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Spacecraft Spin Rate Change due to Propellant Redistribution Between Tanks*

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Abstract

A bubble trapped in the liquid manifold of INTELSAT IV F-7 spacecraft caused a mass imbalance between the System 1 propellant tanks and a wobble half angle of 0.38 degree to 0.48 degree. A maneuver on May 14, 1980 passed the bubble through the axial jet and allowed propellant to redistribute. A 0.2 rpm change in spin rate was observed with an exponential decay time constant of 6 minutes.

In this paper, moment of inertia, tank geometry and hydrodynamics models are derived to match the observed spin rate data. The values of the total mass of the propellant considered were 16, 19 and 20 Kgs with corresponding mass imbalances of 14.3, 15 and 15.1 Kgs, respectively. The result shows excellent agreement with observed spin rate data but it was necessary to assume a greater mass of hydrazine in the tanks than propellant accounting indicated.

I. INTRODUCTION

On May 14, 1980, an orbital inclination change maneuver was performed on INTELSAT IV F-7 spacecraft using the System 1 axial jet. We expected that a bubble trapped in the liquid manifold would pass through the jet during the maneuver.

The bubble passed through the jet about 220 seconds after the maneuver started. About 9 seconds after bubble passage the maneuver was terminated and propellant redistribution became evident from the reduction in spin speed. Propellant moves from tank 2 to tank 1 so that the moment of inertia of the spacecraft increases. The spin rate of the spacecraft decreases by the conservation of angular momentum.

The actual spin rate change for INTELSAT IV F-7 spacecraft is plotted in Figure 1. The spin rate shows an exponential decay with a time constant of 6 minutes. The spin rate decreases from 55.4334 rpm to a final value of 55.2334 rpm in 25 minutes.

The spin speed change gives evidence of bubble removal and allows the mass of propellant

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in each tank to be determined. This paper presents the analysis techniques of spin speed change and their application to INTELSAT IV F-7 spacecraft.

II. OBSERVATIONAL DATA

Figure 1 shows telemetered spin rate value following the inclination maneuver for INTELSAT IV F-7 spacecraft on May 14, 1980. The spin rate decreased by about 0.2 rpm in twenty-five minutes.

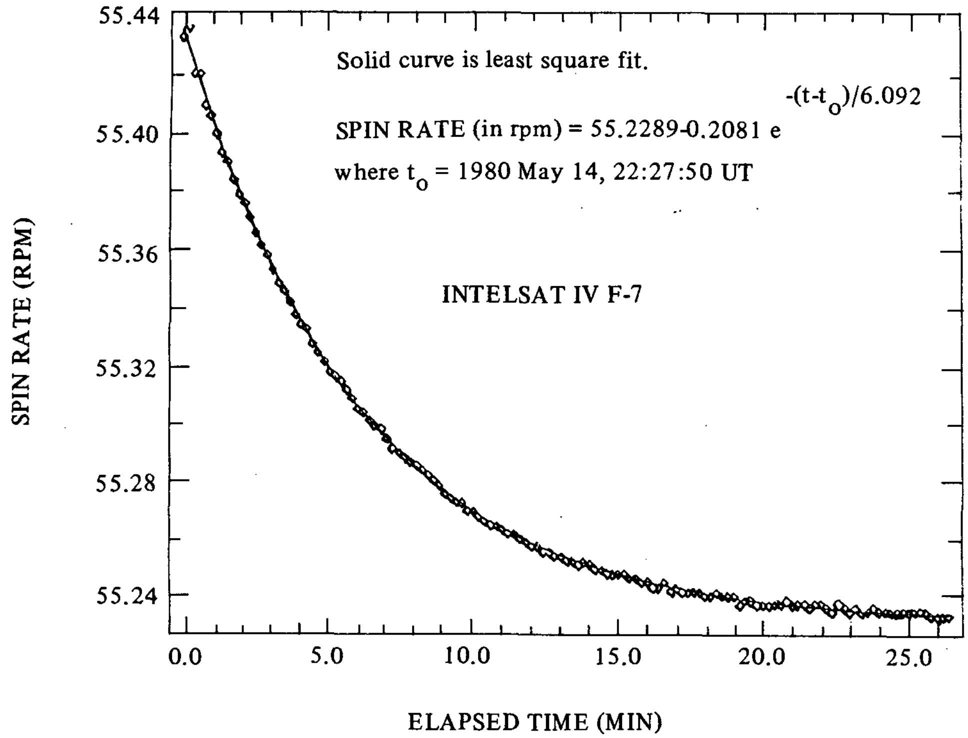

Fig. 1. Spin rate vs. time during redistribution

An exponential decay to an asymptotic value gives a good fit to the data. The assumed form for the curve is

$$\omega = A + Be^{-(t - t_0)/T} \qquad (1)$$

where ω is the spin rate (in rpm), A is its asymptotic value, t is time, t_0 is the initial value of time, and T is the time constant for the spin rate change. B gives the spin rate deviation from the asymptotic value at time t_0. A least squares fit to the set of 134 data points gives the values

A = (55.2289 ± 0.0009) rpm
B = (0.2081 ± 0.0008) rpm
C = (6.092 ± 0.059) min.

The wobble's half cone angle (α) prior to redistribution was computed by bias changes in sun and earth sensor measurements obtained from attitude determinations before and after the event. The angle was estimated to be between 0.38 degree and 0.48 degree.

III. AN APPROXIMATE THEORY

a) Moment of Inertia for Conispheric Propellant Tanks

The INTELSAT IV F-7 spacecraft propellant tanks are mounted symetrically about the spin axis of the spinning rotor section. The rotor showing positioning and orientation system of INTELSAT IV F-7 spacecraft is shown in Figure 2.

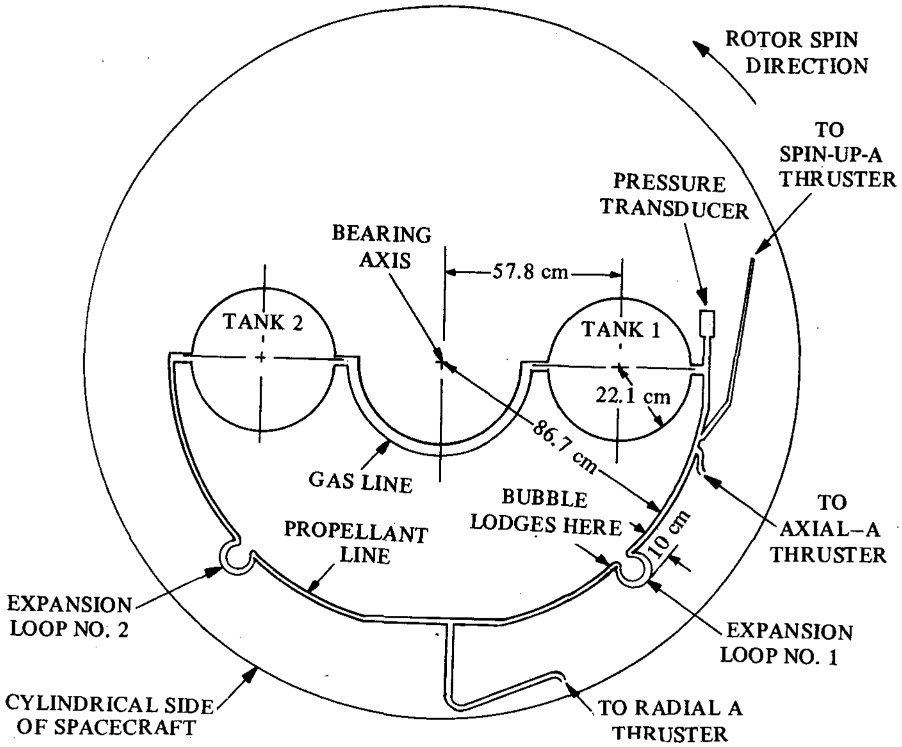

Fig. 2. Rotor showing positioning & orientation system a viewed along the spin axis from the despun antenna platform [Adapted from Vonnegut, 1972]

The propellant tanks consist of a spherical and a conical section, as shown in Figure 3. The included angle of the conical section is 85 degrees. The internal radius of sphere and cone are

22.08 cm and 16.278 cm, respectively.

The moments of inertia of the hydrazine for System 1 about the axis through the center of mass of the spacecraft are listed as functions of propellant mass in Choi (1980).

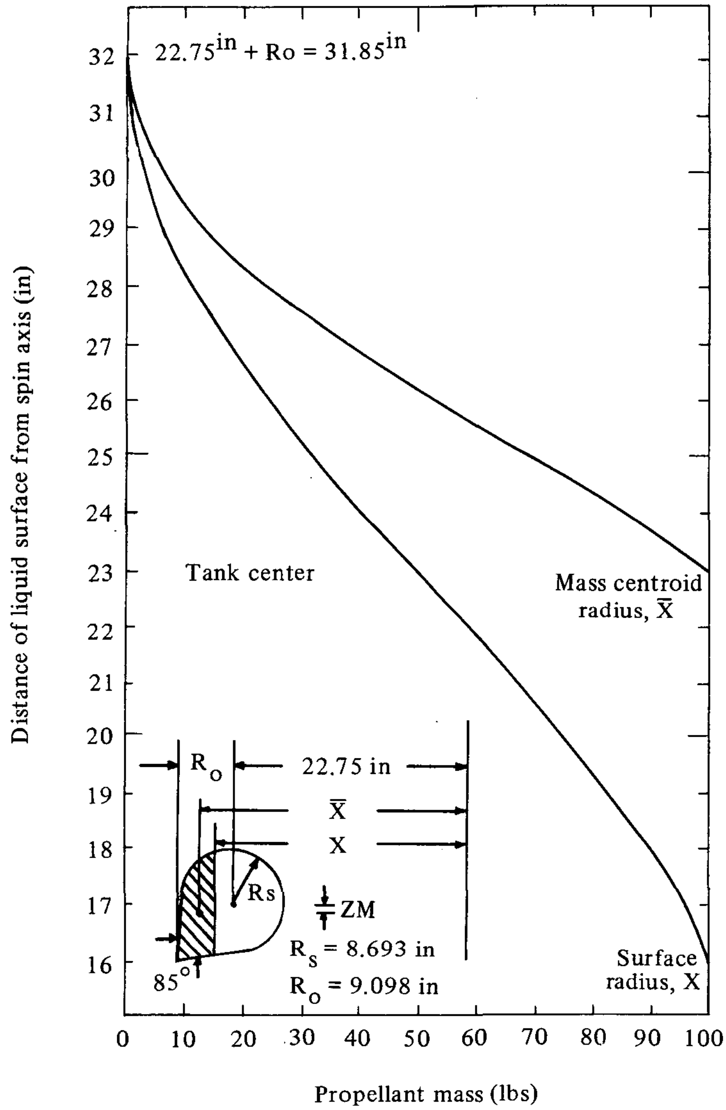

Fig. 3. Distance of liquid surface from spin axis vs. propellant mass

b) Rotor Moment of Inertia Change

Figure 4 shows the effect of mass difference on the spacecraft's coordinate system. Initially, the spacecraft's center of mass is located at 0, with the z-axis along the bearing axis of the despun platform and the center of tank 1 in the xz plane. Due to mass imbalance, the center of mass is displaced from 0 to 0′. The principal axis shift through an angle α to x′z′ and z′ becomes the current spin axis.

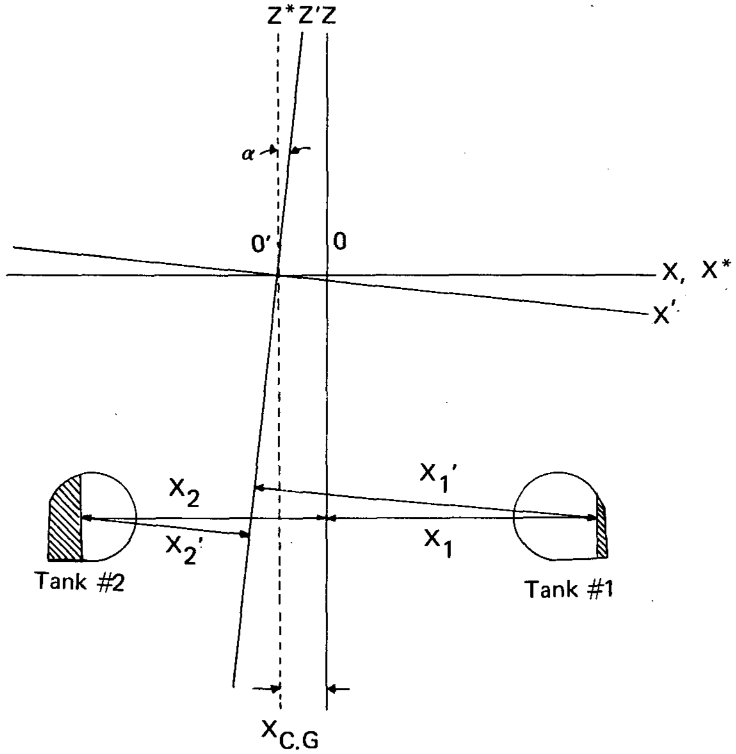

Fig. 4. Mass imbalance effect on spacecraft principal axis

For simplicity, the following assumptions are made for the analysis:

i) neglect the curvature of the free liquid surface and approximate the liquid surface by a level plane perpendicular to the x-axis.

ii) after redistribution, the x and y (transverse) moments of inertia of the spacecraft are equal.

iii) the spacecraft spins about the current principal axis (z-axis) during the redistribution.

iv) the antenna platform is despun on the earth so ω_p = 1 rev/day which is negligible compared to the rotor rate of 55 rpm, but we consider $\omega_p \simeq 0$.

v) neglect the effect of propellant flow in the pipeline.

Before the redistribution, the liquid surfaces in tanks 1 and 2 lie at axial distances x_1 and x_2 shown in Figure 4, respectively. Using the parallel axis theorem, the moments and products of inertia of spacecraft are expressed with respect to the center of mass in the x*y*z* coordinates

$$
\begin{aligned}
I_{xx}^* &= I_{xx} \\
I_{yy}^* &= I_{yy} - M \cdot X_{CG}^{\ 2} \\
I_{zz}^* &= I_{zz} - M \cdot X_{CG}^{\ 2} \\
I_{zx}^* &= I_{zx} \\
I_{yz}^* &= I_{xy}^* = 0
\end{aligned}
\qquad \text{.......} \qquad (2)
$$

where $I_{zx} = (u_2 - u_1)\,\bar{x}\bar{z}$, $(u_2 - u_1)$ is the mass difference between the two Propellant tanks, M is the total mass of the spacecraft (631.96 Kg), $\bar{x}$ and $\bar{z}$ are the location of mass difference, and x_{CG} is the shift in the spacecraft center of mass caused by mass difference (about 1.5 cm).

Let us consider a rotation of coordinate axes to find the principal axis of a system with only one non-zero product of inertia. Suppose the inertia matrix in non-rotated coordinates is of the form

$$
I^* = \begin{pmatrix} I_{xx}^* & 0 & I_{zx}^* \\ 0 & I_{yy}^* & 0 \\ I_{zx}^* & 0 & I_{zz}^* \end{pmatrix} \qquad \text{.......} \qquad (3)
$$

A positive rotation of axes through an angle about the y axis results in a direction-cosine matrix

$$
L = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix} \qquad \text{.......} \qquad (4)
$$

Performing the transformation

$$
I' = L\, I^*\, L^T \qquad \text{.......} \qquad (5)
$$

yields the following results

$$
\begin{aligned}
I_{xx}' &= I_{xx}^* - (I_{xx}^* - I_{zz}^*)\sin^2\alpha - I_{zx}^*\sin(2\alpha) \\
I_{yy}' &= I_{yy}^* \\
I_{zz}' &= I_{zz}^* + (I_{xx}^* - I_{zz}^*)\sin^2\alpha + I_{zx}^*\sin(2\alpha) \\
I_{zx}' &= \tfrac{1}{2}(I_{xx}^* - I_{zz}^*)\sin(2\alpha) + I_{zx}^*\cos(2\alpha)
\end{aligned}
\qquad \text{.......} \qquad (6)
$$

The primed system becomes a set of principal axes if $I_{zx}' = 0$; that is, if α is chosen such that

$$\tan 2\alpha = \frac{2 I_{zx}^*}{I_{zz}^* - I_{xx}^*} \qquad (7)$$

This becomes

$$\tan 2\alpha = \frac{2 I_{zx}}{I_{zz} - I_{xx} - M \cdot X_{CG}^2} \qquad (8)$$

Equation (8) expresses the angular displacement of the spin axis in terms of parameters measured in original coordinates.

For a propellant imbalance, we expect the spacecraft to spin about a new principal axis (z′-axis). This spin axis passes through the spacecraft mass center. Applying the conservation of spacecraft angular momentum to this axis gives

$$I_{zz}' \omega = \text{const} \qquad (9)$$

Because propellant moves from tank 2 to tank 1, the moment of inertia of the spacecraft increases. Therefore, the spin rate of the spacecraft decreases by the above equation (9).

c) Propellant Flow Mechanics During Redistribution

Let us now consider the motion of the liquid between the two tanks, rotating about z′-axis with angular velocity ω. The difference in the radial distances of the liquid level of the tanks produces a static pressure difference between the two tanks as given by

$$\Delta P = \tfrac{1}{2}\rho\omega^2 \left[(x_2')^2 - (x_1')^2 \right] \qquad (10)$$

where ρ is the hydrazine density, and x_1' and x_2' are defined in Figure 4. The above equation (10) includes the shift in spacecraft center of mass caused by the shift in propellant.

From Poiseuille's formulae of flow through a pipe (Landau and Lifshitz, 1959) we obtain

$$\Delta P_f = \frac{128\, \Gamma L W}{\pi \rho D^4} \qquad (11)$$

where Γ is the dynamic viscosity (1.04×10^{-2} g/cm.sec at 20° C), L is the equivalent length of the pipe, W is the mass flow rate ($\frac{du}{dt}$), and D is the tube diameter (0.53 cm).

According to Huson *et al.* (1973), the equivalent length L = 407 cm is the geometric length with a correction added for the bends in the tubing.

The pressure difference of equation (10) must equal to the difference in frictional pressure drop of equation (11) between the two tanks: then

$$\frac{1}{2}\,\rho\,\omega^2[(X_2')^2-(X_1')^2] = -\frac{128\,\Gamma L}{\pi\,\rho\,D^4}W \qquad (12)$$

The equations for the rate of change of propellant tank level are given by

$$\frac{dx_1}{dt} = +\frac{dx_1}{du_1}\frac{du_1}{dt} = -\frac{1}{AREA_1\cdot\rho}W \qquad (13)$$

$$\frac{dx_2}{dt} = +\frac{dx_2}{du_2}\frac{du_2}{dt} + \frac{1}{AREA_2\cdot\rho}W \qquad (14)$$

x_1 and x_2 are equivalent to x in Table 1 and are obtained by linear interpolation as functions of u. $AREA_1$ and $AREA_2$ are the area of the free liquid surface in each tank.

To solve equations (13) and (14) by numerical integration, we used the following steps:

i) assume values of u_1 and u_2 with $u_1 + u_2$ = const.

ii) for each u, the corresponding x is taken from Choi (1980).
Then

$$x_{CG} = \frac{1}{M}(-x_2u_2 + x_1u_1).$$

iii) We compute I_{zx} from equation (2).

iv) IXX, IYY, and IZZ for each u_1 and u_2 in Choi (1980) are added to I_{xx}, I_{yy} and I_{zz} to give total moment of inertia of spacecraft. I^*_{xx}, I^*_{yy}, I^*_{zz} and I^*_{zx} are computed from equation (2). Then α can be found from equation (7).

v) α and I_{zz} in equation (6) determine ω from equation (4).

vi)
$$x_1' = (x_1 + x_{CG})\cos\alpha - \bar{z}\cdot\sin\alpha$$
$$x_2' = (-x_2 + x_{CG})\cos\alpha - \bar{z}\cdot\sin\alpha$$

vii) W can be found from equation (12).

viii) AREA in equations (13) and (14) can be found from equation (A-6) or (A-10).

ix) The differential equations (13) and (14) are then integrated numerically.

x) Then $\Delta u = W\,\Delta t$

$$u_1 = u_1 + \Delta u \text{ and } u_2 = u_2 - \Delta u.$$

xi) To obtain values at the next time step, we repeat the above steps.

IV. NUMERICAL EVALUATIONS OF THE MODEL

For fixed initial values of spin rate and propellant mass ($u_1 + u_2$) in the tanks, the variation of the spin rate was calculated and the final value of spin rate obtained. The appropriate individual values of u_1 and u_2 were determined by a trial and error method until a spin rate change of 0.2 rpm was achieved. The numerical integration was performed using a fourth order Runge-Kutta algorithm.

We list our redistribution model results for INTELSAT IV F-7 spacecraft as follows:

Case 1. u_1 = 0.01 Kg and u_2 = 15.2 Kg

This case corresponds to the predicted values for (u_1 + u_2) from COMSAT's propellant accounting. The mass imbalance of 14.3 Kg corresponds to a half angle of 0.361 degrees. The spin rate time constant is 5.33 minutes. Figure 5 shows that the computed spin rate variation is in poor agreement with the observed data.

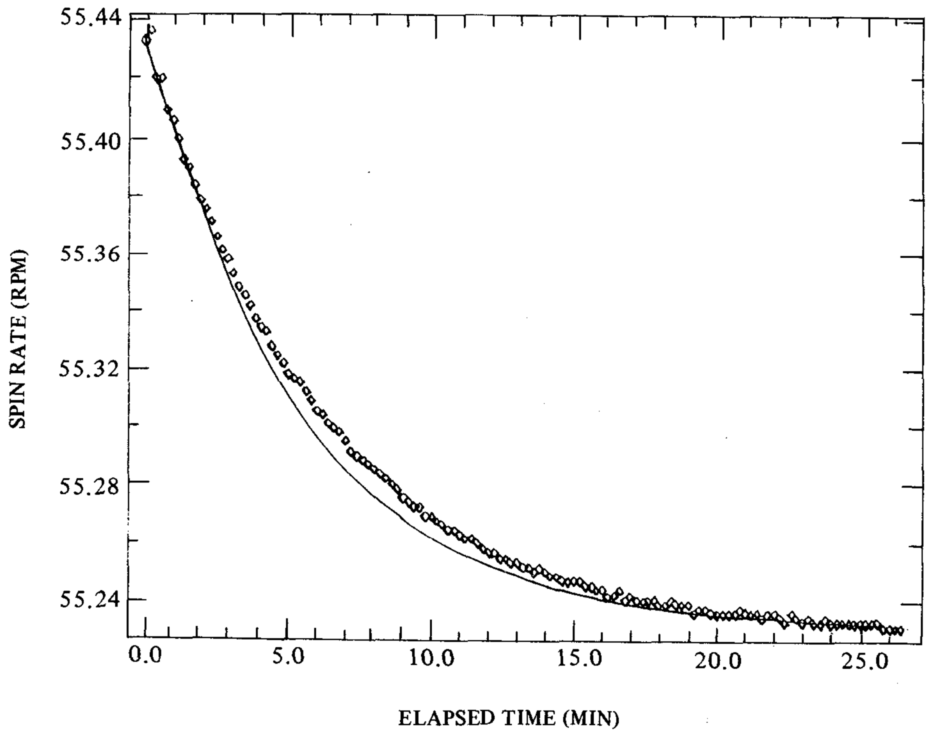

Fig. 5. Comparison of observed data and the model for U_1 = 0.9 Kg and U_2 = 15.2 Kg

Case 2. u_1 = 2 Kg ana u_2 = 17 Kg

This case includes 3 Kg more propellant than the predicted value from COMSAT's propellant accounting. The mass imbalance of 15 Kg corresponds a half angle of 0.370 degrees. The spin rate time constant is 6 minutes. Figure 6 shows that the computed spin rate variation for this case is in excellent agreement with the observed spin rate.

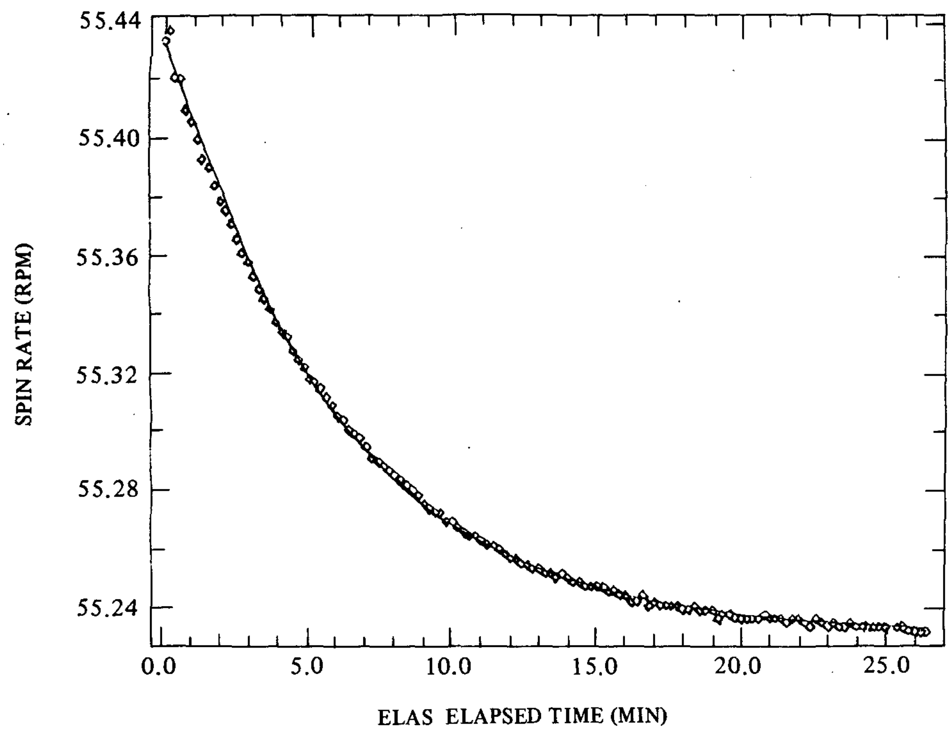

Fig. 6. Comparison of observed data and the model for $U_1 = 2$ Kg & $U_2 = 17$ Kg

Case 3. $u_1 = 2.4$ Kg and $u_2 = 17.5$ Kg

This case includes 3.9 Kg more propellant than the predicted value from COMSAT's propellant accounting. The mass imbalance of 15.1 Kg corresponds a half angle of 0.372 degrees. The spin rate time constant is 6.17 minutes. Figure 7 shows that the computed spin rate variation is in better agreement with the observed spin rate data.

The approximate time constant, T, can be calculated through some algebraic manipulation of equations (12), (13) and (14) to give

$$T \propto L \times AREA \qquad (15)$$

For the case of Gordon *et al.* (1974), AREA is accurately known because for a 50% full tank, AREA does not change appreciably with propellant. They derive a good value L = 407 cm to get a good time constant for a good fit. We adopt this value.

For our case, AREA is uncertain because changing propellant in the tank changes area. We change AREA or equivalently propellant mass until we get a good fit. Equation (15) shows that the small value of time constant results from a small amount of propellant in the tank.

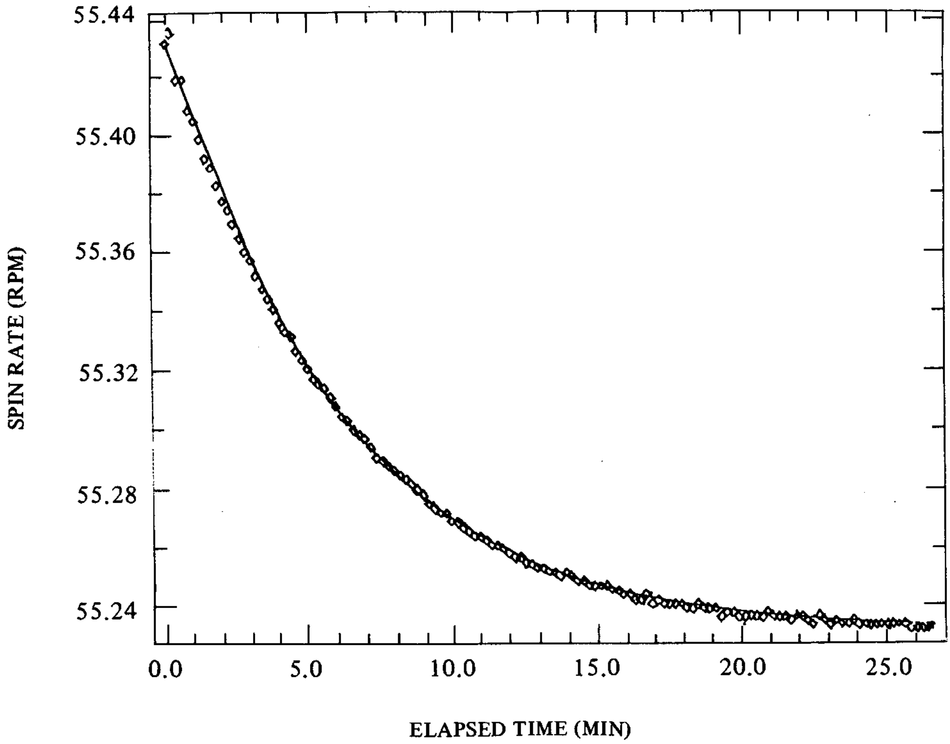

Fig. 7. Co Comparison of observed data and the model for $U_1 = 2.4$ Kg and $U_2 = 17.5$ Kg

V. CONCLUSION

The computer simulation of Gordon *et al.* (1974) gives excellent agreement with the observed spin rate for INTELSAT IV F-3 spacecraft. If the equivalent length of 407 cm is assumed, the model for the predicted propellant accounting is *not* in accordance with the observed spin rate. There is excellent agreement between the model for System 1 with a total propellant mass of 19 Kg and the observed spin rate time constant of 6 minutes.

For INTELSAT IV F-2 Spacecraft radial jet inclination maneuver on April 17, 1980, System 1 ran out of propellant when the predicted value from COMSAT's accounting indicated 3.4 Kg of propellant remained in the System 1. This represents a 5% error in propellant accounting since 68 Kg was initially loaded in the system.

A 5% error in propellant accounting for INTELSAT IV F-7 spacecraft would explain the extra 3.2 Kg of propellant in System 1 found in this paper's analysis.

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