Korean J. Ecology 7(4) : 194~202, 1984

Mathematical and Simulation Models for the Orientation of the Terminal Cladodes of Platyopuntia

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부채仙人掌類의 頂端葉狀莖의 方位에 관한 數學 및 數値模寫 model

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ABSTRACT

The mathematical and simulation models to estimate the monthly average daily solar irradiance on the terminal cladodes of platyopuntia were established. An east-west facing cladode showed maximum irradiance from March to October, while south-north facing one did from November to February from the model. The orientations and the tilt angles were practically measured on Hallim-eup, Cheju-do. They tended to face east-west, but the overall distribution was deviated at about 10° from the direction facing accurate east-west to that facing southern west-northern east. It is suggested that east-west facing cladodes receive maximum solar radiation during the growing season, from the spring to the summer, on the experimental area, and that the deviation of 10° was owing to the southern east wind blowing strongly at that time. The most cladodes inclined to the north or the west rather than erected vertically to the ground. It is thought that the tilt angles were also affected by the southern east wind.

INTRODUCTION

The orientation of the plant leaf critically affects on the interception of photosynthetically active radiation (PAR) and its photosynthesis. The orientation is not determined by inherent genetic information only, but by a tropism according to a vector relation between the sense organ and the external stimuli such as light direction, terrestrial gravity, and etc. (Gillespie & Thimann, 1963; Pickard & Thimann, 1964). Cacti have a special significance for their orientations because their photosynthetic organs are the rigid stems through which PAR cannot pass, while thin leaves of C_3 and C_4 plants are relatively flexible and their orientations can be changed by the wind or by diurnal tracking movements (Gates, 1980). Photosynthetic organ of cacti erects vertically to the ground, and east-west facing cladodes of *Opuntia amyclaea* produced more dry matter in the fall than north-south facing ones at 16°N in Mexico (Rodriquez *et al.*, 1976).

On the survey of the orientation of platyopuntia cladodes, a preferred east-west one has been suggested for *O. compressa* (Baskin & Baskin, 1973) but this was apparently not supported by direct measurement (Abrahamson & Rubinstein, 1976). However, a subsequent study showed statistically significant orientation. Cladodes of *O. echios* in Gal'apagos Islands, *O. stricta* in Florida, and *O. chlorotica* at eastern Sonoran site tended to face east-west, while *O. chlorotica* tended to face south-north in a Western Sonoran site and the Mojave Desert (Nobel, 1980; Nobel, 1981b).

In the areas with a tendency of east-west, most of the rainfall occurred in the summer, on the other hand in the area showing a tendency of north-south in the winter. The preferred orientation maximized PAR interception at times of the year when rainfall favored growth (Nobel, 1982).

In this paper, a model to estimate the monthly solar irradiance on a vertical cladode is to be established. From this model an orientation where a cladode receive maximum solar radiation can be determined. The number of cladodes of the orientation is to be practically examined and compared with that of other orientation.

MATERIALS AND METHODS

The orientation of terminal cladodes of *Opuntia lanceolata* Haw. was examined at Wolyong-ri, Hallim-eup, Cheju-do, Korea (126°9′E, 33°23′N) on 21 May, 1982 (Fig. 1). Cheju-do is a volcanic island, and its ground is composed of basalt. The seashore site and the top of the basalt wall of which height was about 2∼2.5 m and width was 2.4 m, were chosen as investigating plots. The top of the stone wall was flat and covered with a little sand. Soil moisture was insufficient for common plant growth in the both areas, so that dense *Opuntia* community was formed only with a few species having a strong tolerance against the water deficiency such as *Chenopodium album, Calystegia soldanella, Polygonum aviculare,* and etc.

The platyopuntias of about 60 cm height were chosen for experiments and the orientation of the terminal unshaded cladodes was examined with a compass. All angles were measured clockwise from true south, so that a cladode with the surface azimuth angle of 0° faces south.

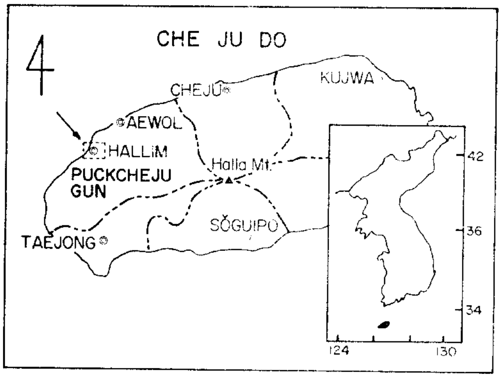

Fig. 1. Geographical map of the studied area.

The tilt was measured as an angle made between the axis of a cladode and the horizontal surface.

Solar irradiance was measured with a temcometer at intervals of 30 minutes on the both sides of *Opuntia* facing north-south (N-S), east-west (E-W), southern east-northern west (SE-NW), and southern west-northern east (SW-NE) individually. The solar radiation interception on a vertical surface was obtained for different orientations by mathematical formula.

Soil water contents were shown as water weight per 100 g of oven dried soil. Soils were dried for 24 hours at 105°C. Salinities of soils were measured by titration with $AgNO_3$.

RESULT AND DISCUSSION

Mathematical and simulation models

The average daily radiation for each calender month on tilted surfaces facing directly towards the equator was estimated by Liu and Jordan (1962). It was then extended to calculate radiation on tilted surfaces oriented east or west of south by Klein (1977). According to these methods, mathematical models to calculate radiation on tilted surfaces of plant leaves depend on measurement of average daily radiation for each month on a horizontal surface of plant leaves, $\bar{I}$, and the calculation of $\bar{I}_0$, the mean daily extraterrestrial radiation for each month, where

$$\bar{K}_T = \bar{I}/\bar{I}_0 \cdots\cdots(1)$$

The average daily radiation on a tilted surface at plant leaves, $\bar{I}_T$, is expressed as

$$\bar{I}_T = R\bar{I} = \bar{R}\bar{K}_T\bar{I}_0$$

where R is the ratio of the daily average radiation on a tilted surface of plant leaves to that on a horizontal surface of plant leaves for each month.

$\bar{I}_T$ is made up of direct beam radiation, sky diffuse radiation, and reflected radiation from the ground. The diffuse and reflected components are assumed to be isotropic. As the method of Liu and Jordan (1962), $\bar{R}$ is given by:

$$\bar{R} = (1-\bar{I}_d/\bar{I}) \cdot \bar{R}_b + (\bar{I}_d/\bar{I})$$
$$(1+\cos S)/2 + \rho(1-\cos S)/2 \cdots\cdots(3)$$
$$\bar{I}_T = (\bar{I}-\bar{I}_d)\bar{R}_b + \bar{I}_d$$
$$(\bar{I}+\cos S)/2 + \bar{I} \cdot \rho(1-\cos S)/2 \cdots\cdots(4)$$

where Id is the monthly average daily diffuse radiation, $\bar{R}_b$ is the ratio of the average beam radiation on the tilted surface of plant leaves to that on the horizontal surface of plant leaves for each month, S is the tilt of the surface of plant leaves from horizonal surface of plant leaves, and ρ is the ground reflectance. In the case of cladodes of platyopuntia erected vertically to a horizontal surface, $\bar{I}-\bar{I}_d$, $\bar{I}_d(1+\cos S)/2$, $\bar{I} \cdot \rho \cdot (1-\cos S)/2$ are assumed to be constant inspite that the orientations of cladodes are different each other. Therefore, the average daily radiation on a surface vertically erected, $\bar{I}_T$, is proportional to the $\bar{R}_b \cdot \bar{R}_b$ is calculated as the ratio of extraterrestrial radiation on a tilted surface of plant leaves to that on a horizontal one for each month, i.e. as an entirely geometric concept.

When a tilt of a surface is 90°, $\bar{R}_b$ is given by

$$\bar{R}_b = (-\sin\delta\cos\phi\cos\gamma)\pi/180(W_{ss}-W_{sr})$$
$$+(\cos\delta\cos\gamma\sin\phi)(\sin W_{ss}-\sin W_{sr})$$
$$-\cos\delta\sin\gamma\,(\cos W_{ss}-\cos W_{sr}))$$
$$/2(\cos\phi\cos\delta\sin W_s +$$
$$\pi/180 W_s \sin\phi\sin\delta) \cdots\cdots(5)$$

where ϕ is the latitude and δ is the solar declination, γ is the surface azimuth angle: the deviation of the normal to the surface from the local meridian, the zero point being due south, east negative, and west positive. W_s is sunset hour angle on a horizontal surface, given by $W_s = \mathrm{Arcos}(-\tan\phi\tan\delta)$. W_{sr} and W_{ss} are the sunrise and sunset hour angles on the tilted surface. If the daily radiation on the opposing sides of a platyopuntia cladode is given as $\bar{I}_{T_1}$, $\bar{I}_{T_2}$ individually, the sum of both incident radiation, $\bar{I}_s$, can be expressed by:

$$\bar{I}_s = \bar{I}_{T_1} + \bar{I}_{T_2} = (\bar{I}-\bar{I}_d)(\bar{R}_{b1}+\bar{R}_{b2}) +$$
$$\bar{I}_d(1+S) + \bar{I} \cdot \rho \cdot (1-\cos S) \cdots\cdots(6)$$

Owing to the same reason in the equation (4), $\bar{I}_T$ proportional to the $\bar{R}_{b1}+\bar{R}_{b2}$. When the azimuth angles of the opposing surfaces of a cladode are γ_1 and γ_2 ($\gamma_2 \geq 0$, $\gamma_1 < 0$), $Rb_1 + Rb_2$ is given by;

$$Rb_1 + Rb_2 =$$
$$(-\cos\gamma_1\sin\delta\cos\phi((W_{ss1}-W_{sr1})$$
$$-(W_{ss2}-W_{sr2}))\pi/180$$
$$+\cos\gamma_1\cos\delta\sin\phi((\sin W_{ss1}-\sin W_{sr1})$$
$$-(\sin W_{ss2}-\sin W_{sr2}))$$
$$-\sin\gamma_1\cos\delta((\cos W_{ss1}-\cos W_{sr1})$$
$$-(\cos W_{ss2}-\cos W_{sr2})))$$
$$/2(\cos\phi\cos\delta\sin W_s$$
$$+\pi/180\, W_s \sin\phi\sin\delta) \cdots\cdots(7)$$

where γ_2 is equal to (γ_1+180), W_{sr1} and W_{ss1} are sunrise and sunset hour angle on the cladode side of negative azimuth angle, and W_{sr2} and W_{ss2} are those of positive azimuth angle, given by

$$W_{sr1} = -\mathrm{arcos}(-\tan\delta\tan\phi)$$
$$W_{sr1} = \gamma_1 + 90°$$
$$W_{sr2} = \gamma_2 - 90°$$
$$W_{ss2} = \mathrm{arcos}(-\tan\delta\tan\phi) \cdots\cdots(8)$$

If the equation (8) is substituted to the equation (7), $\bar{R}_{b1}+\bar{R}_{b2}$ is given by:

$$\bar{R}_{b1}+\bar{R}_{b2} = (-\cos\gamma_1\sin\delta\cos\phi(\gamma_1+90)\pi/180$$
$$+\cos^2\gamma_1\cos\delta\sin\phi$$
$$+\sin^2\gamma_1\cos\delta - \sin\gamma_1\tan\delta\tan\phi)$$
$$/(\cos\phi\cos\delta\sin W_s$$
$$+\pi/180\, W_s\sin\phi\sin\delta) \cdots\cdots(9)$$

The data of $\bar{R}_{b1}+\bar{R}_{b2}$ were calculated by FORTRAN program (Appendix I).

As shown in Fig. 2, the simulated data of the monthly average radiation ratios for all of the

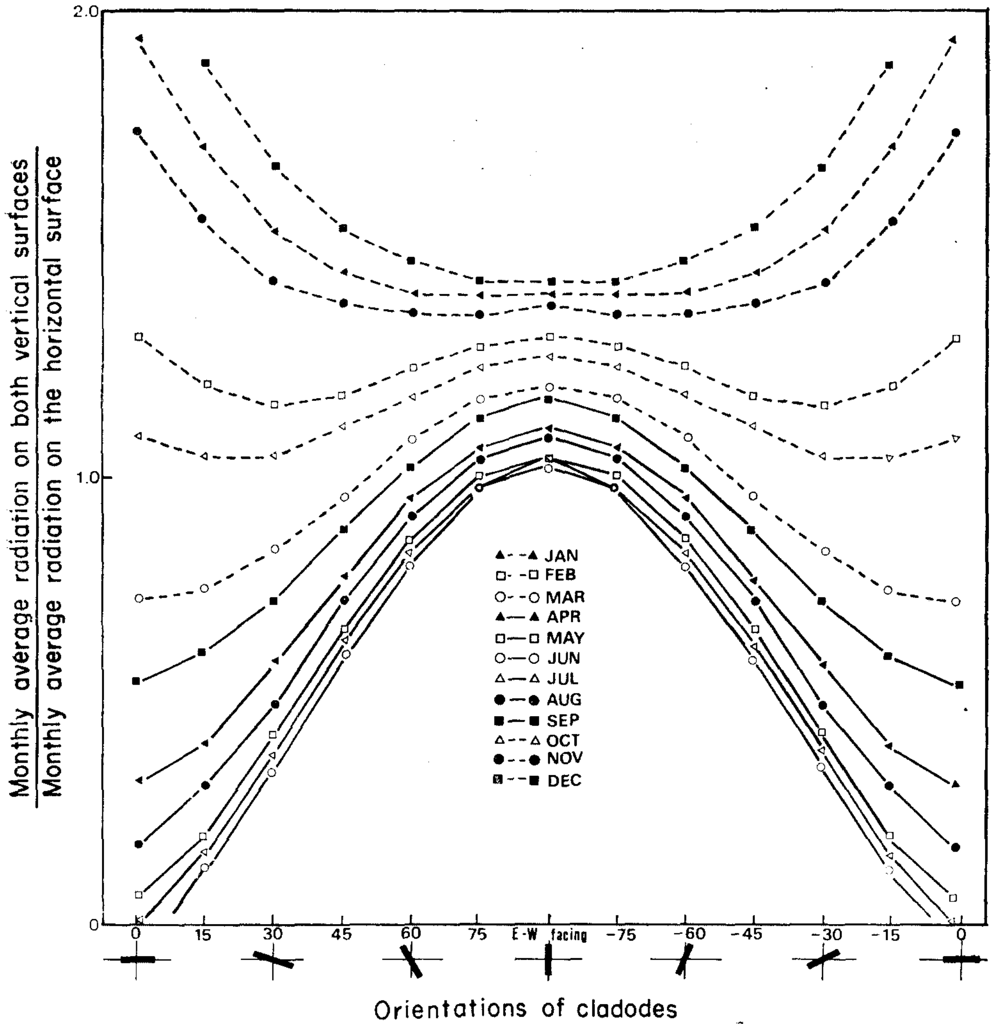

Fig. 2. The simulated diagram of the monthly average radiation ratio, $R_{b1}+R_{b2}$, for all of the months of a year.

months of the year were graphed by FORTRAN.

Experimental results

Investigating area, Cheju-do is a volcanic island located in southern end of Korea (Fig. 1), which belongs to the warm temperature zone. Monthly averages of daily mean temperatures and total precipitation on this area are represented in Fig. 3. The climate of the investigating area is not arid but the basalt ground on which *O. lanceolata* was growing had a small amount of dry surface soil. The surface soil showed high salinity in addition to its dryness because the sites locate near the seashore. The high salinity of soil may add water stress to plants.

The soil water contents were 0.90% and 16.12%, and the salinities were 5.22% and 3.48% on the top of the basalt wall and on the seashore site, respectively. These results suggest that *Opuntia* could grow as a main species on these sites owing to the dry condition of the soil.

PAR is a limiting factor on the growth of platyopuntia because vertical photosynthetic surfaces generally intercept considerably less incident PAR than horizontal surfaces, as well as often receive no direct PAR at all (Gates, 1980). Mathematical model to calculate radiation on vertical surfaces of the cladodes was established by modifing Klein's equation (1977) in order to decide an orientation where the cladode receives the maximum radiation. The monthly ratio of the solar irradiance on the cladode surface to that on the horizontal one was computed from the equation (9). E-W facing cladode showed maximum irradiance from March to October in all of different orientations. From November to February was the irradiance maximized on the surface of S-N facing cladodes (Fig. 2). The results calculated from our mathematical equation agrees to Nobel's (1980, 1981a, 1981b, 1982). When the growing season of *Opuntia* was from March to October the cladodes tended to face east-west, while the tendency to face north-south occurred in the area where the growing period was from November to February. The monthly average radiation ratios, $R_{b1}+R_{b2}$, not only explain convincingly the tendency of orientation, but solar radiation on any tilted surface can be also calculated by extending the equation.

According to the experimental data of the solar radiation on vertical surfaces and the orientation of *Opuntia* cladodes, the total daily amount of solar radiation was the most on surfaces of E-W facing cladodes (Table 1). This result corresponds with the theoretical ratio of solar radiation taken on the vertical surfaces of cladodes from the equation (9); the E-W facing cladode showed the maximum ratio and the ratio was decreased for the cladode to rotate toward the S-N facing orientation on May (Fig. 2).

Table 1. Experimental total daily amounts of solar radiation by the orientations of the cladodes of *O. lanceolata* at Wolyong-ri, Hallim-eup, Cheju-do on 21, May, 1982.

Cladode orientation	S-N facing	NE-SW facing	E-W facing	NW-SE facing
Daily solar irradiance ($cal \cdot cm^{-2} \cdot day^{-1}$)	213.48	371.04	398.28	379.92

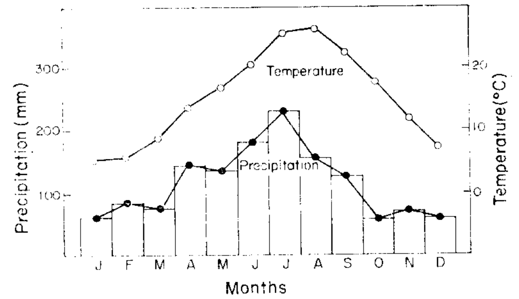

Fig. 3. Monthly means of daily average temperatures and total precipitation at Hallim-eup, Cheju-do.

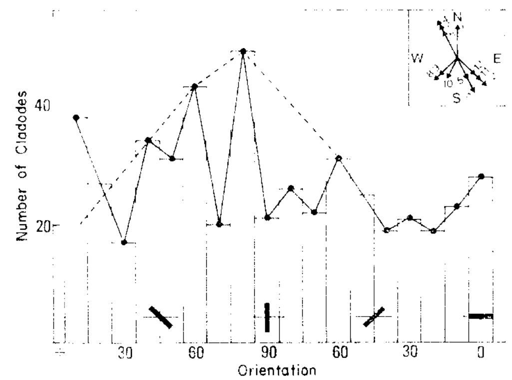

Fig. 4. The orientations of the terminal cladodes of *O. lanceolata* at Wolyung-ri, Hallim-eup, Cheju-do (126°9′, 33°23′N). The heavy line at the lower part represents the cladode orientation. The arrows and the figures at the upper right corner show the wind direction and the month, respectively.

The orientations of terminal cladodes were measured with the compass on the survey sites (Fig. 4). The terminal cladodes tended to face east-west, but the over-all distribution was deviated at about 10° from the direction facing east-west to that facing southern west-northern east. Because most growth of *Opuntia* occurred from April through September on Cheju-do and E-W facing cladodes receive maximum solar radiation for the period, E-W facing tendency of cladodes can be explained well. The relation between the productivity and the orientation of the

Table 2. Wind speed(m/sec) at Hallim-eup, Cheju-do (Central Meteorological Office, Annual Report)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Daily mean	2.6	3.2	3.1	3.1	2.0	2.3	2.9	2.3	1.9	1.6	2.6	2.5
Maximum	8.5	10.0	12.0	14.0	14.5	13.0	9.5	8.0	11.0	10.0	9.0	14.5
Wind direction	ESE	NW	ESE	SSE	SE	ESE	ESE	ENE	ENE	SE	NW	SE
The most wind direction	N	NW	NNW	SSE	NNW	SSE	SSE	NE	NE	NNE	NW	NE

cladodes were studied by Rodriquez *et al.* (1976); E-W facing cladodes of *O. amyclaea* produced more dry matter than S-N facing ones. More dry matter were produced at the cladode oriented to receive more light energy. Besides *Opuntia* orientation, the effect of off-south orientation on the performance of vertical solar panels was investigated to promote the efficiency of solar heat gain (Lorsch & Niyogi, 1971).

It was seen in vertical collectors that the effectiveness actually increase with increasing azimuthal angle (Felske, 1978), and this is consistant with the ASHRAE heat gain factors for vertical windows (ASHRAE, 1972). This occurrs because the vertical collector will capture more of the summer radiation when oriented off-south since in the off-south position the summer sun is more normal nearly to the collector during more hours of the day than in the due-south position.

The number of the cladodes to face SWW-NEE deviated at 10° from E-W facing ones were the most, and the number of SE-NW facing cladodes was remarkably fewer than SW-NE facing ones.

It is thought that the tendency might be owing to a southern east wind which blew strongly in the growing season. Cheju-do is an island to be famous for the abundant stone and wind. Particularly wind blows strongly almost throughout an year. The direction and the speed of the wind were represented monthly in Fig. 4 and Table 2. A southern east wind in April, June and July was very strong with a northern west wind in March and May. The maximum wind velocity reached 14.5m/sec on May. Wind affects plant growth, reproduction, distribution, death, and ultimately plant evolution (Nobel, 1981a). SE-NW facing cladodes became knocked so violently against these winds because they faced almost normally to the wind direction that cladodes of this orientation might remain at a low frequency. It is suggested that the cladodes orient with the wind to the direction reducing the force if they had a strong pressure to wind flux.

The tilt angle against a horizontal ground was also investigated on each cladode according to its orientation(Fig. 5). The tilt angles of S-N facing cladodes are shown in Fig. 5.

Cladodes inclining to the north at about 7.5° against the vertical angle were more than those erected vertically to the ground. Stems of plants have a geotropism growing in the opposite direction to the gravity by plant hormonal action in order to intercept much sun light (Oppenoorth, 1941). Nobel (1981b) reported that longitudinal axis of cylindrical stems tended to tilt toward the south in the northern hemisphere and toward the north in the southern hemisphere. But the axis of most platyopuntia cladodes tended to incline toward the north in our investigation though Korea locates in northern hemisphere. It may be the effects of a strong southern east wind. Most of SE-NW facing cladodes inclined toward west at about 5° against the normal axis to the horizontal ground, and E-W facing ones tended to tilt toward the west. It is thought that the tilt angles of cladodes were affected by the southern east wind belowing strongly at the growing season. The SW-NE facing cladodes which stood avoiding the southern east wind inclined toward the southern west. It may be an effect of the northern east wind belowing strongly in August, September, and October.

The orientation of cladode is mainly determined by the phototropism to intercept the maximum

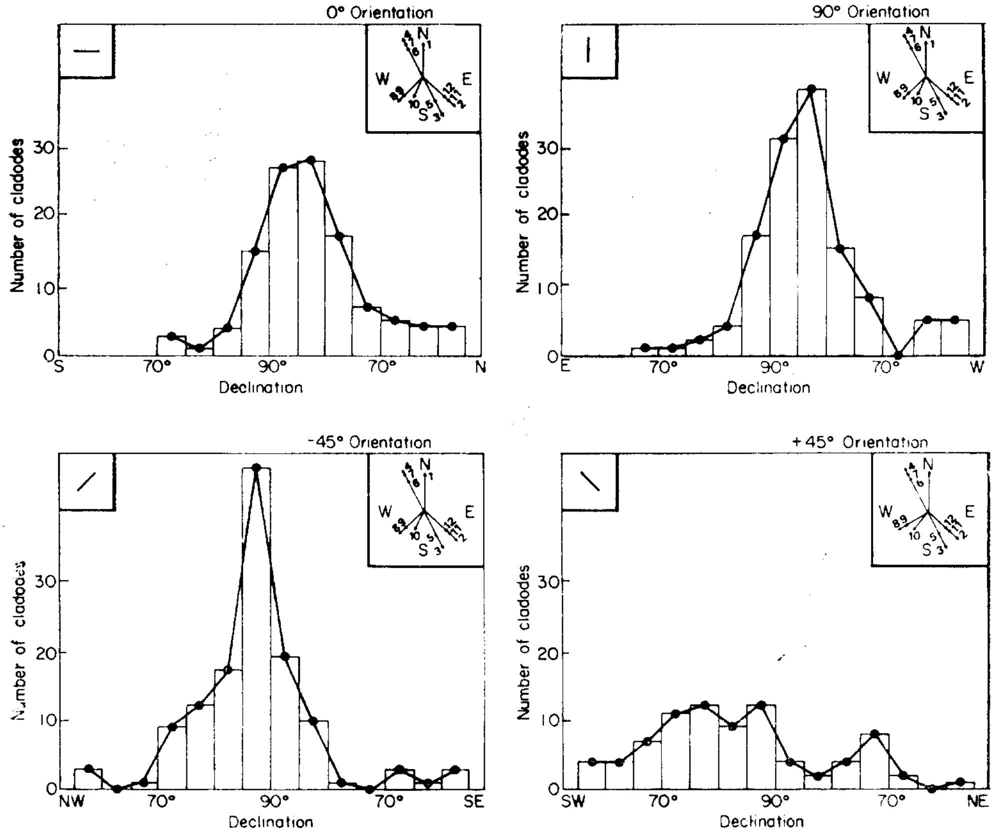

Fig. 5. Distribution of the declination for the cladodes facing N-S(0°), E-W(90°), NW-SE (−45°), NE-SW(+45°). The arrows and the figures in the upper right box represent the wind direction and the month respectively, and the line at the upper left corner shows the cladode orientation.

radiation, and other environmental factors, such as barrier, wind, and etc., are also to influence the orientation of cladode considerably.

摘　要

垂直으로 자라고 있는 부채仙人掌類의 頂端葉狀莖에 관한 月 平均 日受光量을 表示하는 數學 및 數値模寫 model을 定立하였다. 그 model에 의하면 3月부터 10月까지는 南-北方位의 葉狀莖에서 受光量이 가장 많았고 11月부터 2月까지는 東-西方位의 葉狀莖에서 가장 많았다.

실제로 濟州道 翰林邑 月令里에서 生育하고 있는 부채仙人掌 群落을 選擇하여 葉狀莖 方位別 分布를 調査해 본 結果, 南-北方位로부터 西北-東南 方位方向으로 約 10° 程度 回轉된 方位의 葉狀莖數가 가장 많았다. 이는 生長期間이 제주도에서는 대체로 봄부터 여름 사이이므로 이 生長期에 가장 受光量이 많은 南-北方位로 分布하고 있으며 10° 程度의 편차를 나타내는 것은 이 時期에 强하게 부는 東南風과 北西風에 기인된다고 사료된다. 또한 葉狀莖의 傾斜角이 垂直으로부터 5°~10° 程度로 기울어진 것도 葉狀莖이 서있는 方位에서 바람에 의해 받게 되는 壓力때문이라고 추정된다.

Appendix I. FORTRAN computer program used to solve the equation (9)

```
C   GRAPH OF THE FUNTION R
    INTEGER GRR(101), D(12), BLANK, UBAR
    DIMENSION T(12), R(12,12)
    DATA D/1H1, 1H2, 1H3, 1H4, 1H5, 1H
    6, 1H7, 1H8, 1H9, 1HA, 1HB, 1HC/,
 *          BLANK/1H/,  UBAR/1HI/
    DATA T/-0.116, -0.072, -.013, .052, .104,
    .128,
 *              .118, .075, .012, -.053, -.105,
    -.128/
    PI=3.14159265
    P=0.185*PI
    OPEN(6, FILE="KIMOUT 2", CARRIAGECON
    TROL="FORTRAN")
    WRITE(6,10)
 10 FORMAT(1H////5X, "X", 13X, 101('-')/1H0,
    4X, '---')
    IX=-180
    DO 100 I=1, 12
    XPI=IX*PI/180.
    DO 200 J=1, 101
    GRR(J)=BLANK
200 CONTINUE
    GRR(1)=UBAR
    DO 300 K=1, 12
    TPI=T(K)*PI
    WS=ACOS(-TAN(P)*TAN(TPI))
    R(I,K)=(-COS(XPI)*SIN(TPI) * COS(P)*
    (XPI+PI/2.)
 *          +COS(TPI)*SIN(P)*COS(XPI)**2
 *          +COS(TPI)*SIN(XPI)**2
 *          +COS(TPI)*SIN(XPI)*COS(WS))
 *          /(COS(P)*COS(TPI)*SIN(WS)+WS
    *SIN(P)*SIN(TPI))
    IF (R(I,K)) 300 6,7
  7 IF(R(I,K), GE, 2,0) GO TO 300
  6 M=R(I,K)*45.0
    GRR(M)=D(K)
300 CONTINUE
    WRITE(6,20) IX, GRR
 20 FORMAT(4X///4X, I4, 10X, 101A1)
100 IX=IX+15
    WRITE(6,30) ((R(I,K), I=1,12), K=1,12)
 30 FORMAT (1H0, 24(//10X, 6F9.5))
    STOP
    END
```

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(*Received September* 5, 1984)