

## 論 文

## 相關가우스 페이딩 채널에서 디지털傳送에 대한 誤率

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Error Probabilities for Digital Transmission in  
Correlated Gaussian Fading ChannelsYoung Yeul HAN\*, *Regular Member*

**要 約** 이온층 신틸레이션 채널(transionospheric scintillation channel)에서 PSK 통신시스템의 誤率을 가우스-쿼드러처積分(Gauss quadrature integration)公式의 方法을 利用하여 계산하였다. 使用한 채널 모델은 Rino의 모델로 受信信號의 포락선이 相關가우스 랜덤 과정으로 連續히 變하는 페이딩 채널이다. 신틸레이션 채널에 대한 誤率은 UHF帶의 傳送에서 실제 이온층 신틸레이션 데이터를 使用하여 計算하였다.

**ABSTRACT** Calculation of error probabilities for a coherent phase-shift keyed communication system operating in a transionospheric scintillation channel is accomplished by means of the Gauss-quadrature integration formula. The channel model used, patterned after Rino's work, is slowly flat fading wherein the envelope of the received signal is modeled as the envelope of correlated Gaussian quadrature random processes. The error probability for the scintillation channel is calculated using actual ionospheric scintillation data for transmission in the UHF region(30-300MHz).

## 1. INTRODUCTION

The scintillation of radio waves passing through the ionosphere has been an observed phenomenon for many years, first by radio astronomers and then as a result of the reception of radio signals from orbiting satellites. The amplitude distribution of ionospheric scintillation has been approximated with varying degrees of success by the Rice-Nakagami, log-normal and bivariate-Gaussian distributions(1). Rino(2) has shown that the amplitude distribution of ionospheric scintillation is described closely as the envelope of bivariate Gaussian quadrature components and through suitable choices of

parameters a good fit is obtained for all amplitudes except at the extremes of the distribution. Although similar to the Rice-Nakagami and log-normal distributions, the bivariate-normal distribution is more general because it allows the components to be correlated and to have unequal variance.

The purpose of this paper is to present a useful way of calculating the average error probability for phased-shift keyed signaling in the presence of bivariate-normal distributed flat fading. The receiver is assumed to track the phase of the received signals exactly. The Gauss-quadrature integration formula is employed in the calculation of the average error probability.

For situations where the received signal envelope,  $A$ , randomly fluctuates, the overall probability of error is obtained by averaging  $p(E/a)$  over  $A$ :

$$p_E = \int_{-\infty}^{\infty} p(a) p(E/a) da \quad (1)$$

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$p(E/a)$  is the probability of error given  $A=a$ ,  $p(a)$  is the probability density function (pdf) of  $A$ , and  $P_E$  is the average probability of error. In those cases where the envelope of the signal randomly fluctuates, the lower limit of the integral becomes zero. If the received signal is assumed to be a general Gaussian process,  $p(a)$  is an envelope pdf obtained from a bivariate Gaussian pdf by transforming rectangular coordinates into polar coordinates :

$$p(a) = \int_0^{2\pi} p(a \cos \theta, a \sin \theta) a d\theta \quad (2)$$

where

$$\begin{aligned} p_{a\theta}(a, \theta) &= ap_{xy}(V_x, V_y) \Big|_{\substack{V_x = a \cos \theta \\ V_y = a \sin \theta}} \\ &= ap(a \cos \theta, a \sin \theta), \\ 0 \leq \theta \leq 2\pi, \quad 0 \leq a < \infty \end{aligned} \quad (3)$$

$p_{xy}(V_x, V_y)$  is a correlated bivariate Gaussian pdf.

The error probability for the phase-shift keyed signal given  $a$ , can be derived by assuming a bi-phase modulated transmitted signal of the form

$$x(t) = A \cos(\omega_0 t + \cos^{-1} m d(t) + \theta) \quad (4)$$

where  $nT \leq t \leq (n+1)T$  in which  $T$  is the signaling interval,  $d(t)$  is the data sequence, and  $\cos^{-1} m$  the modulation index. Synchronous detection in white, Gaussian noise backgrounds requires a correlation or matched filter detector to give minimum probability of error. Here, the noise is additive, white, and Gaussian with zero mean and one-sided power spectral density  $N$ . In the special case  $m=0$ , assuming that the receiver tracks the phase exactly and the fading is slow so that  $a = \text{constant}$  within a  $T$ -second signaling interval, the probability of error, given  $a$ , is

$$p(E/a) = \frac{1}{2} \operatorname{erfc}(\sqrt{za}) \quad (5)$$

where  $z$  is the signal-energy-to-noise-spectral-density ratio. To perform the integration in Eq.(1) with the lower limit of the integral zero, the Gauss-quadrature integral formula(6) is applicable if we know the moments of  $p(a)$ . The moments may be used to calculate the weights and abscissas in the integration formulæ

$$\int_b^c f(x) w(x) dx = \sum_{k=1}^m W_k f(x_k) + E^1 \quad (6)$$

where  $f(x) \geq 0$  in  $(b, c)$ ,  $W_k$  are the weights  $x_k$  are the abscissas,  $m$  is the number of weights and abscissas, and  $E^1$  is the error of the approximation. Here,  $f(x)$  can be replaced by the amplitude pdf  $p(a)$  and  $w(x)$  by the error probability  $p(E/a)$ . Also,  $b$  is zero and  $c$  is plus infinity. If we define the  $r$ th moments  $M_r$ , associated with  $w(x)$  over  $(b, c)$ , by the equation

$$\int_b^c x^r f(x) dx = M_r \quad (7)$$

and  $M_r$  is represented by

$$\sum_{k=1}^m W_k x_k^r = M_r, \quad r=0, 1, \dots, M-1 \quad (8)$$

it is said that the integral is approximated with degree of precision  $2m-1$ . If we let  $x_1, x_2, \dots, x_m$  be the zeros of  $g(x)$ , i.e.,

$$\begin{aligned} g(x) &= (x-x_1)(x-x_2)\dots(x-x_m) \\ &= x^m + \alpha_1 x^{m-1} + \dots + \alpha_m, \end{aligned} \quad (9)$$

the  $\alpha$ 's can be obtained from equation(8) by transforming. Finally, the abscissas  $x_k$  and weights  $W_k$  are obtained. It is stated that there is no guarantee that the zeros of  $g(x)$  will be real and distinct and that they lie in  $(b, c)$  even though  $f(x) \geq 0$ . If the roots are not real and distinct, the desired formula does not exist.

The moments of  $p(a)$  are given by

$$E[a^n] = \int_0^\infty a^n p(a) da = \int_0^{2\pi} g(\theta) d\theta$$

where  $g(\theta)$  is the integral (see Appendix)

$$\int_0^\infty p(a \cos \theta, a \sin \theta) a^{n-1} da = \frac{1}{2\sqrt{2D}}$$

$$\begin{aligned} & B \left\{ \sum_{\substack{k=0 \\ \text{even}}}^{n-1} \binom{n+1}{k} \left( \frac{\bar{v}_x E}{D} \right)^{n+1-k} \left( \frac{2A}{D} \right)^{k/2} \right. \\ & \left. \left[ \gamma \left( \frac{k-1}{2} + 1 \right) + \gamma \left( \frac{k-1}{2} + 1, E^2 \right) \right] \right. \\ & \left. + \sum_{\substack{k=1 \\ \text{odd}}}^{n-1} \binom{n+1}{k} \left( \frac{\bar{v}_x E}{D} \right)^{n+1-k} \left( \frac{2A}{D} \right)^{k/2} \right. \end{aligned}$$

Table I. Data Sets

	$\sigma_x^2$	$\sigma_y^2$	$C_{xy}$	$\theta$	$\bar{v}_x$
Data Set I	0.3625	0.0074	0.0037	0.6°	1
Data Set II	0.2925	0.0374	0.0174	3.9°	1
Data Set III	0.1580	0.0719	0.036	18.6°	1

$$\left[ \gamma\left(\frac{k-1}{2} + 1\right) - \gamma\left(\frac{k-1}{2} + 1, F^2\right) \right] \quad (10)$$

where

$$A = \sigma_x^2 \sigma_y^2 - C_{xy}^2$$

$$B = \exp \frac{\bar{v}_x^2 E^2 - \bar{v}_x^2 \sigma_y^2 D}{2AD}$$

$$D = \cos^2 \theta \sigma_y^2 - 2C_{xy} \cos \theta \sin \theta + \sin^2 \theta \sigma_x^2$$

$$E = \cos \theta \sigma_y^2 - C_{xy} \sin \theta$$

$$F = \frac{-\bar{v}_x E}{\sqrt{2AD}}$$

$\gamma(q)$  is a gamma function and  $\gamma(s, t)$  is the incomplete gamma function,  $\sigma_x^2$  and  $\sigma_y^2$  are the variances of  $V_x$  and  $V_y$ , respectively,  $C_{xy}$  is the covariance of  $V_x$  and  $V_y$ , and  $\bar{v}_x$  is mean of  $V_x$ . It is assumed that the coherent signal is the phase reference; hence the mean of  $V_y$  becomes zero.

Now it remains to compute the integration for-

mula for each signal-to-noise ratio according to Eq. (6) to obtain the error probability.

### 2. CALCULATION OF $P_E$

The data used are from Rino. With the values of the variances of  $V_x$  and  $V_y$ , the covariance, and the mean of  $V_x$ , moments of  $p(a)$  were calculated up to order 20. With the moments thus obtained, the weights and abscissas of the Gauss-quadrature integration formula were calculated. In finding the roots of the algebraic equation(9) an attempt was made to calculate the roots to tenth by Graffe's root-square method.

Once the weights and abscissas are determined, the error probability is found by the equation

$$P_E = \int_0^\infty p(a) p(E; a) da = \sum_{k=1}^m \frac{1}{2} W_k \operatorname{erfc}(\sqrt{za_k}) \quad (11)$$

For each set of weights and abscissas are determined, the error probability was calculated for the signal-to-noise-ratio,  $z$ , in the range 0 to 20 dB.

### 3. RESULTS

The technique for calculating  $P_E$  just described was tested by employing the data used by Rino(2).

Table II. Value of moments (Data set I)

Order of moment	Value of moment
0	1.000
1	1.030
2	1.370
3	2.105
4	3.587
5	6.634
6	1.313 × 10
7	2.754 × 10
8	6.082 × 10
9	1.406 × 10 <sup>2</sup>
10	3.386 × 10 <sup>3</sup>
11	8.471 × 10 <sup>3</sup>
12	2.194 × 10 <sup>3</sup>
13	5.869 × 10 <sup>3</sup>
14	1.618 × 10 <sup>4</sup>
15	4.575 × 10 <sup>4</sup>
16	1.335 × 10 <sup>5</sup>
17	3.882 × 10 <sup>5</sup>
18	1.218 × 10 <sup>6</sup>
19	3.335 × 10 <sup>6</sup>
20	1.216 × 10 <sup>7</sup>

Value of moments (Data set II)

Order of moment	Value of moment
0	1,000
1	1.039
2	1.330
3	1.939
4	3.114
5	5.398
6	9.982
7	1.951 × 10
8	4.007 × 10
9	8.597 × 10
10	1.920 × 10 <sup>2</sup>
11	4.447 × 10 <sup>2</sup>
12	1.065 × 10 <sup>3</sup>
13	2.633 × 10 <sup>3</sup>
14	6.702 × 10 <sup>3</sup>
15	1.751 × 10 <sup>4</sup>
16	4.706 × 10 <sup>4</sup>
17	1.261 × 10 <sup>5</sup>
18	3.644 × 10 <sup>5</sup>
19	9.500 × 10 <sup>5</sup>
20	3.082 × 10 <sup>6</sup>

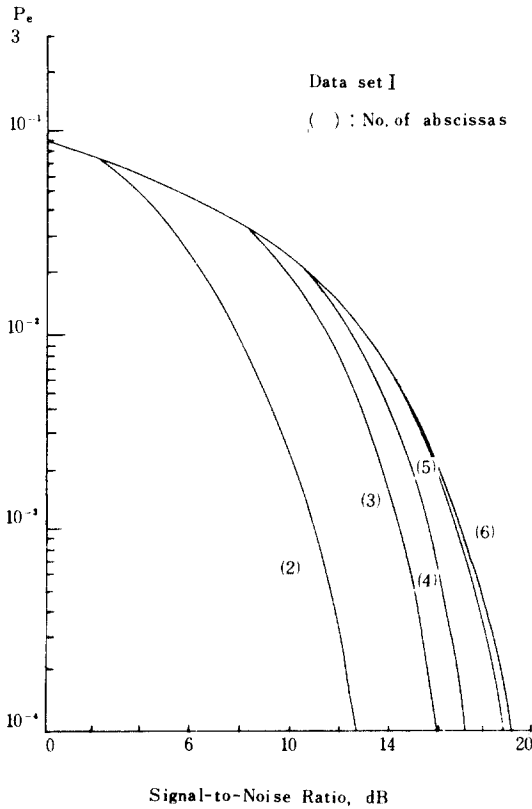


Figure 1. Average probability of error (Data set I)

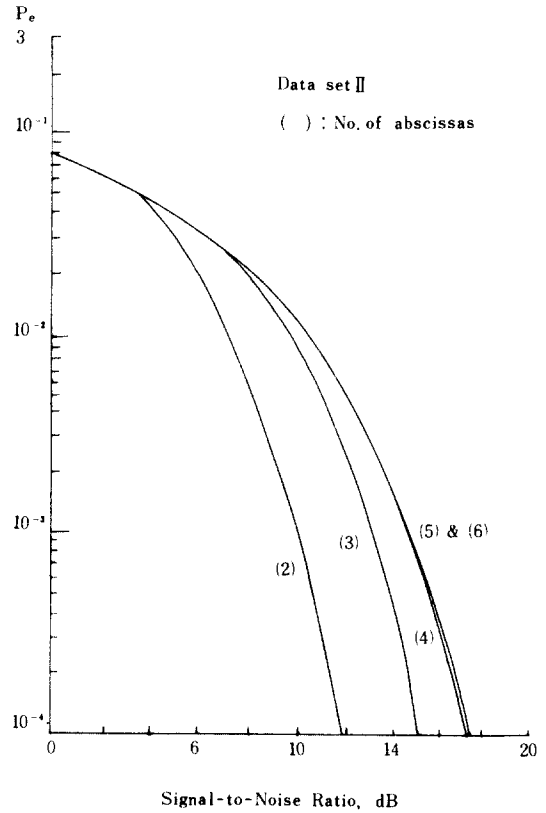


Figure 2. Average probability of error (Data set II)

Value of moments (Data set III)

Order of moment	Value of moment
0	1.000
1	1.043
2	1.230
3	1.589
4	2.209
5	3.267
6	5.095
7	8.331
8	1.421 × 10
9	2.519 × 10
10	4.626 × 10
11	8.775 × 10
12	1.716 × 10 <sup>2</sup>
13	3.450 × 10 <sup>2</sup>
14	7.125 × 10 <sup>2</sup>
15	1.508 × 10 <sup>3</sup>
16	3.270 × 10 <sup>3</sup>
17	7.213 × 10 <sup>3</sup>
18	1.642 × 10 <sup>4</sup>
19	3.3083 × 10 <sup>4</sup>
20	8.938 × 10 <sup>4</sup>

The parameter values for the bivariate Gaussian approximation to the envelopes for three separate data sets are given in Table I. These values were calculated by Rino.

The moments up through order 20 were then calculated using Eq.(10) and Simpson's one-third integration rule with 100 integration intervals. They are given in Table II for data sets I - III, respectively. The weights and abscissas are calculated next. The error probability is then calculated using Eq.(11). The resulting curves are shown in Figures 1-3.

#### 4. CONCLUSIONS

A method has been discussed where by the probability of error for a fading channel can be approximated by a series by knowing the moments of the envelope of the received signal. Using the Rino bivariate Gaussian model to approximate the envelope distribution of three data sets, the moments

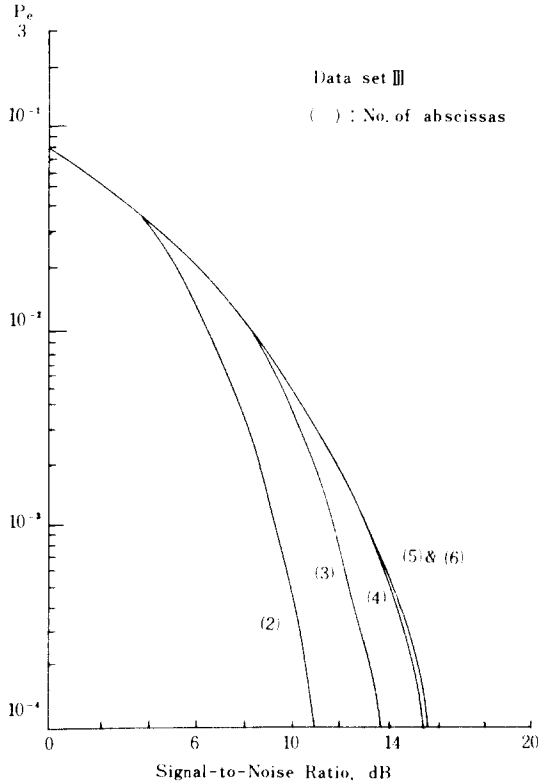


Figure 3. Average probability of error (Data set III)

were analytically calculated from the approximating bivariate Gaussian density function.

The results presented in this paper were for the case of phase-shift-keyed signaling. The application of the method to other coherent and noncoherent signaling schemes would be straight-forward calculation.

## APPENDIX

To obtain the  $n$ th moment of the amplitude pdf from the bivariate Gaussian pdf we perform a change of variables from rectangular to polar coordinates.

If the polar coordinates  $a$  and  $\theta$  are used instead of the rectangular coordinates  $v_x$  and  $v_y$ , the Jacobian is simply  $a$  and the joint pdf of  $a$  and  $\theta$  is

$$\begin{aligned} p_{a\theta}(a, \theta) &= ap_{xy}(v_x, v_y) \Big| \begin{array}{l} v_x = a \cos \theta \\ v_y = a \sin \theta \end{array} \\ &= ap(a \cos \theta, a \sin \theta) \end{aligned} \quad (\text{A } 1)$$

where  $0 \leq \theta \leq 2\pi$ ,  $0 \leq a \leq \infty$

If we integrate  $p_{a\theta}(a, \theta)$  over  $\theta$  and  $a$  to get the phase and amplitude pdfs, respectively, we obtain

$$p_{\theta}(\theta) = \int_0^{\infty} p_{a\theta}(a \cos \theta, a \sin \theta) a da \quad (\text{A } 2)$$

$$p_a(a) = \int_0^{2\pi} p_{a\theta}(a \cos \theta, a \sin \theta) a d\theta \quad (\text{A } 3)$$

Now the  $n$ th moment of the amplitude pdf becomes

$$\begin{aligned} E[a^n] &= \int_0^{\infty} a^n p_a(a) da \\ &= \int_0^{\infty} \left[ a^n \int_0^{2\pi} p(a \cos \theta, a \sin \theta) a d\theta \right] da \end{aligned} \quad (\text{A } 4)$$

By interchanging the integrals, it may be written as

$$E[a^n] = \int_0^{2\pi} \left[ \int_0^{\infty} a^{n+1} p(a \cos \theta, a \sin \theta) da \right] d\theta \quad (\text{A } 5)$$

The inner integral over  $a$  can be obtained in analytic form. The bivariate Gaussian pdf can be written in another form by noting that

$$\begin{aligned} \sigma_x^2 \sigma_y^2 (1 - r^2) &= \sigma_x^2 \sigma_y^2 \left( 1 - \frac{C_{xy}^2}{\sigma_x^2 \sigma_y^2} \right) \\ &= \sigma_x^2 \sigma_y^2 - C_{xy}^2 \end{aligned} \quad (\text{A } 6)$$

Here  $\sigma_x^2$  and  $\sigma_y^2$  are variances of  $v_x$  and  $v_y$ , respectively,  $r$  is the correlation coefficient and  $C_{xy}$  is the covariance of  $v_x$  and  $v_y$ . Now the bivariate Gaussian pdf of  $v_x$  and  $v_y$  becomes

$$\begin{aligned} p(v_x, v_y) &= \frac{1}{2\pi \sqrt{\sigma_x^2 \sigma_y^2 - C_{xy}^2}} \\ &\exp \left[ -\frac{1}{2(\sigma_x^2 \sigma_y^2 - C_{xy}^2)} \right. \\ &\quad \left. x[(v_x - \bar{v}_x)^2 \sigma_y^2 - 2C_{xy}(v_x - \bar{v}_x)v_y \right. \\ &\quad \left. + v_y^2 \sigma_x^2] \right] \end{aligned} \quad (\text{A } 7)$$

Let  $\sigma_x^2 \sigma_y^2 - C_{xy}^2 = A$

The inner integral of Equation (A 5) becomes

$$\int_0^\infty p(a \cos \theta, a \sin \theta) a^{n-1} da = \int_0^\infty \frac{1}{2\pi\sqrt{A}} \exp\left[-\frac{1}{2A} x\left\{(a \cos \theta - \bar{v}_x)^2 \sigma_y^2 - 2C_{xy} (a \cos \theta - \bar{v}_x) a \sin \theta + (a \sin \theta)^2 \sigma_x^2\right\}\right] a^{n-1} da \quad (\text{A } 8)$$

Expanding the square in the exponent as

$$a^2 \cos^2 \theta \sigma_y^2 - 2a \cos \theta \bar{v}_x \sigma_y^2 + \bar{v}_x^2 \sigma_y^2 - 2C_{xy} a^2 \cos \theta \sin \theta + a \sin \theta 2C_{xy} \bar{v}_x + a^2 \sin^2 \theta \sigma_x^2$$

the exponent can be rearranged into the form

$$a^2 (\cos^2 \theta \sigma_y^2 - 2C_{xy} \cos \theta \sin \theta + \sin^2 \theta \sigma_x^2) - 2a \bar{v}_x (\cos \theta \sigma_y^2 - C_{xy} \sin \theta) + \bar{v}_x^2 \sigma_y^2 = a^2 D - 2a \bar{v}_x E + \bar{v}_x^2 \sigma_y^2 \quad (\text{A } 9)$$

where

$$D = \cos^2 \theta \sigma_y^2 - 2C_{xy} \cos \theta \sin \theta + \sin^2 \theta \sigma_x^2 \quad (\text{A } 10)$$

$$\text{and } E = \cos \theta \sigma_y^2 - C_{xy} \sin \theta \quad (\text{A } 11)$$

Thus Eq. (A 8) becomes

$$g(\theta) = \int_0^\infty p(a \cos \theta, a \sin \theta) a^{n-1} da = \int_0^\infty \frac{1}{2\pi\sqrt{A}} \left\{ \exp\left[-\frac{D}{2A}\right] x\left\{a^2 - \frac{2a\bar{v}_x E}{D} + \frac{\bar{v}_x^2 \sigma_y^2}{D}\right\}\right\} a^{n-1} da \quad (\text{A } 12)$$

On completing the square on  $a$  in the exponent

$$a^2 - \frac{2a\bar{v}_x E}{D} + \frac{\bar{v}_x^2 \sigma_y^2}{D} = \left(a - \frac{\bar{v}_x E}{D}\right)^2 + \frac{\bar{v}_x^2 \sigma_y^2}{D} - \frac{\bar{v}_x^2 E^2}{D^2} \quad (\text{A } 13)$$

The integral may be written as

$$\frac{1}{2\pi\sqrt{A}} \exp\left\{\frac{\bar{v}_x^2 E^2 - \bar{v}_x^2 \sigma_y^2 D}{2AD}\right\} \int_0^\infty \exp\left[-\frac{D}{2A}\left(a - \frac{\bar{v}_x E}{D}\right)^2\right] a^{n-1} da \quad (\text{A } 14)$$

which, if we substitute

$$w = \sqrt{\frac{D}{2A}} \left(a - \frac{\bar{v}_x E}{D}\right), \quad dw = \sqrt{\frac{D}{2A}} da$$

$$\text{or } a = \sqrt{\frac{2A}{D}} w + \frac{\bar{v}_x E}{D}$$

and put

$$B = \exp\left\{\frac{\bar{v}_x^2 E^2 - \bar{v}_x^2 \sigma_y^2 D}{2AD}\right\}$$

Eq. (A14) become

$$\frac{1}{2\pi\sqrt{A}} B \sqrt{\frac{2A}{D}} \int_{-\frac{\bar{v}_x E}{\sqrt{2AD}}}^\infty e^{-w^2} \left(\sqrt{\frac{2A}{D}} w + \frac{\bar{v}_x E}{D}\right)^{n-1} dw \quad (\text{A } 15)$$

Expanding the term in parentheses by using the binomial theorem results in

$$\int_0^\infty p(a \cos \theta, a \sin \theta) a^{n-1} da = \frac{1}{2\pi\sqrt{A}} B \sqrt{\frac{2A}{D}} \sum_{k=0}^{n-1} \binom{n-1}{k} x \left(\frac{2A}{D}\right)^{k/2} \left(\frac{\bar{v}_x E}{D}\right)^{n-1-k} \int_{-\frac{\bar{v}_x E}{\sqrt{2AD}}}^\infty w^k e^{-w^2} dw \quad (\text{A } 16)$$

However, we notice the integral in Eq. (A16) can be written in terms of the gamma and incomplete gamma functions, where the gamma function and incomplete gamma function are defined by

$$\Gamma(p) = \int_0^\infty t^{p-1} e^{-t} dt \quad (\text{A } 17)$$

$$\text{and } \gamma(p, x) = \int_0^x t^{p-1} e^{-t} dt \quad (\text{A } 18)$$

respectively.

$$\begin{aligned} \text{Let } x &= w^2 & dx &= 2w dw \\ w &= \sqrt{x} & dw &= \frac{1}{2\sqrt{x}} dx \\ F &= \frac{-\bar{v}_x E}{\sqrt{2AD}} \end{aligned}$$

The integral may be written

$$\int_{-F}^\infty w^k e^{-w^2} dw = \int_0^\infty w^k e^{-w^2} dw + \int_{-F}^0 w^k e^{-w^2} dw = \int_0^\infty w^k e^{-w^2} dw - \int_0^F w^k e^{-w^2} dw \quad (\text{A } 19)$$

The first integral in Eq. (A19) can be expressed in terms of the gamma function by a change of variable;

$$\int_0^\infty w^k e^{-w^2} dw = \int_0^\infty (x)^{k/2} e^{-x} \frac{1}{2\sqrt{x}} dx$$

$$= \frac{1}{2} \gamma\left(\frac{k-1}{2} + 1\right) \quad (A20)$$

The second integral in Eq. (A19) can be expressed in two ways depending on whether  $k$  is even or odd.

$$-\int_0^{-F} w^k e^{-w^2} dw = \int_0^{F^2} (x)^{k/2} e^{-x} \frac{1}{2\sqrt{x}} dx$$

$$= \frac{1}{2} \gamma\left(\frac{k-1}{2} + 1, F^2\right)$$

$k = \text{even} \quad (A21)$

When  $k$  is odd the integral becomes

$$-\int_0^{-F} w^k e^{-w^2} dw = -\int_0^{F^2} (x)^{k/2} e^{-x} \frac{1}{2\sqrt{x}} dw$$

$$= -\frac{1}{2} \gamma\left(\frac{k-1}{2} + 1, F^2\right),$$

$k = \text{odd} \quad (A22)$

Putting all the above results together, we obtain

$$\int_0^\infty p(a \cos \theta, a \sin \theta) a^{n-1} da = \frac{1}{2\pi\sqrt{2D}}$$

$$B \left[ \sum_{\substack{n-1 \\ k=0 \\ \text{even}}} \left(\frac{n+1}{k}\right) \left(\frac{\bar{v}_x F}{D}\right)^{n-1-k} x \left(\frac{2A}{D}\right)^{k/2} \right.$$

$$\left[ \gamma\left(\frac{k-1}{2} + 1\right) + \gamma\left(\frac{k-1}{2} + 1, F^2\right) \right.$$

$$+ \sum_{\substack{n-1 \\ k=1 \\ \text{odd}}} \left(\frac{n+1}{k}\right) \left(\frac{\bar{v}_x F}{D}\right)^{n-1-k} x \left(\frac{2A}{D}\right)^{k/2}$$

$$\left. \left. \gamma\left(\frac{k-1}{2} + 1\right) - \gamma\left(\frac{k-1}{2} + 1, F^2\right) \right] \right] \quad (A23)$$

where

$$A = \sigma_x^2 \sigma_y^2 - C_{xy}^2$$

$$B = \exp \frac{\bar{v}_x^2 F^2 - \bar{v}_x^2 \sigma_y^2 D}{2AD}$$

$$D = \cos^2 \theta \sigma_y^2 - 2C_{xy} \cos \theta \sin \theta + \sin^2 \theta \sigma_x^2$$

$$E = \cos \theta \sigma_y^2 - C_{xy} \sin \theta$$

$$F = \frac{-\bar{v}_x E}{\sqrt{2AD}}$$

The right side of Eq. (A23) is now a function of  $\theta$  only. We will denote it by  $g(\theta)$ . When  $n$  is zero, it is the phase pdf and reduces to

$$p(\theta) = \frac{\sqrt{A}}{2\pi D} \exp\left[-\frac{\bar{v}_x^2 \sigma_y^2}{2A}\right] + \frac{\bar{v}_x E}{\pi^{1/2} 2^{3/2} D^{3/2}}$$

$$x \exp\left[\frac{\bar{v}_x^2 E^2 - \bar{v}_x^2 \sigma_y^2 D}{2AD}\right] \operatorname{erfc}\left[\frac{-\bar{v}_x E}{\sqrt{2AD}}\right] \quad (A24)$$

### REFERENCE

- (1) Fremouw, E. J., "Modeling and prediction of ionospheric scintillation," AIAA Paper no. 74-54, AIAA 12-th Aerospace Sciences Meeting, Washington, DC, Jan. 1974.
- (2) Hatfield, V. E. and C. S. Rino, "Non-Rician statistics and their implications for modeling effects of scintillation on communication channels," Paper 4-1, Ionospheric Effects Symposium Proceedings, Washington, DC, Jan. 1975.
- (3) Ziemer, R. E. and W. H. Tranter, "Principles of communications; systems modulation, and noise," Houghton-Mifflin, 1976.
- (4) Carnahan, B., H. A. Luther and J. O. Wilkes, "Applied numerical methods," John Wiley & Sons, Inc. 1969.
- (5) Bello, P. A., "Characterization of randomly time-variant linear channels," IEEE Trans. on Comm. Systems vol. CS11, pp. 360-393, Dec. 1963.



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