論文 84-21-3-11

The Error Rate Performance of APK System in the Presence of Interference and Noise

(간섭과 잡음의 존재하에서 APK 시스템의 오율 특성)

蔡 鍾 元*, 孔 炳 玉**, 趙 成 俊**

(Jong Won Chai, Byung Ock Kong and Sung Joon Cho)

要 約

본 논문에서는 동일 채널 간섭과 잡음 환경하에서의 L레벨 ASK 및 多相 PSK, APK 시스템의 성능을 오율의 관점에서 究明하였다. ASK 신호와 PSK 신호에 대한 오율식을 이용하여 반송파대 잡음 전력비, 반송파대 간섭파 전력비 및 간섭파의 포락선 분포의 함수로서 APK 시스템의 오율 특성을 밝히고 PSK 시스템 및 ASK 시스템, QAM 시스템, APK 시스템의 성능을 비교 검토하였다.

Abstract

In this paper, the error rate performance of L-level amplitude shift keying (ASK), M-ary phase shift keying (PSK) and amplitude phase keying (APK) systems have been studied in the presence of interference and noise.

Using the derived error probability equations, the error rate performance of each L-level ASK and M-ary PSK system has been evaluated in terms of carrier-to-noise power ratio (CNR), carrier-to-interferer power ratio (CIR), and envelope distribution of interferer.

These results are combined and then the error rate performance of APK signal has been found. Finally, the error rate performance is compared and discussed.

I. Introduction

The increasing demands of data transmission with limited bandwidth point to the need for modulation techniques that are highly efficient utilization of various media for transmission of digital data, e.g., the M-ary PSK is one of such technique requiring less bandwidth than FM or biphase PSK (BPSK).

In addition to spectrum efficient angle modulation technique such as PSK, digital radio transmission may use quadrature amplitude modulation(QAM) or combined amplitude and phase modulation(APK) when the bandwidth efficiencies of 3 bits/s/Hz or higher is needed.[1]

APK requires less power than PSK for the same error probability and alphabet size. Therefore, APK is a potentially attractive modulation method for satellite communication and the investigation have been undertaken to examine its performance for a transponder channel.

*正會員, 三星半導体通信(株)研究所

(Telecom. Research Institute, SS Semiconductor & Telecom. Co., Ltd.)

**正會員, 韓國航空大學 通信工學科

(Dept. Comm. Eng., Hankuk Aviation College)

接受日字：1983年 12月 9日

The ever increasing demand and supply for communication channel in RF bands causes a serious problem of electromagnetic interference. The most important sources of interference are cochannel and adjacent channel interference. But now in this paper, especially cochannel interference which is generated by the reuse of existing band in use is considered.

In most case of analyzing interference, constant envelope of interferer is assumed. But most radio signals are affected by ionospheric and tropospheric modes of propagation. So the envelopes of signals undergo a change by fading channel.

In this paper, we derived the error rates of L-level ASK and M-ary PSK in the environment of Gaussian noise and cochannel interference whose envelope suffers a change by fading channel. And combining these results, we evaluated and discussed the error rate performance of APK system.

II. Analysis Model

The analysis model is presented in Fig. 1. The transmission channel is assumed to be transparent so that the ASK or PSK signal arrives undistorted at the front end of ASK or PSK receiver, respectively.

There are both a stationary white Gaussian noise and interferer of m-distributed envelope. Then the errors will be due to the combined effect of interference and noise. Here we assumed the receiver consisted of an ideal bandpass filter and detector. The detector is ideal L-level detector in the case of ASK signal and ideal phase detector in the case of PSK signal.

A 2L bit ASK signal received during the N'th interval with signaling interval T and carrier angular frequency ω_c can be represented as[2]

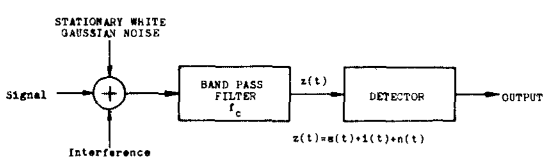


Fig. 1. Analysis model.

$$s_A(t) = S_i \cos \omega_c t \quad NT \leq t \leq (N+1)T \quad (1)$$

where $S_i \in \pm d, \pm 3d, \ldots, \pm(2L-1)d$;
the amplitude of signal,
2d; the distance between two adjacent signal points,
L; the number of levels of ASK signal.

PSK signal can be represented as

$$s_p(t) = S \cos (\omega_c t + 2\pi \frac{\lambda}{M}) \quad (2)$$

where S is the amplitude of signal, ω_c the carrier angular frequency, M the number of levels, and λ (=0, 1, ..., M-1) the M-ary information. The probability of occurrence of each information is assumed to be same.

In recent years, radio engineering requirements have become more stringent and necessitate not only detailed information median signal intensity but also much more exact knowledge on fading statistics in both ionospheric and tropospheric modes of propagation. Such tendencies have forced a large number of extensive experiments and theoretical investigations to be performed on the intensity distribution of fading under various conditions.

In this paper, we introduced the Nakagami's m-distribution equation as interferer's envelope. This formula was deduced by Nakâgami(1943) from his large-scale experiments on rapid fading with long distrances propagation in HF band and Fig. 2 shows the m-distribution.

The interferer can be written as

$$i(t) = I \cos (\omega_c t + \phi) \quad (3)$$

where ϕ is the phase difference between signal and interferer, then, the propability density function(p.d.f.) of envelope and phase are represented as follows;[3]

$$p(I) = \frac{2m^m I^{2m-1}}{\Gamma(m)\, \Omega^m} \exp(-m \frac{I^2}{\Omega}) \quad (4)$$

$$p(\phi) = \frac{1}{2\pi} \quad 0 \leq \phi \leq 2\pi \quad (5)$$

where $m = \frac{(\overline{I^2})^2}{\overline{(I^2 - \overline{I^2})^2}} \geq \frac{1}{2}$ (fading figure)

$$\Omega = \overline{I}^2$$

As shown in Fig. 2, m=1/2 and m=1 present the one sided Gaussian and Rayleigh p.d.f., respectively, and $m \to \infty$ corresponds to no fading.

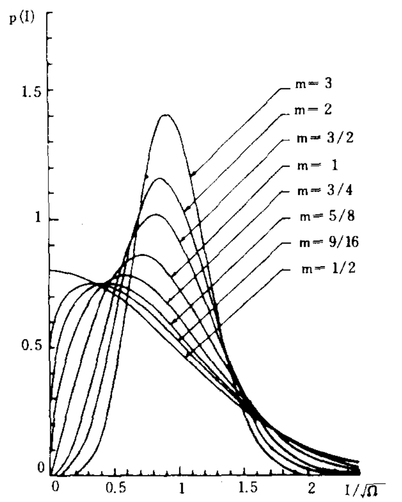


Fig. 2. The m-distributed p.d.f.

In this paper, we consider the Gaussian noise only. The assumption of narrow band noise(bandwidth $B \ll f_c$) enables us to state that the inphase term $n_c(t)$ and the quadrature term $n_s(t)$ in the expression

$$n(t) = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t \qquad (6)$$

varied "slowly enough" with respect to the carrier that an envelope could be defined. The p.d.f. of $n_c(t)$ and $n_s(t)$ are represented by Gaussian form.

$$P(n_c) = \frac{1}{\sqrt{2\pi}\,\sigma_n} \exp\left(-\frac{{n_c}^2}{2\sigma_n^2}\right) \qquad (7)$$

$$P(n_s) = \frac{1}{\sqrt{2\pi}\,\sigma_n} \exp\left(-\frac{{n_s}^2}{2\sigma_n^2}\right) \qquad (8)$$

III. The Error Rate Performances

1. L-level ASK System

We show in Fig. 3 the phasor diagram of the received L-level ASK signal corrupted by Gaussian noise and interferer whose envelope is m-distributed.

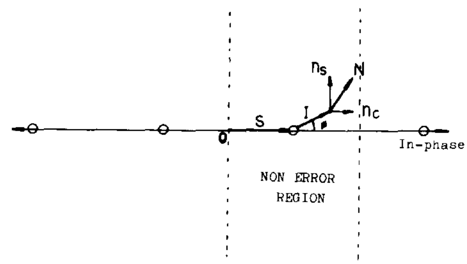


Fig. 3. Phasor diagram.

When we denote the received composite signal z(t), then

$$\begin{aligned} z(t) &= s_A(t) + i(t) + n(t) \\ &= S_i \cos \omega_c t + I \cos(\omega_c t + \phi) + n(t) \qquad (9) \end{aligned}$$

When equal thresholds are assumed, an error occurs under the following conditions:

$$\begin{aligned} &\text{if } S_i \neq (2L-1)d \text{ and } |Z_1| > d \\ &\text{if } S_i = (2L-1)d \text{ and } Z_1 < -d, \text{ and} \\ &\quad S_i = -(2L-d)d \text{ and } Z_1 > d \qquad (10) \end{aligned}$$

where $Z_1 = I \cos\phi + n_c$ (11)

Therefore we can obtain the error probability conditioned on the amplitude and phase of interferer, i.e.,

$$P_E(I, \phi) = \frac{2L-1}{2L} \text{ Prob.} \,|_{I,\phi} (|Z_1| > d) \qquad (12)$$

Eq.(12) can be rewritten by using Eq.(11) as

$$\begin{aligned} P_E(I,\phi) = \frac{2L-1}{2L} & [\text{Prob.}(-\infty < n_c < -d - I\cos\phi) \\ & + \text{Prob.}(d - I\cos\phi < n_c < \infty)] \qquad (13) \end{aligned}$$

Integrating Eq.(13) over the error region, we get

$$\begin{aligned} P_E(I,\phi) = \frac{2L-1}{4L} & \left[\text{erfc}\left(\sqrt{\alpha} + \frac{I\cos\phi}{\sqrt{2}\,\sigma_n}\right)\right. \\ & \left. + \text{erfc}\left(\sqrt{\alpha} - \frac{I\cos\phi}{\sqrt{2}\,\sigma_n}\right)\right] \qquad (14) \end{aligned}$$

where $\alpha = \dfrac{d^2}{2\sigma_n^2}$ = CNR (carrier-to-noise power ratio). The average error probability is the average of conditional error probability over I and ϕ.

$$P_E = \int_0^{\infty} \int_0^{2\pi} P_E(I,\phi) d\phi dI \qquad (15)$$

Carrying out the integration by the use of Eq.(14), we get

$$P_E = \frac{2L-1}{4L} \left[2\, \mathrm{erfc}(\sqrt{\alpha}) + \frac{4}{\sqrt{\pi}} \exp(-\alpha) \cdot \sum_{\ell=1}^{\infty} H_{2\ell-1}(\sqrt{\alpha}) \cdot \left(\frac{\alpha}{\gamma}\right)^{\ell} \frac{1}{4^{\ell}\Gamma^2(\ell+1)} \frac{\Gamma(\ell+m)}{\Gamma(m)\, m^{\ell}} \right. \qquad (16)$$

where $\alpha = \frac{d^2}{2\sigma_n^2} = \mathrm{CNR}$,

$\gamma = \frac{d^2}{\Omega} = \mathrm{CIR}$,

m ; fading figure,

$\Gamma(\cdot)$; gamma function,

$H_{2\ell-1}$; Hermite polynomial.

Eq.(16) is the general equation of error probability for L-level ASK signal.

2. M-ary PSK System

At first, the equation of error probability for biphase PSK(BPSK) is derived, and extended to the case of M-ary PSK signal. We show in Fig.4 the phasor diagram of the received BPSK signal corrupted by interference and noise.

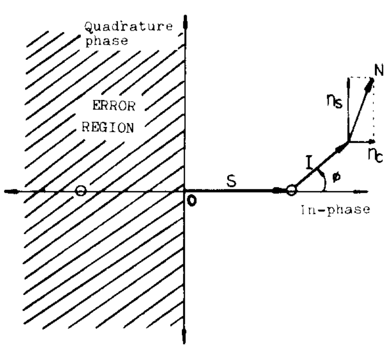


Fig. 4. Phasor diagram of the received BPSK signal corrupted by interference and noise.

The conditional error probability is

$$P_E(I,\phi) = \mathrm{Prob.}|_{I,\phi} (-\infty < n_s < -\infty) \cdot \mathrm{Prob.}|_{I,\phi} (-\infty < n_c < -S - I\cos\phi) \quad (17)$$

Hence the sine term of noise (n_s) has no effect in error probability, Eq.(17) becomes

$$P_E(I,\phi) = \mathrm{Prob.}|_{I,\phi} (-\infty < n_c < -S - I\cos\phi) \quad (18)$$

Let's adopt the Eq.(7). Then we can rewrite Eq.(18) as

$$P_E(I,\phi) = \int_{-\infty}^{-S-I\cos\phi} P(n_c)\, dn_c \qquad (19)$$

Using the Eq.(15), we get the error probability P_E,[4] then

$$P_E = \frac{1}{2}\mathrm{erfc}(\sqrt{\alpha}) + \frac{1}{\sqrt{\pi}} \exp(-\alpha) \sum_{\ell=1}^{\infty} H_{2\ell-1}(\sqrt{\alpha}) \cdot \left(\frac{\alpha}{\gamma}\right)^{\ell} \cdot \frac{1}{4^{\ell}\Gamma^2(\ell+1)} \cdot \frac{\Gamma(\ell+m)}{\Gamma(m)\, m^{\ell}} \qquad (20)$$

The BPSK case may be extended to the M-ary case by using the Glave bound with the following relations:[1]

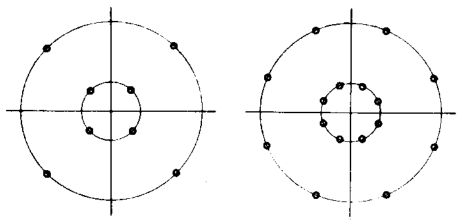
$$P_E\,(\text{M-ary}) \text{ at } (E/N) = 2\, P_E\,(\text{BPSK}) \text{ at } \left\{ \left(\frac{E}{N}\right) + 10\log_{10}\left(\sin^2 \frac{\pi}{M}\right) \right\} \text{ for a given}$$

$$\mathrm{CIR}(\text{M-ary}) = \mathrm{CIR}(\text{BPSK}) - 10\log_{10}\left(\sin^2 \frac{\pi}{M}\right) \qquad (21)$$

3. APK System

Combining the results obtained previously, ASK error rate and M-ary PSK error rate, we can derive the error rate of circular array case of APK system which is the one of the several cases of APK arrays. Fig. 5 shows the signal space diagrams of (4,4) circular array and (8,8) circular array as examples of APK system. The derived error rate equation above can be applied to this circular APK system.

Fig. 5(a) is the (4,4) circular array. This



(a)(4,4) circular array (b)(8,8) circular array

Fig. 5. Signal space diagram of circular APK signal.

type of signal array is hybrid of 2-level ASK and quaternary PSK. Therefore, we can get the error rate of APK system by substituting the value of L of L-level ASK and that of M of M-ary PSK.

The error rates of (4,4) array and (8,8) array APK systems in this paper are

$$P_E\{(4,4)\ \text{APK}\} = P_E\ (\text{2-level ASK}) + P_E\ (\text{4-PSK}) \tag{22}$$

and

$$P_E\{(8,8)\ \text{APK}\} = P_E\ (\text{2-level ASK}) + P_E\ (\text{8-PSK}). \tag{23}$$

IV. Discussions and Conclusions

Using the derived equations of L-level ASK and M-ary PSK signal, we have numerically calculated and evaluated the error rate performance of APK system compared to ASK, PSK and QAM systems with the variations of carrier-to-noise power ratio (α), carrier-to-interferer power ratio (γ), and fading figure (m).

The numerical results are shown in Fig.6.

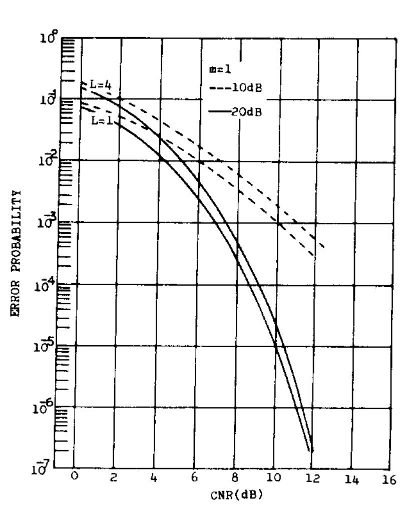


Fig. 6. Error rate of L-level ASK signal with interferer and Gaussian noise.

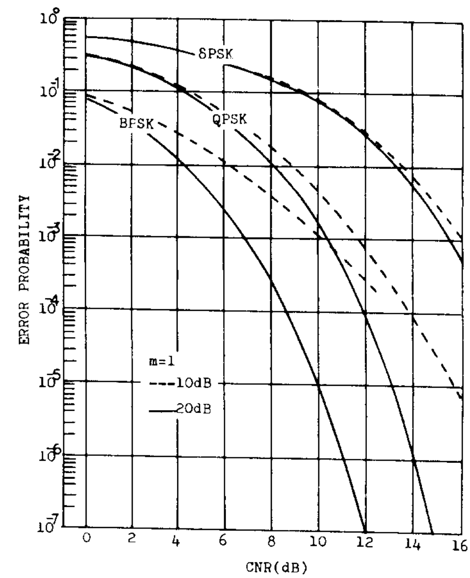


Fig. 7. Error rate of M-ary PSK signal with interferer and Gaussian noise.

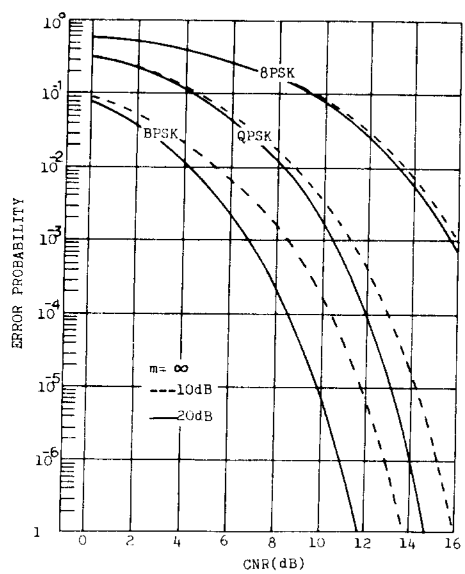


Fig. 8. Error rate of M-ary PSK signal with interferer and Gaussian noise.

As shown in this figure the more errors occur in larger L. These results agree with the physical facts.

For an M-ary PSK system the calculated results are plotted in Fig.7~Fig.8 as a function of CNR with CIR as a parameter. In these figures, the less errors occur in larger m.

In Fig.9~Fig.10, we plotted the error performance of BPSK with CIR as a parameter. The curves are in excellent agreement with those obtained above results.

Fig.11~Fig.12 show that the larger value of CIR the smaller variation of the error performance is.

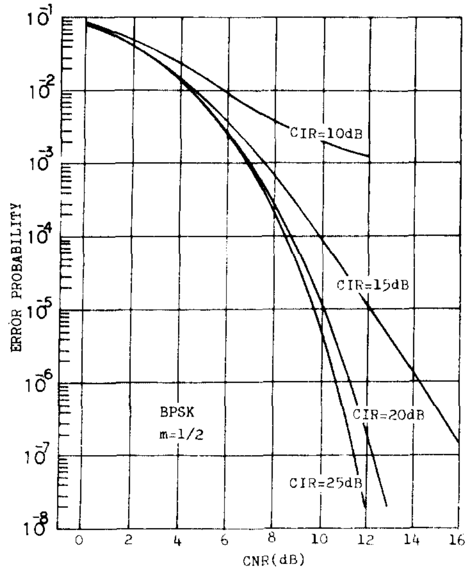


Fig. 9. Error rate of BPSK signal with respect to the change of CIR.

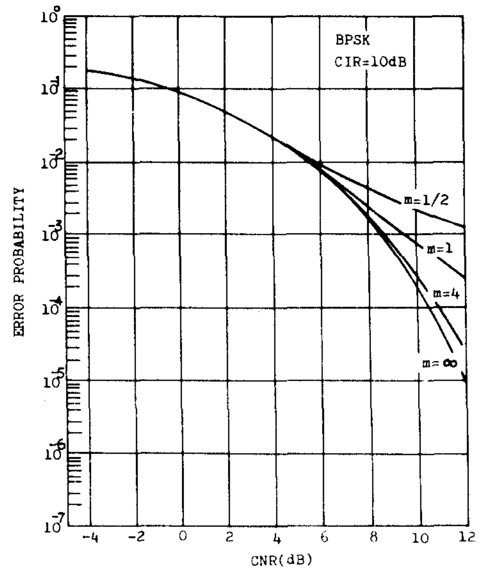


Fig. 11. Error rate of BPSK signal with respect to the change of m.

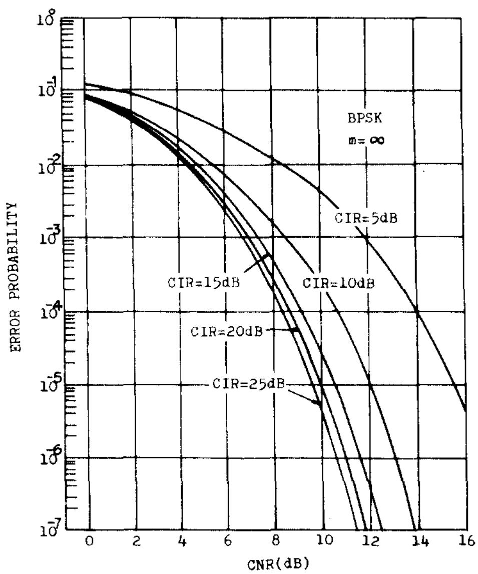


Fig. 10. Error rate of BSPK signal with respect to the change of CIR.

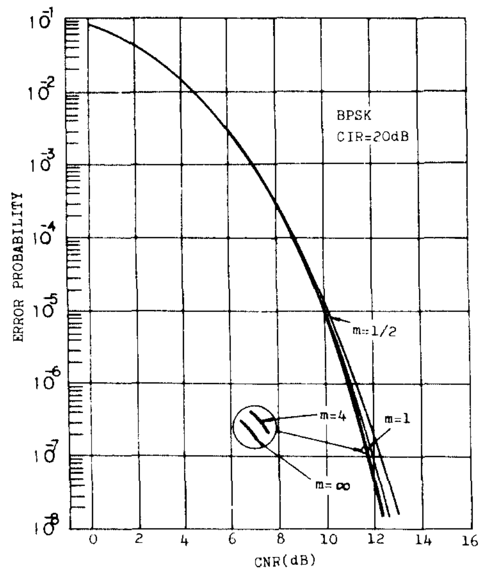


Fig. 12. Error rate of BPSK signal with respect to the change of m.

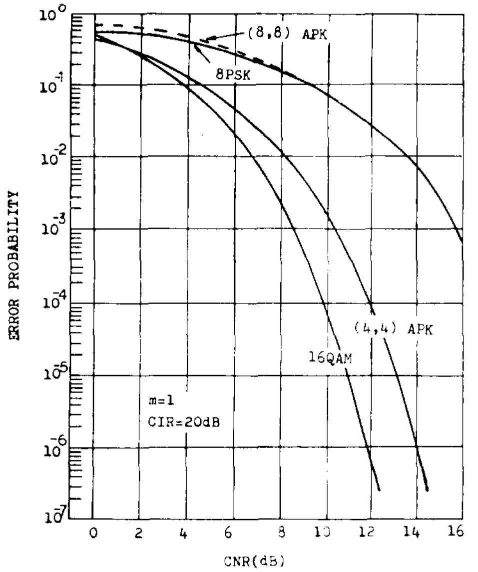


Fig. 13. Comparison of error performance of several types of modulation.

Finally we compare the error rate performances of ASK, PSK, QAM, and APK in Fig. 13. For M≥8, APK offers an advantage in CNR related to PSK that increases with alphabet size.

References

[1] K. Feher, *Digital Modulation Techniques in an Interference Environment.* Gainesville, VA: Don White Consultants, 1977.

[2] A. Javid and R.P. Tetarenko, "Error probability upper bound for coherently detected QASK signals with cochannel interference," *IEEE Trans. on Commun.*, vol. COM-27, no.12, Dec. 1979.

[3] N. Nakagami, *The M-Distribution–A General Formula of Intensity Distribution of Rapid Fading.* in Statistical Methods in Radio Wave Propagation, W. Hoffmann, Ed., Pergamon Press, 1960.

[4] J.W. Chai and S.J. Cho, "Effects of interference and noise on APK system," *Symposium of General Science in KIEE,* vol.6, no.1, July 1983.