 3 months. Filter coefficients h_k of 1-year running average are 1/12 for k from −5 to 5 and 1/24 for k of −6 and 6, and those of 3-months running average are 1/3 for k from −1 to 1.

with respect to their performance and efficiency. It will be shown that the convolution filter with Hamming window is easy to understand and its design and use are simpler than the recursive filter, but it is less efficient than the recursive one. As an example of oceanographic application of the filter, data of the alongshore wind at Pusan and the sea surface temperature (SST) at Gampo in summers of 1973∼1979 (Lee, 1983) are filtered and analyzed by computing cross-correlation functions between them.

CONVOLUTION FILTER

Digital filters frequently employed in these days can be grouped into convolution (moving average, MA) and recursive (autoregressive moving average, ARMA) types (Robinson and Silvia, 1978, p. 244-268). In this section we review the convolution filter briefly.

The convolution filter is popular in many scientific fields because it is easy to design and to handle. The design of the convolution filter is based on the Fourier transform of a desired transfer function in frequency domain. Its filtering operation is usually carried out in time domain by convolving filter coefficients to input time series.

Let x_t be an input series and y_t be the filtered output series. A finite version of discrete convolution can be represented by

$$y_t=\sum_{k=-M}^{M} h_k x_{t-k}, \tag{1}$$

where h_k is called a kernel of convolution and M is the one-side length of filter (total filter length is $2M+1$). The z-transformation of (1) is

$$Y(z)=H(z)X(z), \tag{2}$$

where $Y(z)$, $H(z)$ and $X(z)$ are z-transforms of y_t, h_t and x_t, respectively. If $z=\exp(-2\pi if\Delta t)$, then the z-transformation is equivalent to discrete Fourier transform, and the frequency response function $H(z)$ can be written by (Jenkins and Watts, 1968, p. 46)

$$H(z)=\sum_{k=-M}^{M} h_k z^k, \tag{3a}$$

or

$$H(f)=\sum_{k=-M}^{M} h_k \exp(-2\pi ifk\Delta t), \tag{3b}$$

where f is the frequency and Δt the sampling interval. If the convolution kernel h_k is an even function (i.e., $h_k=h_{-k}$), then (3b) becomes

$$H(f)=\sum_{k=-M}^{M} h_k \cos(2\pi fk\Delta t)$$

$$=h_0+2\sum_{k=1}^{M} h_k \cos(2\pi fk\Delta t). \tag{4}$$

From (4) we see that for symmetric convolution the frequency response function has no imaginary part, and so there is no phase shift.

Filter coefficients h_k can be found from the Fourier transform of the frequency response function $H(f)$. That is,

$$h_k=\Delta t\int_{-f_N}^{f_N} H(f)\ \exp\ (2\pi ifk\Delta t)df$$

$$=2\Delta t\int_0^{f_N} H(f)\cos(2\pi f k\Delta t)df, \quad (5)$$

where $f_N=1/2\Delta t$ is the Nyquist frequency. The frequency response function of a ideal low-pass filter is given by

$$H^{LP}(f)=\begin{cases}1, & 0\leq f\leq f_L,\\ 0, & f_L\leq f\leq f_N.\end{cases} \quad (6)$$

From (5) and (6), we get filter coefficients

$$h_0^{LP}=2\Delta t\int_0^{f_L} df=2f_L\Delta t, \quad (7a)$$

and

$$h_k^{LP}=2\Delta t\int_0^{f_L}\cos(2\pi f k\Delta t)df$$
$$=\frac{1}{\pi k}\sin(2\pi f_L k\Delta t),\ k=1, 2, \ldots, M. \quad (7b)$$

In the case of a high-pass filter, the desired frequency response function is

$$H^{HP}(f)=\begin{cases}0, & 0\leq f\leq f_H,\\ 1, & f_H\leq f\leq f_N,\end{cases} \quad (8)$$

and the filter coefficients are

$$h_0^{HP}=2\Delta t(f_N-f_H), \quad (9a)$$

and

$$h_k^{HP}=-\frac{1}{\pi k}\sin(2\pi f_H k\Delta t). \quad (9b)$$

The frequency response function and filter coefficients of a band-pass filter are

$$H^{BP}(f)=\begin{cases}0, & 0\leq f\leq f_L,\\ 1, & f_L\leq f\leq f_H,\\ 0, & f_H\leq f\leq f_N,\end{cases} \quad (10)$$

$$h_0^{BP}=2\Delta t(f_H-f_L), \quad (11a)$$

and

$$h_k^{BP}=\frac{1}{\pi k}[\sin(2\pi f_H k\Delta t)-\sin(2\pi f_L k\Delta t)]. \quad (11b)$$

The corresponding ones of a band-stop (or band-reject) filter are

$$H^{BS}(f)=\begin{cases}1, & 0\leq f\leq f_L,\\ 0, & f_L\leq f\leq f_H,\\ 1, & f_H\leq f\leq f_N,\end{cases} \quad (12)$$

$$h_0^{BS}=2\Delta t(f_N+f_L-f_H), \quad (13a)$$

and

$$h_k^{BS}=\frac{1}{\pi k}[\sin(2\pi f_L k\Delta t)-\sin(2\pi f_H k\Delta t)]. \quad (13b)$$

Comparisons of (7), (9), (11) and (13) show that once the coefficient of low-pass filter is known, then the other coefficients can be determined simply by complementary designs.

A truncation of the filter coefficients in time domain after a certain number of terms and also a sudden transition between pass and stop bands in frequency domain bring the oscillation and overshooting of frequency response function, known as Gibb's effect. To reduce this undesirable effect, it is convinient to apply a certain window to the filter coefficient. Among various window functions suggested, a simple and effective one is the Hamming window W_k given by (Golden, 1973, p. 544)

$$W_k=0.54+0.46\cos(\pi k/M),$$
$$k=0, 1, \ldots, M. \quad (14)$$

In the windowing method the filter coefficient h_k is multiplied by the window function W_k.

Fig. 2 shows the Gibb's effect and the effect

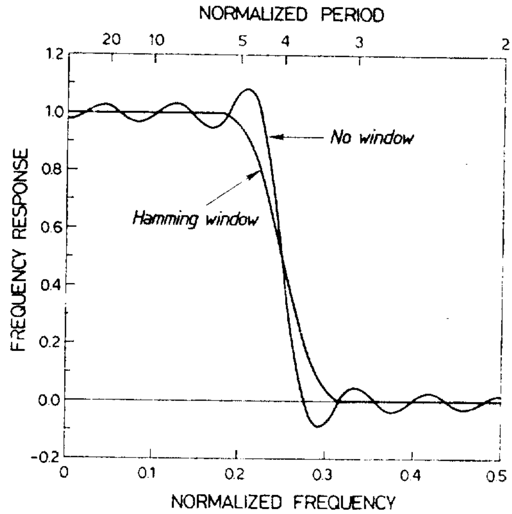

Fig. 2. Frequency responses of low-pass convolution filters. The filter length is 25 ($M=12$ in (1)) and the specified normarized cut-off frequency is 0.25. Note that the filter without window has a remarkable Gibb's effect, but the filter with Hamming window reduces it sufficiently.

of Hamming window for the low-pass filter with $M=12$. From Fig. 2 we see that the Hamming window is very effective in reducing the Gibb's effect, but the cut-off performance is not so sharp. This problem can be solved by using a longer filter coefficients. However a long convolution filter will need a longer computation time and loose many data both in the head and in the tail of the input time series. When we apply a $2M+1$ long filter to an input time series, required computations for one filtered output are $2M+1$ times of multiplication aod add, and M numbers of data in both the head and the tail are lost.

RECURSIVE FILTER

It is well known that recursive filters are more efficient than convolution ones, because the recursive filter can have a very short transition band for a comparatively short span of filter (Hamming, 1977, p. 216). In this section we review the recursive filter briefly.

Using z-transformtion, a transer function $H(z)$ can be expressed as the ratio of two ploynomials of $A(z)$ and $B(z)$ as

$$H(z)=A(z)/B(z)=\sum_{k=0}^{M} a_k z^k / \sum_{k=0}^{N} b_k z^k. \quad (14)$$

where a_k and b_k are the filter coefficients and M and N are the corresponding filter lengths. If we substitute (14) into (2), we get

$$B(z)Y(z)=A(z)X(z). \quad (15)$$

In time domain (15) is represented by

$$b_0 y_t=\sum_{k=0}^{M} a_k x_{t-k}-\sum_{k=1}^{N} b_k y_{t-k}. \quad (16)$$

Here we assume a priori $x_t=y_t=0$ for $t<0$. Since (16) is a recursive equation with respect to y_t, this type of filter is called a recursive filter.

Let's construct a desired recursive filter from a serial coupling of several simple recursive filters with two poles and two zeros, i.e.,

$$H(z)=G_0\prod_{k=1}^{L} H_k(z), \quad (17a)$$

where

$$H_k(z)=(1+a_{1k}z+a_{2k}z^2)/(1+b_{1k}z+b_{2k}z^2), \quad (17b)$$

a's and b's are constants and G_0 is the scale factor resulted from $a_0=1$. Here we refer $H_k(z)$ to a basic filter. (17) shows that the output from k-th filter becomes the input of $(k+1)$-th filter. That is,

$$y_{t,k}=y_{t,k-1}+a_{1k}y_{t-1,k-1}+a_{2k}y_{t-2,k-1} -b_{1k}y_{t-1,k}-b_{2k}y_{t-2,k} \quad (18)$$

where $y_{t,k}$ represents the output from k-th basic filter, and $y_{t,0}$ is the input series, i.e., $y_{t,0}=x_t$.

By letting $z=\exp(-2\pi i f\Delta t)$, we get the frequency response $H(f)$ of recursive filter as

$$H(f)=G_0\prod_{k=1}^{L} H_k(f), \quad (19a)$$

where

$$H_k(f)=\frac{a_{1k}+(1+a_{2k})\cos\omega+i(1-a_{2k})\sin\omega}{b_{1k}+(1+b_{2k})\cos\omega+i(1-b_{2k})\sin\omega}, \quad (19b)$$

and $\omega=2\pi f\Delta t$ is the non-dimensional angular frequecy. (19) shows that the recursive filter accompanies phase shifts. A zero phase shift filter, however, can be easily obtained by a bi-directional cascaded operation, i.e., apply filtering operation in both forward and backward directions (Kanasewich, 1973, p. 199). If there is a phase shift at a given frequency in the first pass through the filter, there is the same phase shift of the opposite sign at the same frequency in the second (i.e., the reversed) pass. Because we are processing the output of the first pass as the input in the reverse pass, the two phase shifts must cancel exactly.

The z-transform of cascaded filter, $H^C(z)$, is given by

$$H^C(z)=G_0\prod_{k=1}^{L} H_k^C(z), \quad (20a)$$

where

$$H_k^C(z)=H_k(z)H_k(-z)=\frac{1+a_{1k}^2+a_{2k}^2+a_{1k}(1+a_{2k})(z^{-1}+z)+a_{2k}(z^{-2}+z^2)}{1+b_{1k}^2+b_{1k}^2+b_{1k}(1+b_{2k})(z^{-1}+z)+b_{2k}(z^{-2}+z^2)}. \quad (20b)$$

By substituting $z=\exp(-i\omega)$, we get the frepuency response, $H_k^C(f)$, of cascaded filter as

$$H_k^C(f)=H_k(f)H_k^*(f)=|H_k(f)|^2=\frac{1+a_{1k}^2+a_{2k}^2+2a_{1k}(1+a_{2k})\cos\omega+2a_{2k}\cos2\omega}{1+b_{1k}^2+b_{2k}^2+2b_{1k}(1+b_{2k})\cos\omega+2b_{2k}\cos2\omega}. \quad (21)$$

Note that the right-hand side of (21) is real-valued, and the cascaded recursive filter has no phase shift. It shoud be noticed that the time series is filtered twice and, therefore, the amplitude response of the cascaded filter is $|H(f)|^2$ instead of $|H(f)|$.

In these days Butterworth, Chebyshev and elliptic filters are in the classical position among various recursive filters. Frequency response of the Butterworth filter, $B_n(\omega)$, is

$$|B_n(\omega)|^2=1/(1+\omega^{2n}), \quad (22)$$

and that of the Chebyshev filter, $C_n(\omega)$, is

$$|C_n(\omega)|^2=1/[1+\{\varepsilon T_n(\omega)\}^2], \quad (23)$$

where ε is a constant and $T_n(\omega)=\cos(n\cos^{-1}\omega)$ is the Chebyshev polynomial of order n. The characteristics of these filters are basically low-pass types but the transformation to the other types such as high-pass is quite easy. Frequency responses of elliptic and another Chebyshev filters are discussed in Hamming(1977, p. 205-207). Digitalization of theses analog filters is carried out by a bilinear z-transform(Kanasewich, 1973, p. 187-192), and details of the transformation can be found in Saito and Ishii (1969), and Ashida and Saito (1970).

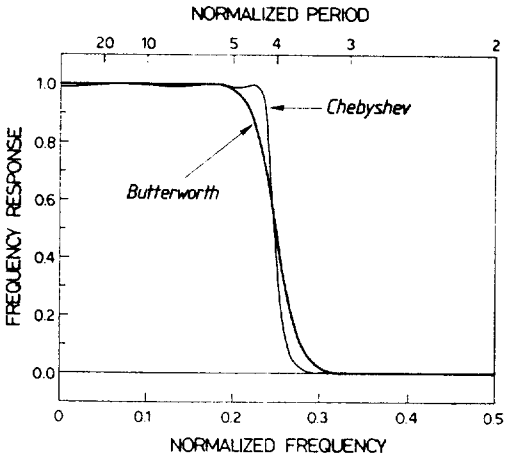

Fig. 3. Frequency responses of low-pass Butterworth and Chebyshev filters. These filters are constructed by serial coupling of three basic recursive filters (L=3 in (20a)) with two poles and two zeros.

Fig. 3 shows filter performances of Butterworth and Chebyshev (equi-ripple in pass band) low-pass types. The frequency responses are not $H(f)$ but $H^C(f)=|H(f)|^2$, i.e., cascaded responses. Their computational efficiencies are nearly same as in Fig. 2, i.e., about 24 times of multiplication and add per one output. A comparison of Figs. 2 and 3 shows that the recursive filters have more narrow transition bands from pass and stop than the convolution filter with Hamming window. If we allow some ripple in pass band, then the sharpness of cut-off of the Chebyshev filter increases greatly. It should be noted that there are distortions in the head and tail parts of two-sided (cascaded) filtered output, because the recursive filtering involves feedback loops.

COEANOGRAPHIC APPLICATION

As mentioned above the filter performance of running average method is not so good (Fig. 2). Both the flatness of pass and stop bands and the sharpness of cut-off are usually required in the digital filtering of oceanographic time series. These requirements can be satisfied by the convolution filter with Hamming window(Fig. 2) or by the recursive filters (Fig. 3). In this section we demonstrate the usefulness of filtering technique by appling it to a set of oceanographic data.

Lee (1983) reported that, in the southeast coast of Korea in summer, the alongshore wind at Pusan and the SST at Gampo are related each other. That is, the SST decreases rapidly due to upwelling when the alongshore wind

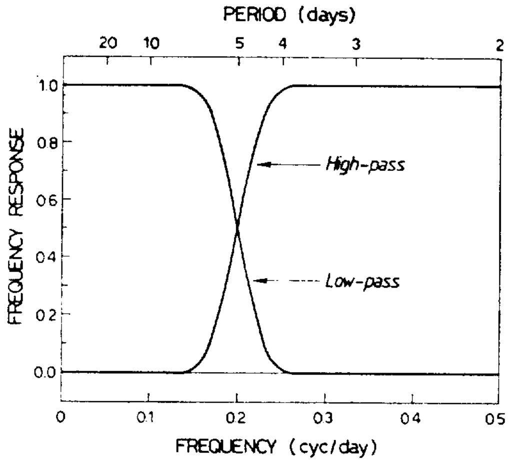

Fig. 4. Frequency responses of low- and high-pass convolution filters with Hamming window. These filters, with filter length of 31(M=15 in (1)) and cut-off periods of 5 days, are applied to the wind and SST data.

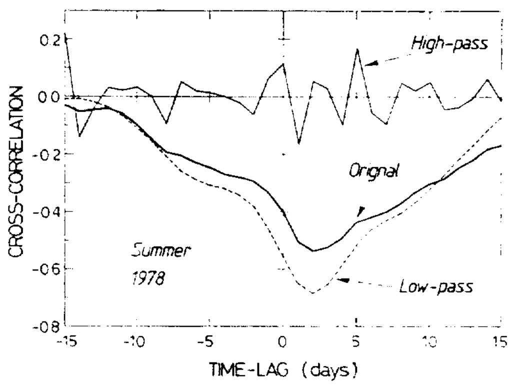

Fig. 5. Cross-correlation functions between alongshore wind at Pusan and SST at Gampo in summer, 1978. The data consist of the original, the low-passed (periods longer than 5 days) and the high-passed series (periods shorter than 5 days), respectively.

blew for more than three days. As an applcation of the filtering technique, we compute the cross-correlation functions between the original, the low-passed and the high-passed series of the wind and SST data.

Fig. 4 shows frequency responses of low- and high-pass convolution filters with Hamming window, which we apply on the wind and SST data. These filters, with 31 coefficients (M= 15), are designed to have narrow transition

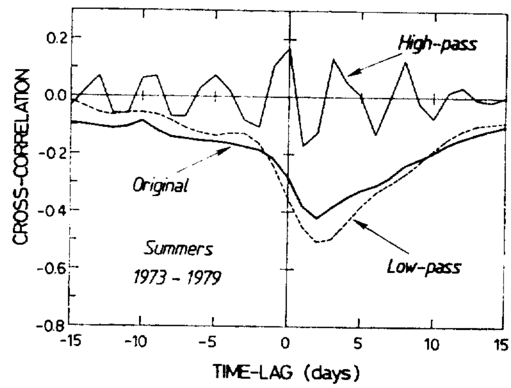

Fig. 6. Average cross-correlation functions between alongshore winds at Pusan and SST at Gampo in summers during 1973 to 1979. The data consist of the original, the low-passed and the high-passed series, respectively.

bands. The midpoint of transition band is 5 days (0.2 cycle/day), and the response is 0.5 at this frequency. Using these filters we filtered the daily data of the alongshore wind at Pusan and the SST at Gampo in summers (4 May~30 September) during 1973 to 1979, and computed cross-correlation functions between them for each year separately.

Fig. 5 shows cross-correlation functions between the alongshore wind and the SST in summer, 1978. The cross-correlation function between the original (no filtering) series shows the negative maximum value at time lags between 2 and 3 days, which means that the alongshore wind leads the SST by 2 or 3 days. However we cannot know what frequency component brings the high correlation. Filtered results give us insights to this question. In fact the low-passed series show higher correlation than the original ones at about the same time lags, but the high-passed series do not show any significant correlation. In other words, the high correlation between the wind and SST is caused mainly due to low-frequency components (periods longer than 5 days), and the long period variations of alongshore wind generate those of SST with time lags of 2 or 3 days.

Table 1. Maximum cross-correlations between alongshore wind at Pusan and SST at Gampo in summers during 1973 to 1979

Year	Original		Low-pass		High-pass	
	Time-lag (days)	Max. correl.	Time-lag (days)	Max. correl.	Time-lag (days)	Max. correl.
1 9 7 3	2	−0.23	2	−0.29	−8	−0.33
1 9 7 4	2.5	−0.43	3	−0.58	−2	−0.28
1 9 7 5	2	−0.39	2	−0.47	10	−0.35
1 9 7 6	2.5	−0.48	3	−0.46	18	−0.20
1 9 7 7	2	−0.50	2.5	−0.55	−3	−0.37
1 9 7 8	2.5	−0.54	2.5	−0.69	−17	−0.17
1 9 7 9	2.5	−0.38	2.5	−0.54	−11	−0.22
Average	2	−0.42	2.5	−0.50	1	−0.17

Fig. 6 shows cross-correlation functions averaged during 1973 to 1979. Table 1 shows the highest cross-correlations and the corresponding time lags of each year. From the table we learn that the low-passed series have higher correlations than the original ones at about the same time largs in every summer except in 1976, but the high-passed series of each year do not show any systematic correlation. Therefore, we can conclude that only the low-frequency component of alongshore wind leads that of SST by 2 or 3 days every year. Without the filtering technique, it might have been impossible to identify whether the correlation between the alongshore wind and the SST was due to high-frequency components or low frequency ones.

DISCUSSION AND CONCLUSIONS

Convolution filters, which are simple to understand and easy to design, are likely to be used in situations where computer time is not a serious problem. Although symmytric convolutions have no phase shift, they are physically unrealizable, i.e., the procisses include "future" signal. Thus a real-time convolution filtering is impossible. On the other hand, recursive filters, which are physically realizable and rapid in computation, can be used even in real-time signal processing. But the recursive filter has a disadvantage of accompanying phase shifts. Although the cascaded recursive filterings can remove that shortcoming, it cannot be used in real-time operation. It is worth noting that both recursive and convolution filters have about the same flexibility in meeting various conditions, and the optimum filter design can be determined from the required conditions of frequency responses. By computing the transfer functions of various filter schemes, we can choose the best one.

In many scientific fields, filtering operations are usually prerequisite for data analysis. Filtered results sometimes give us more clear physical insights. Based on the filtered results, for example, we get a better understanding for the upwelling phenomenon in the southeast coast of Korea in summer as shown in the last section. From the cross-correlation analysis for the original, the low-passed and the high-passed data, we recognized that the long period variation of the alongshore wind at Pusan and that of the SST at Gampo are highly correlated with each other at 2 or 3 days of time lags. The running average method, although frequently used in oceanography, is not suitable for a precise analysis because its frequency response is poor.

ACKNOWLEDGEMENTS

We wish to thank Jae Chul Lee for providing us the wind and SST data used in this work, and Byung-Gul Lee for assisting us in the treatment of the data.

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