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<**Original**>

Optimal Design of a Vibration Absorber Against Machine Tool Chatter

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공작기계 채터 방지를 위한 진동흡수기의 최적설계

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초 록

대부분의 동적댐퍼들은 주구조물의 진동진폭을 정해진 주파수 변위내에서 최대로 줄이는 것을 목표로 한다. 그러나 공작기계의 안정성은 시편과 공구사이의 상대변위와 절삭력에 의해 결정되는 전달함수의 최대크기에 의해서보다는 실수부분의 최소치에 의해 결정된다는 것이 잘 알려져 있다.

본 논문에서는 이 사실에 착안하여 공작기계에서 발생하는 채터를 흡수하기 위한 최적의 댐퍼를 설계하는 절차를 제시하고 1자유도로 대표될 수 있는 구조물의 경우에 대하여 구체적인 방법을 예시하였다. 종래의 최적 댐퍼의 성질을 구하는 방법에 비해 수학적인 절차가 약간 복잡해지기는 하나 전산기를 이용하여 큰 어려움이없이 최적의 설계자료를 얻을 수 있다.

댐퍼 질량이 정해졌을 때 감쇠율과 스프링 계수를 변수로하는 목표함수가 하나의 식으로 유도될 수 없기 때문에 간단한 최적화 방법으로 이변수 황금분할법을 사용하였다. 수치적인 예를 통하여 종래의 다른 방법에 의한 결과와 비교하고 제안된 방법론의 타당성을 입증하였다.

1. Introduction

The relative vibration of a machine tool structure between workpiece and toolholder must be maintained as small as possible because it affects not only smoothness of cut surface of workpiece but also stability of machining process. If it is found, after a machine tool structure is built, that the relative vibration between the two elements mentioned above is too serious to continue machining, the structural design should be changed.

Since, however, redesign of a structure is very expensive and takes time, attachment of an additional vibratory system, i.e., a damper to a suitable point of the structure is more desirable. Usually dynamic dampers are designed so that the magnitude of vibration of main structure may be minimized. Such dampers can be applied to the machine tool structure to reduce to some extent the magnitude of vibration between workpiece and toolholder. Noting that the vibration which occurs more often and is detrimental in machine tools is chatter vibration rather than forced vibration and that the stability against chatter vibration depends upon the minimum value of real part of the transfer function of the main structure, and not the magnitude of vibration[1], in this paper the vibration absorber to be attached to the main structure is optimized so as to minimize

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the minimum real part of transfer function so that the maximum chip width or stability can be achieved. In section 2, basic theory in stability of machine tool structure is presented. In section 3, it is shown how to simplify the problem, how to formulate the optimization problem, and what kind of optimization method to choose. In section 4, a design region is shown to get an idea for optimization and to see how complicated it is. The design chart to obtain the optimum damping coefficient and the stiffness of the absorber is also given in nondimensionalized terms. Optimum results obtained by minimization of minimum value of real part of transfer function is compared with those by other authors[2,3,4,6].

2. Basic Theory in the Stability of Machine Tool Structure

The machining process in machine tool is basically a closedloop system including two fundamental parts, i.e., cutting process and machine tool structure, which can be schematically represented as shown in Fig. 1.

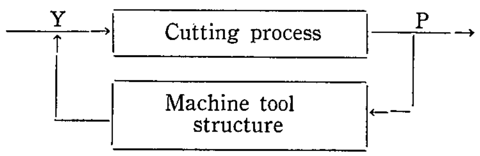

Fig. 1 Basic diagram of machining process

For simplicity of the analysis a few assumptions are made[1] :

(a) The vibratory system of the machine is linear.

(b) The direction of the variable component of the cutting force is constant.

(c) The variable component of the cutting force depends only on the vibration in the direction of the normal to the cut surface.

(d) The value of the variable component of the cutting force varies proportionately and instantaneously with the vibrational displacement Y.

(e) The frequency of the vibration and the mutual phase of undulations in subsequent overlapping cuts are not influenced by the relationship of wavelength to the length of cut.

Under these assumptions the machining process described in Fig. 2. can be mathematically represented by:

$$P=-b\cdot r\cdot(Y-Y_0) \qquad (1)$$

$$Y=P\cdot\Phi(\omega) \qquad (2)$$

where $\Phi(\omega)$ is the transfer function of the machine tool structure between the workpiece and tool holder, b the chip width, and r a real coefficient.

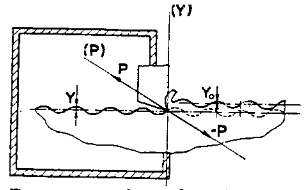

Fig. 2 Representation of a dynamic machining process

After some manipulation the ratio of the magnitude of vibration in the previous cut to that in the current cut can be expressed as:

$$\frac{Y_0}{Y}=\frac{\frac{1}{br}+\Phi}{\Phi} \qquad (3)$$

The condition for the limit of stability is given as:

$$\left|\frac{Y_0}{Y}\right|=\left|\frac{\frac{1}{br}+\Phi}{\Phi}\right|=1 \qquad (4)$$

Rewriting eq. (4) in terms of real part $G(\omega)$ and imaginary part $H(\omega)$ of the complex function $\Phi(\omega)$ gives:

$$\left|\frac{\frac{1}{br}+G+jH}{G+jH}\right|=1 \qquad (5)$$

Since the imaginary parts of the numerator and the denominator in the above equation are equal, the condition(5) can be substituted by:

$$\left|\frac{\frac{1}{br}+G}{G}\right|=1 \qquad (6)$$

It can also be seen easily that condition (6) is satisfied only if:

$$\frac{1}{2br}=-G \qquad (7)$$

The final form of the lowest or critical limit of stability condition in the basic theory can be expressed as:

$$\frac{1}{2b_{\lim}r}=-G_{\lim} \qquad (8)$$

Eq. (8) means that the value of the maximum chip width $b_{\lim}$ is dependent on the value of the minimum of the real part of the transfer function G of the machine tool structure.

3. Statement of Problem

3.1. Derivation of the Relative Transfer Function between Workpiece and Toolholder with a Vibration Absorber Attached as Two Degrees of Freedom System

It was explained in the previous section that the stability or the maximum width of cut in machining process depends upon the minimum value of the real part of the transfer function of machine tool structure between workpiece and toolholder. Although the machine tool structure as shown in Fig. 3 is an infinite degrees of freedom system, in many cases it can be represented by a few degrees of freedom system for practical purposes. In this paper the number of significant mode in the relative transfer function is assumed to be one for simplicity of the analysis, which means the relative movement between workpiece and tool holder can be described by an equivalent single degree of freedom system shown in Fig. 4.

The simplified model of the structure with a damper attached on it is shown in Fig. 5, where m_1, c_1, and k_1 are the equivalent mass, damping coefficient, and stiffness of the main structure and m_2, c_2, and k_2 are the mass, damping coefficient and stiffness of the vibration absorber.

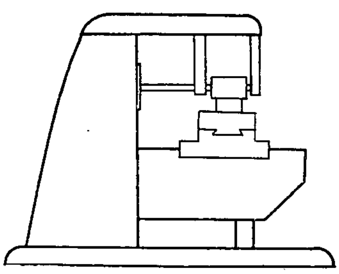

Fig. 3 An example of machine tool structure

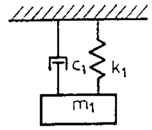

Fig. 4 A single degree of freedom system

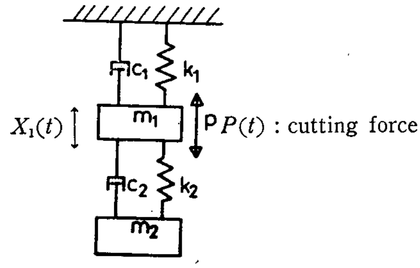

Fig. 5 Simplified model of machine tool structure with an absorber attached

The equations of motion of the system in Fig. 5 are:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} P(t) \\ 0 \end{Bmatrix} \tag{9}$$

Which can be easily solved to obtain the transfer function $\Phi(\omega)$ between the vibration of main structure and the cutting force:

$$\Phi(\omega) = \frac{X_1}{P}(j\omega) = G(\omega) + jH(\omega) \tag{10}$$

$$= \frac{k_2 - m_2\omega^2 + jc_2\omega}{k_1k_2 - (c_2c_1 + k_2m + k_1m_2 + k_2m_2)\omega^2 + \omega^4 + j[(c_1k_2 + c_2k_1)\omega - (c_1m_2 + c_2m_1 + c_2m_2)\omega^3]}$$

Introducing the nondimensional parameters:

$$\mu = \frac{m_2}{m_1},\quad \omega_1 = \sqrt{\frac{k_1}{m_1}},\quad \omega_2 = \sqrt{\frac{k_2}{m_2}},\quad f = \frac{\omega_2}{\omega_1}$$

$$\zeta_1 = \frac{c_1}{2\sqrt{k_1m_1}},\quad \zeta_2 = \frac{c_2}{2\sqrt{k_2m_2}} \text{ and } g = \frac{\omega}{\omega_1}$$

into eq. (10)

$$\Phi(\omega) = \frac{1}{k_1} \cdot \frac{f^2 - g^2 + 2j\zeta_2 fg}{[f^2 - (1 + 4\zeta_1\zeta_2 f + f^2 + \mu f^2)g^2 + g^4] + 2j[(\zeta_1 f^2 + \zeta_2 f)g - (\zeta_1 + \zeta_2 f + \mu\zeta_2 f)g^3]} \tag{11}$$

3.2. Statementof Optimization Problem

Rewriting eq. (8), the limit value of chip width b is represented by

$$b_{\lim} = -\frac{1}{2r \cdot G_{\lim}}, \tag{12}$$

which means that the minimum value of the real part of the transfer function, eq. (10), should be minimized to obtainthe maximum chip width. Therefore the optimization problem can be formulated as:

Given : m_1, c_1, and k_1

Find : m_2, c_2, and k_2 (13)

To minimize : G_{lim}

where G_{lim} is the minimum value of $G(\omega)$, the real part of $\Phi(\omega)$. From eq. (11) $G(\omega)$ can be represented in nondimensionalized terms:

$$G(\omega)=-\frac{1}{k_1}\frac{g^6-(A+f^2-4\zeta_2 fC)g^4-(f^2+Af^2-4\zeta_2 fB)g^2-f^4}{g^8+(-2A+4C^2)g^6+(2f^2+A^2-8BC)g^4+(-2Af^2+4B^2)g^2+f^4}$$

where (14)

$A=4\zeta_1\zeta_2 f+(1+\mu)f^2+1$

$B=\zeta_1 f^2+\zeta_2 f$

$C=\zeta_1+\zeta_2 f+\mu\zeta_2 f$

In most real circumstances m_2 is given rather than to be optimized and so the number of design parameters can be reduced by one. Confining the damping factor of the absorber ζ_2 and the ratio of the natural frequencies of the main structure and absorber $f=\frac{\omega_2}{\omega_1}$ within some range, the optimization problem can be reformulated in terms of nondimensionalized parameters:

Given : ω_1, ζ_1, and μ

Find : f and ζ_2

To minimize : $G_{lim}=\mathrm{Min}[G(\omega)]$ (15)

Subject to : $\zeta_{min}<\zeta_2<\zeta_{max}$

$f_{min}<f<f_{max}$

3.3. Method of Solution

Since the objective function, G_{lim}, is not given in closed form, it should be calculated numerically. For that reason, it can be expected that, the problem of optimization would be rather complicated if Gradient or Optimal Gradient Search is to be applied. Although, however, the mathematical formulation of the objective function is complicated, the design region would not be complex because the objective function is related simply to the ratio of two polynomials. Based on these thought and the actual shape of design region an example of which will be shown in next section, the Univariate Golden Section Search which does not need the computation of the derivatives of the objective function is applied.

4. Results

4.1. Design Region

The isomerit contour of the objective function in eq. (15), as an example, is shown in Fig. 6, where the given numerical conditions of the main structure and damper follow.

$m_1=20$, $c_1=0.8485$, $k_1=10$

$m_2=1$

Since the design region is relatively simple as expected, a good approximation of the optimum design parameters can be made from it.

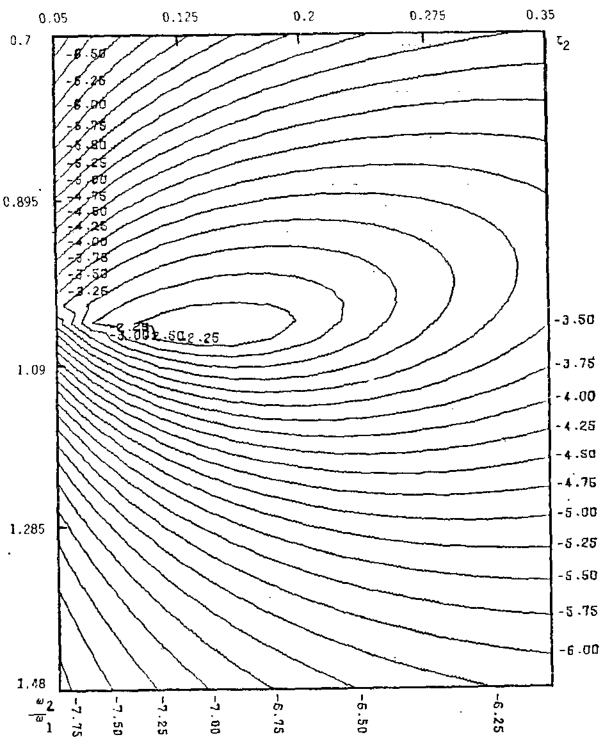

Fig. 6 An example of isomeric contour of the objecte function($m_1=20$, $c_1=0.8475$, $k_1=10$, $m_2=1$)

4.2. Optimum Damping Factor and Stiffness of the Absorber

The numerical values of the optimum natural frequency ratio of the primary and secondary system and the optimum damping factors of the secondary system (absorber) calculated using the computer program in the appendix are shown in Table 1. The design chart is shown in Fig. 7. For the purpose of comparison the optimum parameters by other authors when $\zeta_1=0$ are also given. In Den Hartog and Brock the parameters are intended to minimize the maximum magnitude of the response of the main structure over the full exciting frequency range. In Seireg they are optimized to minimize the maximum response for conditions of white noise random excitation.

Table 1 Optimum design parameters of vibration absorber

ζ_1		μ	0.001	0.005	0.01	0.05	0.1
0.01		ω_2/ω_1	1.0128	1.0129	1.0348	1.0429	1.0328
		ζ_2	0.0500	0.0500	0.0631	0.1385	0.1926
0.03		ω_2/ω_1	1.0318	1.0454	1.0506	1.0559	1.0438
		ζ_2	0.0500	0.0500	0.0657	0.1409	0.1949
0.05		ω_2/ω_1	1.0505	1.0621	1.0664	1.0689	1.0556
		ζ_2	0.0500	0.0500	0.0678	0.1430	0.1877
0.08		ω_2/ω_1	1.0779	1.0868	1.0899	1.0880	1.0709
		ζ_2	0.0500	0.0519	0.0704	0.1458	0.1995
0.1		ω_2/ω_1	1.0957	1.1031	1.1054	1.1005	1.0814
		ζ_2	0.0500	0.0541	0.0732	0.1475	0.2010
0.15		ω_2/ω_1	1.1389	1.1426	1.1431	1.1310	1.1071
		ζ_2	0.0500	0.0579	0.0765	0.1509	0.2042
0.2		ω_2/ω_1	1.1805	1.1807	1.1794	1.1604	1.1317
		ζ_2	0.0500	0.0596	0.0783	0.1543	0.2074
0.0	*Den artog	ω_2/ω_1	0.9990	0.9950	0.9900	0.9524	0.9091
		ζ_2	0.0194	0.0428	0.0597	0.1212	0.1526
	**Seireg	ω_2/ω_1	0.9990	0.9950	0.9900	0.9535	0.9129
		ζ_2	0.0224	0.0500	0.0707	0.1581	0.2236
	***Brock	ω_2/ω_1	0.9990	0.9950	0.9900	0.9524	0.9091
		ζ_2	0.0194	0.0426	0.0591	0.1151	0.1373

* $\left.\frac{\omega_2}{\omega_1}\right)_{opt}=\frac{1}{1+\mu}$ $\zeta_{2opt}=\frac{\omega_1}{\omega_2}\cdot\sqrt{\frac{3\mu}{8(1+\mu)^3}}$

** $\left.\frac{\omega_2}{\omega_1}\right)_{opt}=\frac{1}{\sqrt{1+2\mu}}$ $\zeta_{2opt}=\sqrt{\frac{\mu}{2}}$

*** $\left.\frac{\omega_2}{\omega_1}\right)_{opt}=\frac{1}{1+\mu}$ $\zeta_{2opt}=\sqrt{\frac{3\mu}{8(1+\mu)^3}}$

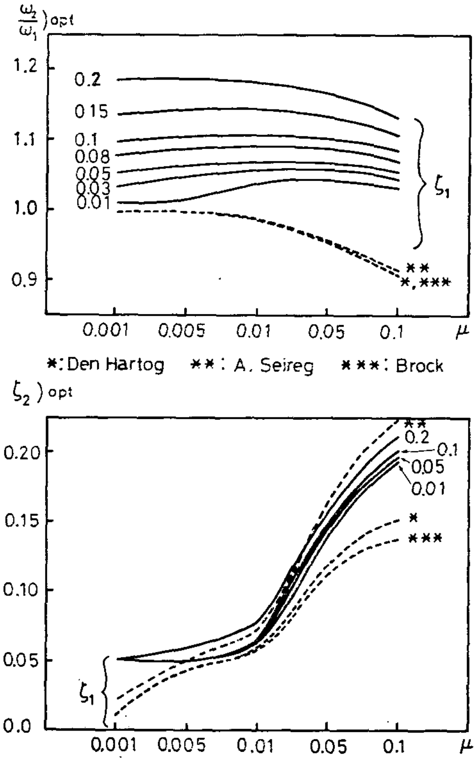

Fig. 7 Optimum design chart of a vibration absorber against chatter

4.3. An Example

The transfer function of the main structure without vibration absorber is shown in Fig. 8 in terms of real and imaginary part and magnitude and phase,

where

$m_1=20,\ c_1=0.8485,\ k_1=10$

$|\Phi(\omega)|_{max}=1.65$

$G(\omega)|_{min}=-0.83$

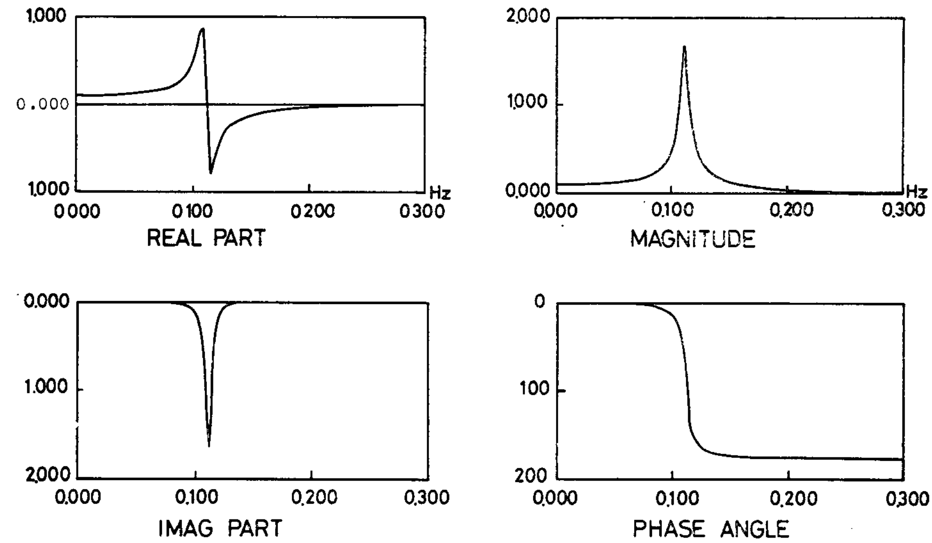

Fig. 8 Transfer function of main structure without absorber ($m_1=20$, $c_1=0.8485$, $k_1=10$, $\zeta_1=0.03$)

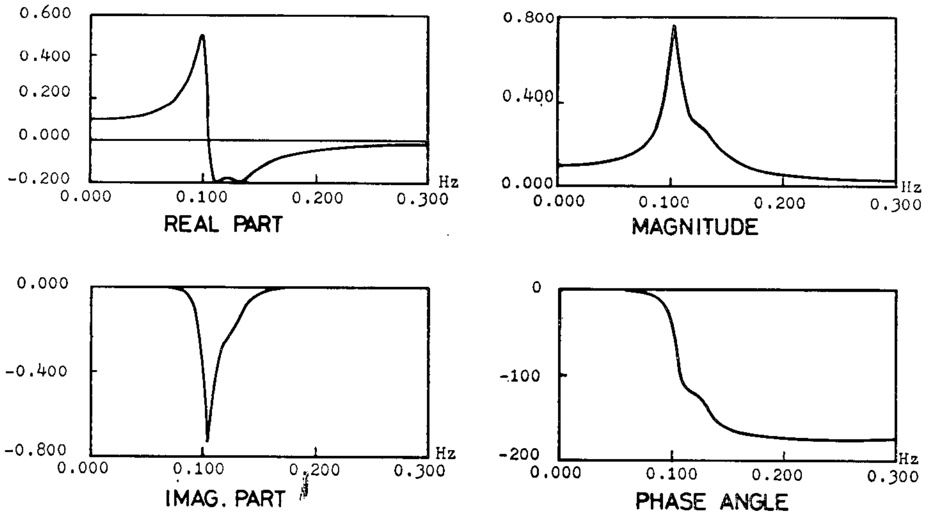

Fig. 9 Transfer function of main structure with absorber optimized by minimizing G_{lim} ($m_1=20$, $c_1=0.8485$, $k_1=10$, $m_2=1$, $c_2=0.2104$, $k_2=0.5575$)

The transfer function of the main structure with the absorber attached on it is shown in Fig. 9, where

$m_2=1$, $c_2=0.2104$, $k_2=0.5575$

$|\Phi(\omega)|_{max}=0.77$

$G(\omega)|_{min}=-0.20$

Hence the maximum chip width in cutting or the stability of the machine tool structure can be increased four times by attaching the optimized damper.

The transfer functions optimized by other authors neglecting the damping in the main structure are also shown in Fig. 10 and Fig. 11, and the values of optimum tuning parameters are given in Table 2. The above results shows that the maximum of $|\Phi(\omega)|$ and the minimum of $G(\omega)$ can not be achieved at the same time.

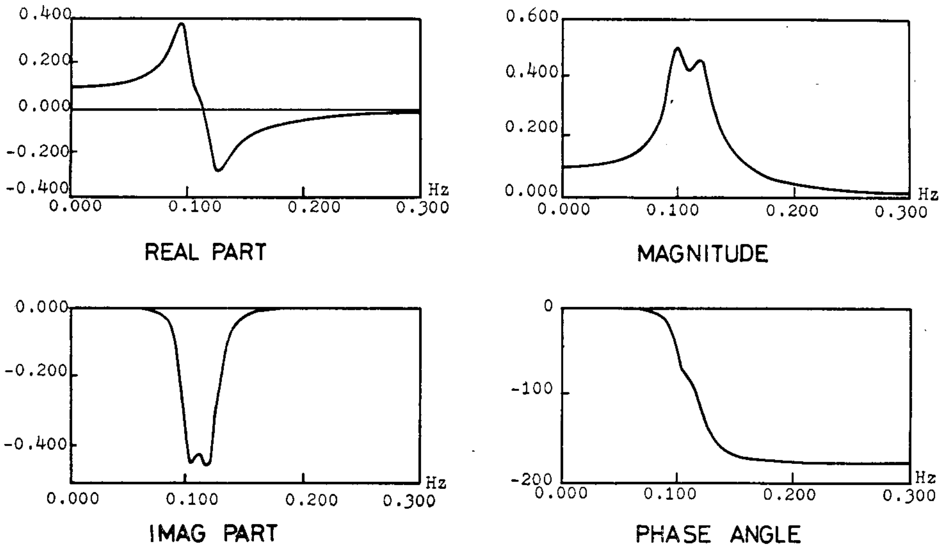

Fig. 10 Transfer function of main structure with absorber optimized by Den Hartog's Formula ($m_1=20$, $c_1=0.8485$, $k_1=10$, $m_2=1$, $c_2=0.18$, $k_2=0.4535$)

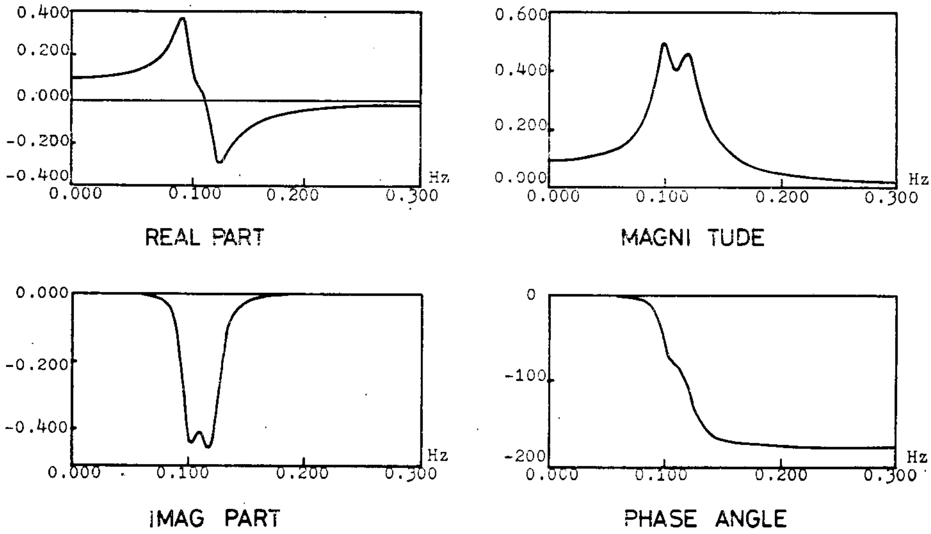

Fig. 11 Transfer function of main structure with absorber optimized using Brock's Formula ($m_1=20$, $c_1=0.8485$, $k_1=10$, $m_2=1$, $c_2=0.1714$, $k_2=0.4535$)

Table 2 Optimum parameter values by other authors

	Den Hartog	Brock	Seireg
m_2	1	1	1
c_2	0.18	0.1714	0.213
k_2	0.4535	0.4535	0.4545

5. Conclusion and Suggestion

It was shown that the stability of machining process can be increased much more effectively by attaching dampers optimized using the procedure proposed in the paper than those by other methods. Under heavy loads, i.e., rough cutting, where the smooth cut surface are not necessarily required, and hence chatter is likely to occur, dampers against chatter are strongly recommended.

The design parameters of such dampers can be easily obtained from the chart in Fig. 7 once if the dynamic characteristics of machine tool structure are identified.

In most studies of forecast or adaptive control to avoid the machine tool chatter so far the cutting conditions such as feed and speed have been changed. Since reducing the feed or speed means the increase of production time, it is not desirable. Up to now the on-line control of dynamic damper against chatter which doesn't need change of cutting conditions could not be realized, because the on-line identification of the machine tool structure whose characteristics are time-dependent under actual working conditions were impossible. The instant spectrum analyzer, which uses the FFT algorithm and are recently available as a commercial product, can very accurately identify the natural frequencies and damping factors. The use of the spectrum analyzer mentioned above and the application of microcomputer to store and manipulate the optimum parameters of the damper would make it possible in near future to control the adjustible damper on-line so that the stability against chatter can be maximized without changing the speed and feed.

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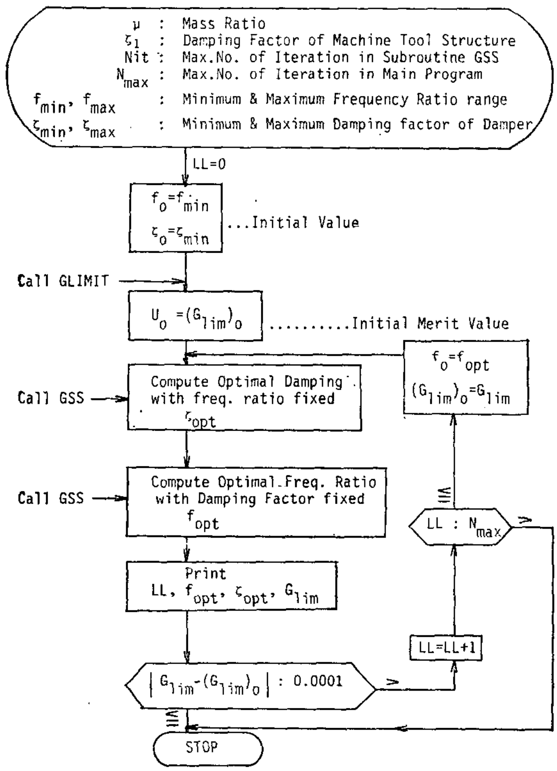
Appendix

Flow chart of main program



Flow chart of subroutine $GSS(x_{min}, x_{max}, y_0,$ Nit, Tol, x_{opt}, U_{opt}, Ind) : Golden section search program to optimize two parameters(damping factor& freq. ratio)

Ind=1	: Search for damping factor
Ind≠1	: Search for freq. ratio
Nit	: Max. No. of iteration
Tol	: Tolerence
x_{opt}	: Optimized variable
U_{opt}	: Optimum merit function