On Performance of Adaptive Array and Sidelobe Canceller

(간섭 신호 제거를 위한 Adaptive Array 및 측엽 제거 기법의 특성 분석)

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要 約

본 논문에서는 adaptive array의 이론해석을 통해 antenna array의 물리적인 현상에 따른 비임폭과 지향 특성을 분석하였으며, 최대 신호대 잡음비(maximum signal-to-noise ratio) 알고리즘에 의한 array 시스템 설계시 유용한 몇가지 요소들을 도출된 관계식과 37-element 비임안테나의 시험 결과를 예로 들어 제시하고 있다.

본 논문은 또한 위성통신, 마이크로파 통신등에서 간섭신호 제거에 유용한 측엽 제거기법의 성능 특성을 표시하는 관계식을 새로이 도출하여 제시하고 있다.

Abstract

This paper examines the array antenna theory, basic relations between the array size (aperture) and its beamwidth and resultant patterns. This paper also provides array antenna system design criteria, mainly maximizing the signal-to-noise ratio (SNR) and its corresponding optimum array structure and weight functions. Explicit new expressions for array performance are also illustrated in terms of the array output SNR. An example is provided for a 37-element planar array to explicitly illustrate the beam-forming and nulling operations of the array.

Fundamentals of sidelobe canceller (SLC) systems have been discussed along with a derivation of new SLC equations for optimum weights.

I. Introduction

Recently, adaptive antenna arrays and sidelobe canceller (SLC) systems have received great interest in a wide variety of applications including satellite communications, sonar systems and military anti-jamming radios and radars.[1] Array antennas consisting of many controllable elements have unique and outstanding features that other antennas (e.g., single feed dish antenna) do not share.

Two primary unique functions of the adaptive array may include the followings; First, the array antenna can provide spatial discrimination by forming a beam in the direction of the desired signal while steering

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nulls and reducing sidelobe levels in the direction of unwanted signals. Second, the adaptive algorithms used for the processing of the element output provide self-optimization, depending upon the design criteria, which is extremely desirable when there exists temporal and spatial uncertainties along with nonstationary environments.

II. Adaptive Array

Consider an array consisting of K controllable elements by means of weighting W at each element's output as shown in Figure 1.

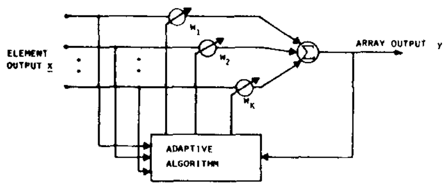


Fig. 1. Functional diagram of an adaptive array.

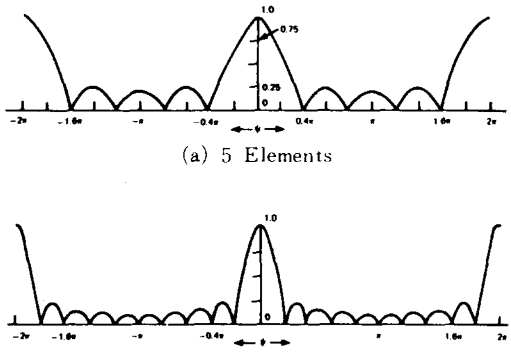
The output of the array y is formed as a linear combination of the array element outputs

$$y=\underline{W}^*\,\underline{X} \tag{1}$$

Where W* is a transpose conjugate of weight vector applied to the element output of an array. The weights W are complex in that both amplitude and phase adjustment and are controlled by feedback from the array output such that the far-field array pattern can adapt to a changing environment. Assuming a quiescent environment, in which no directional interference signals exist and the internal noise components are uncorrelated at each element output, the gain at angle θ with respect to the array normal, $A(\theta)$ for linear arrays with an equal spacing is well known as

$$A(\theta)=\frac{\sin\dfrac{K\pi d}{\lambda}(\sin\theta-\sin\theta s)}{K\sin\dfrac{\pi d}{\lambda}(\sin\theta-\sin\theta s)} \tag{2}$$

where θs is the arriving angle of the desired signal with respect to the array normal, λ is the wavelength of the desired signal, and d is the element spacing. The quiescent gain pattern, also called the array factor, is shown in Figure 2 for five and ten element arrays with $\psi=(2\pi d/\lambda)(\sin\theta-\sin\theta s)$. As shown in Figure 2, only the lobe at $\psi=0$ (or $\theta=\theta s$) is the desired mainlobe.



(a) 5 Elements

(b) 10 Elements

Fig. 2. Array patterns for 5 and 10 element linear arrays with half-wavelength spacing.

Any additional main lobes that appear in the visible region are called grating lobes. Appropriate selection of array element spacing and element gain normally eliminates grating lobes from the desired angular sector of scan. For example, in an array steered to $\pm\pi/2$ measured from the array normal, the condition which avoids grating lobes in the region of $\pm\pi/2$ is

$$d \leq \lambda/2. \tag{3}$$

another important factor that characterizes the array is its beamwidth. The 3 dB beamwidth, which is most commonly used, is defined as the beamwidth at the half-power and shapes are mainly determined by the total antenna length L, where L=Kd for linear arrays, and zero crossings of the array pattern (nulls) under quiescent (non-jamming conditions) are at multiples of λ/L so that the second lobe appears at λ/d, as do other lobes at all positive and negative multiples of this value.

When the noise environment contains undesired directional signals, the observed waveform at the input of the array shown in Figure 1 consists of a signal plus noise where noise is a combination of directional interferences and internal thermal noise. Thus the input vector $\underline{X}(t)$ can be expressed as a summation of the signal and noise

$$\underline{X}(t) = \underline{S}(t) + \underline{N}(t) \tag{4}$$

where

$\underline{S}(t)$ and $\underline{N}(t)$ can be expressed in vector forms.

$\underline{S}(t) = s(t)\,\underline{u}$, $s(t)$: signal

$\underline{N}(t) = g(t)\,\underline{v} + \underline{n}(t)$, $g(t)$: interference

$n(t)$: thermal noise

where $\underline{u}$ and $\underline{v}$ are (KX1) pointing vectors associated with the incoming directions of the desired signal and the interference and this can be easily expandable to multiple jammer case.

If the power spectral density of the signal is S for the frequency band of interest, then the (KXK) covariance matrix R_{SS} is conveniently defined as

$$\begin{aligned} R_{SS} &= E\left|\underline{S}(t)\ \underline{S}^*(t)\right| \\ &= S\underline{u}\ \underline{u}^*. \end{aligned} \tag{5}$$

For noise fields consisting of a spatially uncorrelated component with spectral density N at each element and interference source with spectral density J, the noise covariance matrix R_{NN} becomes

$$\begin{aligned} R_{NN} &= E\left|\underline{N}(t)\ \underline{N}^*(t)\right| \\ &= NI + Jvv^*. \end{aligned} \tag{6}$$

III. Array Structure and Performance

Several different criteria for array performance have been used to generate control algorithms for optimum weights of the array. Applebaum[2] used the maximum signal-to-noise ratio criteria, Widrow et al[3] and Griffiths[4] have considered algorithms based upon the minimum mean-square error criterion. Lacoss[5] and Frost[6] have presented unbiased minimum noise variance procedures. In this appendix only the maximum output signal-to-noise ratio (SNR) is considered as a performance measure.

The output SNR is defined as the expected output signal power divided by the output noise power

$$SNR = \frac{E\ |\underline{W}^*\underline{S}|^2}{\underline{W}^* R_{NN} \underline{W}} \tag{7}$$

solving for $\underline{W}$ that maximizes the SNR, we have[2]

$$\underline{W} = S\,R_{NN}^{-1}\,\underline{u} \tag{8}$$

and the maximum SNR becomes

$$SNR = S\underline{u}^*\,R_{NN}^{-1}\,\underline{u}. \tag{9}$$

The computation of the maximum SNR expressed above is not simple because of the matrix inversion involved in the formula. However, using the property of Hermitian matrix and Bartlett's formula[11], the inversion of the noise covariance matrix can be simplified and the resultant SNR becomes

$$SNR = \alpha K \left(1 - \frac{\beta|\underline{u}^*\underline{v}|^2}{K(1+K\beta)}\right) \tag{10}$$

where $\alpha = S/N$ and $\beta = J/N$ and K is the number of element of array. When the interference power is dominant, i.e., $\beta \gg 1$, then SNR can be approximated as

$$SNR \simeq \alpha K \left(1 - \frac{|\underline{u}^*\underline{v}|^2}{K^2}\right) \tag{11}$$

Note that the directional correlation factor between the desired and interference signals $|\underline{u}^*\underline{v}|$ plays an important role in determining the array performance. When $\underline{u}$ and $\underline{v}$ are orthogonal, the array can point the beam in the desired signal direction and steer a null in the interference direction simultaneously and no performance loss is encountered. In contrast, no nulling can be provided if the desired signal and interference are in the same angular direction, in which case $|\underline{u}^*\underline{v}| = K$.

The SNR expression above also shows an interesting fact that the output SNR becomes independent of the input interference power for $\beta >> 1$. The above expression also indicates that the interference can be completely nulled out unless the interference is aligned with the incoming desired signal direction.* However, the mainbeam again also degrades at the expense of complete nulling by a factor of $1 - |\underline{u}^*\underline{v}^2/K^2|$ as shown above. Multiple interference source cases can also be derived easily but are considered beyond the scope of this paper.

1. An Example

A typical 37-element phased array is considered here as an example to illustrate the array performance discussed in this section. Figure 3 shows the configuration of the 37-element array and each element represents a hexagonal-shaped horn antenna with an element gain of 24.3 dB (thus the total antenna gain is 40 dB). The array pattern or the quiescent gain pattern of this 37-element phased array is shown in Figure 4 where X denotes $X = \sqrt{G}\sin\theta$ where G is the total gain of 40 db and θ is the angle from the array normal. For earth coverage θ is assumed to vary between $\pm 8.5^\circ$ and thus X is in the range of (-14.8, +14.8).

Three interferences are assumed located at X = -5.5, 2.5, and 3.5 with all the same interference-to-thermal noise ratio (KJ/N) of 20 dB each. The input SNR (KS/N) here is assumed 20 dB. The maximum output SNR is plotted in Figure 5 for desired signal positions varying from X = -10 to 10 (assuming optimum weight setting as a function of signal angle). It is noted that when the desired signal is spatially separated from interferences sufficiently, no significant SNR loss is shown. However, when the desired signal is at the interference position, i.e., both incoming directions are the same, then no nulling can be provided and the resultant SNR is minimized.

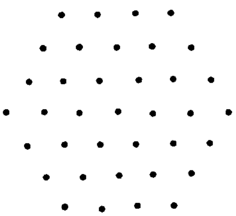


Fig. 3. A 37-element planar array.

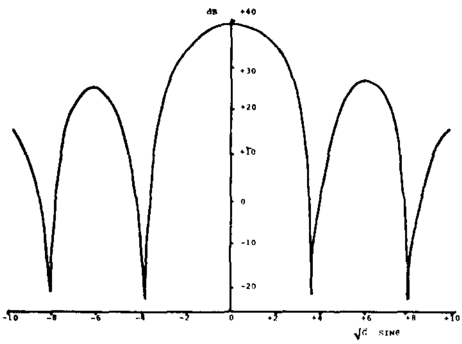


Fig. 4. Quiescent gain pattern.

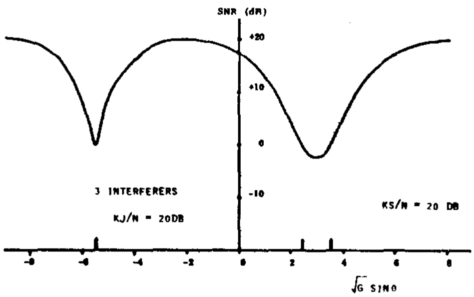


Fig. 5. SNR performance for interferers at X=-5.5, 2.5, 3.5

The antenna power gain patterns after the optimum weights are applied for fixed signal positions are shown in Figures 6 and 7. It is noted that the mainbeam gain (gain at desired signal position) degrades significantly when the desired signal is spatially close to the interference (Figure 7). This shows the contribution of the spatial correlation between the disired signal and interference to the performance degradation.

* Assumes narrowband siguals

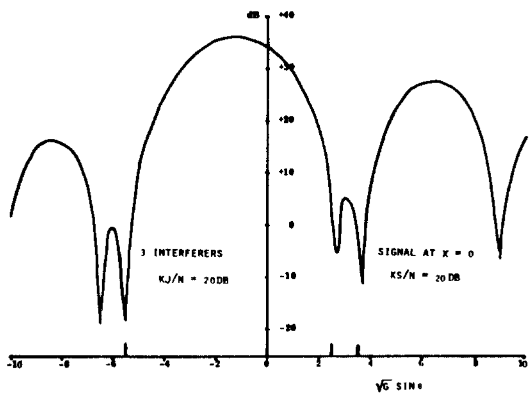


Fig. 6. Gain performance for the signal at X=0, interferers at X=−5.5, 2.5, 3.5.

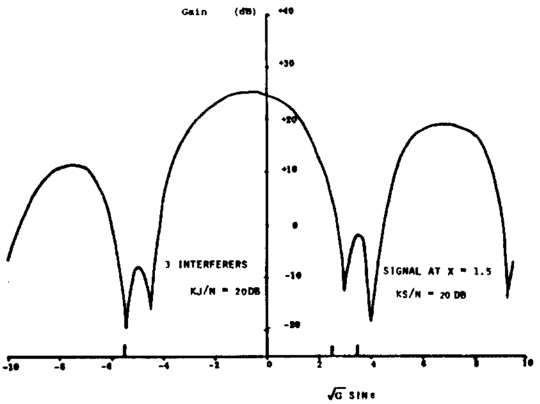


Fig. 7. Gain performance for the signal at X=1.5, interferers at X=−5.2, 2.5, 3.5.

IV. Sidelobe Cancellation (SLC)

When a large dish antenna is used for signal reception, the beamwidth of antenna is normally very narrow and the possibility of receiving interference sources by the mainlobe is quite unlikely. However, if the J/S ratio is much greater than the difference between the mainlobe peak and the envelope of the sidelobes, the residual interference power can still overwhelm the disired signal power and thus requires some additional AJ protection. In such a case, sidelobe cancellation techniques of the antenna. Sidelobe cancellation may be viewed as a special application of the adaptive array. The SLC system as shown in Figure

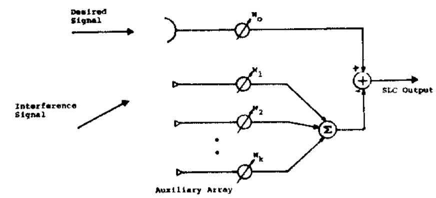


Fig. 8. Sidelobe cancellation system.

8 consists of a main, high gain antenna and K auxiliary antennas. The gain of the auxiliary antenna is normally at the level of sidelobe of the main antenna gain. When the J/S ratio is large, the desired signal received at the auxiliary antenna is negligible and the replica of the jamming signal is provided by the auxiliary antenna for the cancellation of the jamming signal received at the main antenna as shown in Figure 9. Optimum weights $\underline{W}$ for

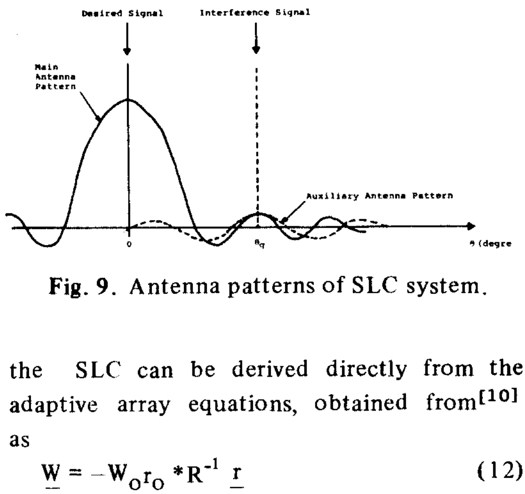


Fig. 9. Antenna patterns of SLC system.

the SLC can be derived directly from the adaptive array equations, obtained from[10] as

$$\underline{W} = -W_o r_o{}^* R^{-1}\, \underline{r} \tag{12}$$

Where

W_o : weight of the main antenna

$r_o{}^*$: directional interference component received by the main antenna (conjugate)

R : covariance matrix of the noise received by the auxiliary antenna

$\underline{r}$: vector of the interference received by the auxiliary antenna elements.

Note that the optimum weight function of the SLC has the same form as that of the adaptive array but differs in that it uses $\underline{r}$, the noise vector, instead of the desired signal vector in order to point a beam toward the interference, so that its resultant beam is used for the cancellation of the interferer in the main antenna output.

V. Factors Affecting the Nulling Capability of Array/SLC

This section provides a brief description of basic factors that degrade the nulling capability of the array and SLC. Some factors do not affect the adaptive array system performance to the same degree as in SLC systems mainly because of the structural differences between arrays and SLCs. In genenal, the performance of spread spectrum AJ techniques increases linearly with increased spreading bandwidth for a fixed data rate. On the other hand spatial nulling protection techniques using adaptive arrays or SLCs have null depth which degrades as the signal bandwidth increases.

Other factors such as; spatial separation between desired and interference signals, desired signal incoming directions, array/SLC configurations including the antenna array shape, element spacing, multi-path dispersion and hardware factors (limited dynamic range of practical component weight jitter, etc.) all contribute to overall achievable null depths in practical systems.

1. Bandwidth Effect

Adaptive array processing for beam forming and null steering is inherently performance limited for large relative bandwidth (bandwidth/center frequency) desired and interference signals.[7],[8] In general it is found that the output SNR performance for array processor is inversely proportional to both J/S ratio and bandwidth. In contrast, the SNR for narrow band signals and jammer becomes independent of the jammer power as J/S ratio increases. Thus increased spread spectrum processing gain via increased bandwidth may not improve the overall performance due to the degraded nulling capability of an array[9] unless the signal bandwidth/signal center frequency is kept relatively small. It is also shown in [9] that degradations to null depth due to bandwidth effects are increased in proportion to the distance between the elements (or distance between the elements and the feed of the main antenna in SLC systems). Thus serious nulling degradation can be experienced when SLC elements are placed on the edge of a large dish antenna [i.e., (diameter/λ)>>1]. Figure 10 provides guidelines on achievable null depth versus SLC element distance from the center of a reflecting dish quoted from[10].

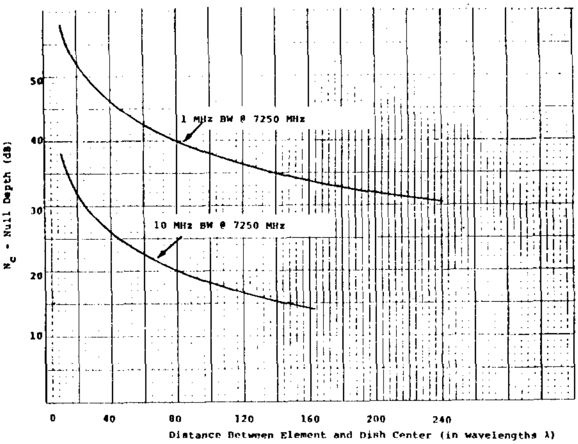


Fig.10. Null depth vs. distance between SLC element and dish center

Note that results are to be used only as guidelines for a single interference source and no tapped delay equalization (or multiple interferers with tapped delay equalization). Thus to achieve null depths better than 30 dB requires reduced separation from the dish center. Hole punching in the dish could permit reduced element spacing from a dish center but could lead to other severe problems (e.g., structural problems or intermodulation products, etc.). One attractive alternative which avoids the above problems recommended in [10] is to use the error horns typical of pseudo-monopulse tracking systems, as an array feed sidelobe canceller to protect the main feed signal. This technique aboids the bandwidth limitation problem without the need for unnecessary complexity required for tapped delay line techniques.

2. Multipath Dispersion for SLC Applications

The multipath dispersion of the interference signals in the main antenna can significantly reduce the nulling capability. Multipath dispersion may occur because of the different incoming paths of the interference signal reception, i.e., direct path to feed, diffraction from the edge of the dish/subreflector, reflection due to the earth surface and scattering, etc. Thus, the interference signals received by the main antenna may differ from those received by the SLC elements, creating misadjusted null patterns by the SLC, which could seriously degrade the nulling performance. Extensive investigation in real applications is required to alleviate this problem by searching for the optimum SLC configurations in terms of the aperture reflector geometry of the main antenna and the resultant dispersion characteristics of signals not received at boresight.

3. Correlated Disired and Interference Signals

In the real world environment, correlation between the desired and interference signals may occur if the interference itself is caused by the multipath phenomenon of the desired signal. However, because of the random phase fluctuation of multipath components, only a partial correlation is seen by the receiver. It is generally known that the array performance degradation is proportional to the correlation coefficient between the desired signal and interference resulting from the multipath phenomenon.

4. Weight Jitter Effects

Even after the weights converge to a steady state, the adaptive algorithm itself may cause random fluctuation of the weights (weight jitter). Finite weight resolution is also a cause of weight jitter. Weight jitter degrades the array nulling performance in a manner inversely proportional to the variance of the weight jitter. Increasing the adaptive processor loop gain (i.e., increase algorithm bandwidth) will cause the loop to adapt faster but will generally result in increased steady state weight jitter thus requiring reduced loop gain in a steady state environment. Weight jitter effect in a SLC system is expected to be similar to that for an adaptive array, but no study has been found in the open literature on this subject.

5. Mismatch of a-Priori Information

The mismatch in a beamformer (which forms a beam in the desired signal direction) occurs when the knowledge of the signal directional property is imprecise. For SLC applications, the loss due to the mismatch is totally dependent upon the shape of the mainlobe of the main antenna. But for the adaptive arry application, the mainbeam is scanned to the informed position of the signal and its performance may suffer if a) the mainbeam is very sharp (large aperture case), and b) the interference source is very close to the desired signal so that the gain pattern near the desired signal becomes very sharp to provide a deep null.

IV. Adaptive Algorithms and Signal Acquistion

Adaptive algorithms differ for various performance criteria because their corresponding optimum weights are different as shown in the previous section. More importantly, proper algorithms should be chosen depending on the environment under consideration and the signal acquisition method. This signal acquisition is required mainly to differentiate the desired signal from interference. For example, when the reference signal (or desired response) is available, the when the reference signal (or desired response) is available, the Widrow type [3] of adaptive algorithm can be easily used. When some a-priori information regarding the signal is available instead of a reference signal, then an adaptive algorithm that utilizes the signal incoming direction and the expected desired signal power can be used. When no information is available regarding the signal statistics, very complicated signal-searching methods may have to be used.

1. Use of a Reference Signal

Least mean square algorithm uses of a reference signal to differentiate the desired signal from interferences, and the error signal, difference between the array output and the reference signal, is used as a feedback control signal to adjust the weights that minimize the MSE. To apply this concept to a practical communications system, it is necessary to obtain a suitable reference signal. Some distortion or delay of the reference signal is acceptable as long as the reference signal is highly correlated with the actual signal and uncorrelated with the interferences. When the reference signal is available, the weight adjustment circuit accepts the antenna signals and the error signal (which is the difference between the reference signal and the array output) and adjusts the weights to minimize the error signal.

The least mean-square (LMS) algorithm suggested by Widrow is an implementation of the method of steepest descent. It searches for the optimum weights that minimizes the error signal adaptively by moving along the gradient. The adaptive equation for the updated weight vector is

$$\underline{W}_{k+1} = \underline{W}_k + \mu \epsilon_k \underline{X}_k \qquad (13)$$

Where

$\epsilon_k = y_k - d_k$

y_k = k-th sample of the array output

d_k = k-th sample of the reference signal

$\underline{X}_k$ = k-th sample of array input vector

μ = stepsize regulating factor

Note that $\epsilon_k \underline{X}_k$ term is the k-th estimate of the gradient. Figure 11 shows a functional diagram of an adaptive array using a reference signal. It has been experienced that the expected value of the weight vector converges to the Wiener weight vector even by starting an arbitrary initial one. The condition for the convergence is that the regulating factor μ should be greater than 0 but less than the reciprocal of the largest eigenvalue λ_{max} of the input convariance matrix,

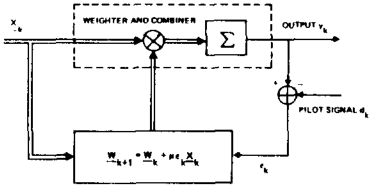


Fig.11. Adaptive processor with pilot signal.

$$0 < \mu < 1/\lambda_{max}\,. \qquad (14)$$

Detailed convergence discussions are shown in [3], [4]

The algorithm suggested by Griffiths uses a mixture of average and instantaneous quantities, described as

$$\underline{W}_{k+1} = \underline{W}_k + \mu[S\underline{u} - \underline{X}_k \underline{X}_k{}^* \underline{W}_k]$$
$$= \underline{W}_k + \mu[S\underline{u} - y_k \underline{X}_k] \qquad (15)$$

A functional diagram of this algorithm is shown in Figure 12. A thorough analysis of the convergence of the algorithm is given in [4] and performance comparison of these algorithms is provided in [13].

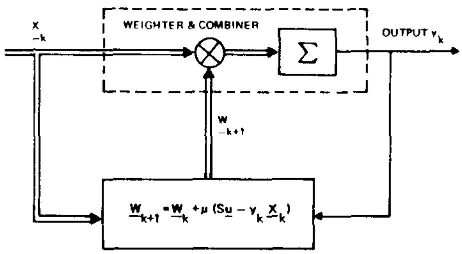


Fig.12. Adaptive processor with prior information

2. Use of Spread-Spectrum Techniques

In some applications the reference signal is readily available. But in many practical communication systems, special signal processing using spread-spectrum techniques may be required to obtain a reference signal. A combination of adaptive arrays and spread spectrum modulation is, however, still in its infancy stage and very few references [9][10][12] are available in the literature. The main concern of using an adaptive array with wideband spread spectrum modulation is the degradation of array nulling capabilities due to the wide bandwidth of interferences and desired signal. Figure 13 shows a functional block diagram of

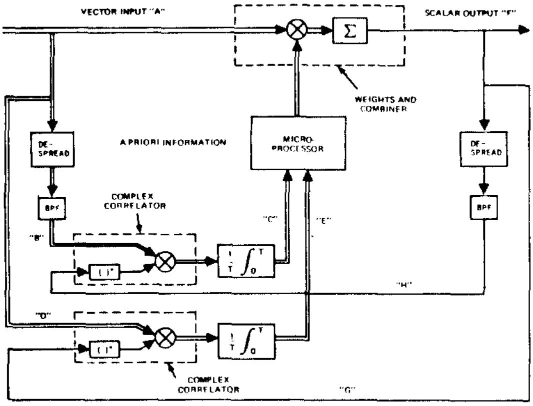


Fig.13. Functional diagram of adaptive processor using spread spectrum.

the adaptive processor using spread spectrum techniques to obtain the reference signal. "A" point in the Figure represents the input data vector and "B" and "H" are unweighted and weighted signal estimate respectively, and their correlation output "C" is containing signal information only because "H" becomes uncorrelated with the interference contained in "B". Since the interference-to-signal ratio (J/S) is assumed much greater than 1, the correlation between "D" and "G" would provide interference information, especially noise covariance matrix R_{NN}. The microprocessor then uses signal and noise information, represented by points "C" and "E" respectively, to computed the optimum weight vector adaptively. Note the bandpass filter in the diagram has the bandwidth of the desired signal before spreading. After the weights converge to a steady state, the array output "F" becomes a clean spread signal and "H" becomes a clean estimate of the signal after despreading.

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