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A Study on the Synthesis of Robot Arms based on a Graphical Analysis of Accessible Region

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作業領域의 圖式的 解析에 의한 多關節 로봇팔의 綜合에 관한 研究

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抄 錄

本 論文에서는 多關節 平面 로봇팔의 作業領域 및 幾何學的 最適設計에 對하여 研究하였다.

關節이 두개인 平面 로봇팔의, 空間的 效率과 關係되는, 作業領域의 面積은 解析的으로 計算할 수 있으나, 세개 以上의 多關節 平面 로봇팔의 作業領域의 面積은 一般的으로 單純하게 解析的으로 解를 求하기가 매우 어렵다. 本 論文에서는 두개의 그래픽 RAM을 가진 마이크로 컴퓨터를 사용하여 作業領域의 面積을 近似的 圖式法으로 求하고 이것을 이용하여 미리 任意의 作業點들이 指定되었을 때 이들 作業點들을 通過, 포함하고, 또 機構學的으로 最適인 作業領域을 갖는 로봇팔을 設計하였다.

Nomenclature

A : Area of accessible region
A_i : Transformation matrix
T_i : Position of the i-th link in base coordinate
l_1 : Length of the first link
l_2 : Length of the second link
l_3 : Length of the third link
P_2 : Link ratio l_2/l_1
P_3 : Link ratio l_3/l_1
θ_1 : Angular displacement of the first link
θ_2 : Angular displacement of the second link
θ_3 : Angular displacement of the third link
O_1 : Center of revolution of the first link
O_2 : Center of revolution of the second link
O_3 : Center of revolution of the third link
x : X-coordinate of end position of robot arm
y : Y-coordinate of end position of robot arm
DX : X-coordinate of prespecified work point
DY : Y-coordinate of prespecified work point

1. Introduction

Researches on robot arms and manipulator may be devided into three parts. One is on the kinematic analysis and another is on the dynamic analysis and the other is control. This paper deals with the kinematic analysis of planar robot arms. The properties which depend upon the time duration of its movement are not considered; such properties can be properly treated from the viewpoint of dynamics and control.

The number of links and joints, as well as their type and sequence, determine the kinematic geometry of robot arm. One of the most important kinematic performance characteristics of robot or manipulator is the work region which covers the totality of

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positions.

There have been a few studies on this subject by B. Roth⁽¹⁾, L.A. Loeff⁽²⁾, A.H. Soni⁽³⁾, A. Kumar and K.J. Waldron⁽⁴⁾. Recently Y.C. Tsai and A.H. Soni⁽⁵,⁶⁾ studied the workspace and synthesis of two-link robot arms. They have limitations that the joint displacements should change from negative values to zero values first and they should change to positive values.

In this paper the work region and its area of three-link robot arms are studied using the graphical method. Following the analysis, syntheses are made for three-link planar robot arms.

2. Accessible Region of Planar Robot Arm

An industrial robot can be considered as a general-purpose manipulator that consists of several rigid bodies, called links, connected in series by joints. One end of the chain is free and equipped with a tool to manipulate objects or perform assembly tasks. Mechanically a robot is composed of an arm and a wrist subassembly plus a tool and is designed to reach a workplace located within its work region. The arm subassembly is the orientation mechanism. The accessible region is a space of influence for a robot whose arm can deliver the wrist subassembly unit to any point within the space.

2.1. Kinematic Equations for the Planar Robot Arm

Using the Denavit-Hartenberg representation of links, the position and orientation of the i-th link in base coordinates are given by the transformation matrix product.

$$T=A_1 A_2 \cdots\cdots A_i$$
$$=\prod_{i=j}^{i} A_i \qquad \text{for } i=1,2,\cdots\cdots n$$
$$=\begin{bmatrix} n_x & s_x & a_x & q_x \\ n_y & s_y & a_y & p_y \\ u_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where the transformation matrix A_i describes the position and orientation of the i-th link with respect to the $(i-1)$-th link⁽⁷⁾.

For a planar two-link robot arm, from Eq. (1), the followings can be written.

$$x=l_1 \sin\theta_1+l_2 \sin(\theta_1+\theta_2) \tag{2}$$
$$y=l_1 \cos\theta_1+l_2 \cos(\theta_1+\theta_2) \tag{3}$$

Eqs. (2), (3) describe the loci of circles. From these equations design charts for general accessible region can be constructed. The shape of the accessible region is a function of link proportions and limiting values of θ_1 and θ_2. The area of the accessible region can be derived analytically from Eqs. (2), (3) :

$$A=\iint dx\, dy$$
$$=\iint J\, d\theta_1 d\theta_2\cdot$$
$$=l_1 l_2(\theta_{1,\,\max}-\theta_{1,\,\min})$$
$$(\cos\theta_{2,\,\min}-\cos\theta_{2,\,\max}) \tag{4}$$

where Wronskian J is $J=l_1 l_2 \sin\theta_2$

For a planar three-link robot arm, from Eq. (1), the followings can written.

$$x=l_1 \sin\theta_1+l_2 \sin(\theta_1+\theta_2)+l_3 \sin(\theta_1+\theta_2+\theta_3) \tag{5}$$
$$y=l_1 \cos\theta_1+l_2 \cos(\theta_1+\theta_2)+l_3 \cos(\theta_1+\theta_2+\theta_3) \tag{6}$$

The accessible region of a three-link robot can be generated from that of a two-link robot. As shown in Fig. 1, the region $PQRR'P'P$ swept by rotating the accessible region of the two-link robot, $PQRP$, about the base point O_1 is the accessible region of the three-link robot. Though present study is restricted to the planar robot arm, it can be extended to the three-dimensional robot of which z-axes of

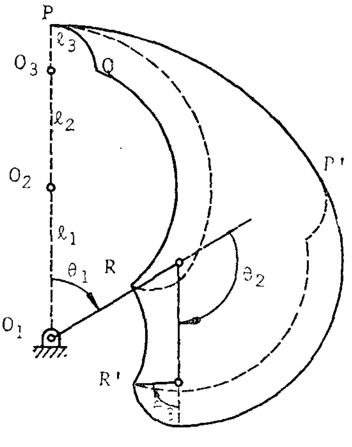


Fig. 1 Accessible region of three-link robot

joints are parallel each other. In other words the accessible region of the three-dimensional robot can be obtained by rotating the accessible region of planar robot about z-axis of base frame.

2.2. Calculation of the Accessible Region Area

In three-link robot the area covered under the accessible region cannot be obtained easily as in two-link robot arms. So the graphical method is used to obtain the area of the accessible region of three-link robot arms.

The shape of the accessible region depends on link dimension and joint angular displacement range. So if each link dimension and relative angular displacement are known, the accessible region of robot can be drawn.

In order to calculate the area of the accessible region in a computer, two graphic RAMs are needed. In this study MZ80B SHARP Microcomputer which has two graphic RAMs with resolution 320×200 dots and 64K memory is used.

Proposed procedure is as follows (Fig. 2 and 3 show the procedure graphically.).

(1) Assign the graphic input mode to graphic area 1.

(2) By rotating the end point P of the last link with radius O_3P through an angle $\theta_{3,\max}-\theta_{3,\min}$, a circular arc PQ is generated.

(3) Store the circular arc PQ in the graphic RAM-1

(4) Assign the graphic input mode to graphic area 2.

(5) Scan the graphic RAM-1 to see whether dots are set or reset. If ther is a set dot S, rotate the dot S with radius O_2S about the joint O_2 through an angle $\theta_{2,\max}-\theta_{2,\min}$. Then another circular arc SS' is generated.

(6) Store the circular arc SS' generated by step 5in the graphic RAM-2.

(7) Continue step 4-6 through all the dots along PQ in graphic RAM-2. Then the work region swept by two-link robot $PP'Q'Q$ is obtained.

(8) In a similar way repeat step 3-7 to rotate the region $PP'Q'Q$ about the joint O_1 and store the results in the graphic RAM-1. Then the accessible region of three-link robot, $PP''P'''Q''Q'QP$, is generated.

(9) Scanning both graphic RAM-1 and RAM-2, enumerate all the dots which are set. This is done by the POINT function which scans graphic areas to determine whether specified dots are set or reset.

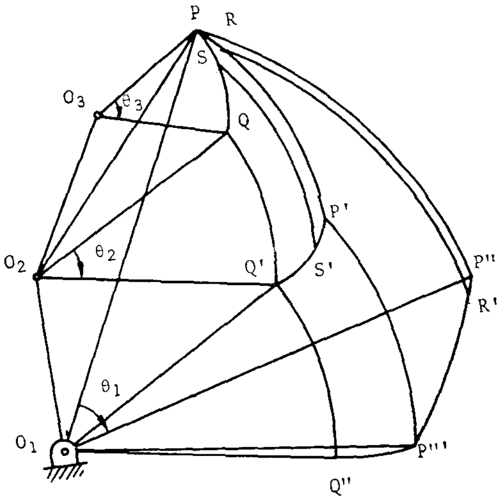
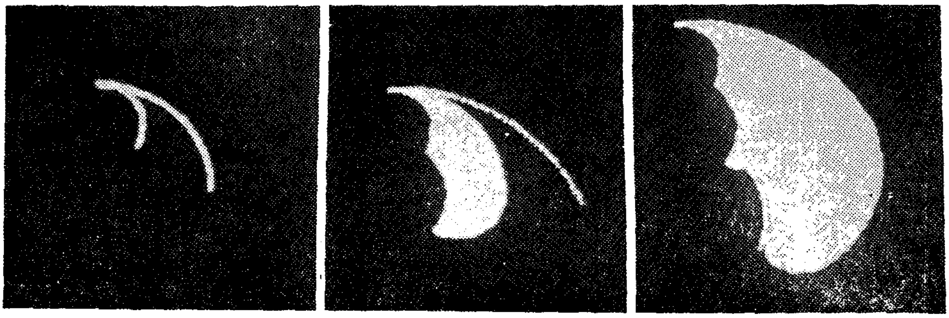


Fig. 2 Demonstration of the synthesis procedure



(a) Step 5 (b) Step 7 (c) Step 8

Fig. 3 Results displayed on CRT

(10) The number of dots which was set in graphic area 1 and 2 becomes equivalent area of accessible region.

3. Synthesis of Planar Robot Arms

A general statement for synthesis of planar robot arms may be related to covering a specified accessible region with optimum link ratios, and with suitable angular displacement of each link.

3. 1. Synthesis of Three-link Robot Arm

Synthesis of two-link robot was suggested previously by A.H. Soni(5). In this study similar procedure as proposed in two-link robot is used to synthesize three-link robot. But some modifications are required.

While link ratio $l_2/l_1=1$ maximizes the accessible region in two-link robot, it is not so in three-link robot. So link ratios among l_1, l_2, l_3 must be treated as unknown variables. Then mobility analysis can be made to determine link lengths. However as referred earlier, accessible region area of robot with more than three-link cannot be obtained in closed forms. So in this synthesis a graphical method is used to calculate the area of the accessible region of three-link robot. Furthermore, in three-link robot inverse kinematic equation has many solutions, while in two-link robot one solution. In other words the way for links to reach a specified work position is not unique but large in number as shown in Fig. 7. So a proper one is to be selected.

A three-link robot can be considered as a four-link closed mechanism as shown in Fig. 4, provided the end position of the last link is fixed at an arbitrary work position (x, y).

From Eqs. (5), (6) we cannot solve inverse kinematic equations, because there are only two equations for three unknowns. But if the angular displacement of the first link θ_1 is given as α, those of the other two links, θ_2 and θ_3 are fixed accordingly.

i.e.

If $\theta_1=\alpha$, then θ_3 lead to

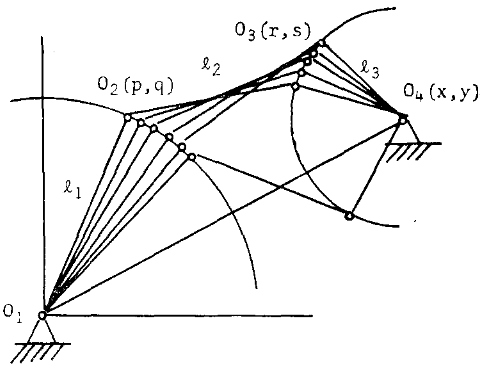


Fig. 4 Equivalent four-link closed mechanism

$$\theta_2=\cos^{-1}\left[\frac{y-q}{\sqrt{(x-p)^2+(y-q)^2}}\right]-\cos^{-1}\left[\frac{(x-p)^2+(y-q)^2+l_2{}^2+l_3{}^2}{2l_2\sqrt{(x-p)^2+(y-q)^2}}\right]-\alpha \quad (7)$$

$$\theta_3=\cos^{-1}\left[\frac{(x-p)^2+(y-q)^2-(l_2{}^2+l_3{}^2)}{2l_2l_3}\right] \quad (8)$$

where $p=l_1\cos\alpha$, $q=l_1\sin\alpha$

As the angle of the first link varies, a series of sets of angles are obtained. Among them a proper one is to be selected.: The set of joint angles are selected such that total sum of joints can be minimum. Here a set in which the sum of the absolute angular displacements for all the work position are found. Then the shape of the accessible region can be determined.

Proposed synthesis procedure is as follows.

(1) Assume base point(x_0, y_0) arbitrily.

(2) Calculate the length between the base point(x_0, y_0) and work points (x, y).

(3) Determine the total lengths of links as the maximum of the lengths obtained in step 2.

(4) Assume link proportions $P2=l_2/l_1$, $P3=l_3/l_1$ arbitrarily. Then lengths of l_1, l_2, l_3 are determined.

(5) Varying the angle θ_1, solve the inverse kinematic equations which satisfy mobility constraints.

(6) Find angular displacement range of each link for all the work points.

(7) Once length and angular displacement of each link is known, the shape of the accessible region is graphically determined and its area is also obtained.

(8) Use GRID4 search method(9) to find optimum values of x_0, y_0, $P2$, $P3$ such that the area of the

accessible region is minimum. The optimum values of x_0, y_0, l_1, l_2, l_3, θ_1, θ_2, θ_3 will provide the necessary data for the synthesized robot.

Fig. 5 shows the arrangement of programs to execute above procedures in a microcomputer with two graphic RAMs.

EXEC is the executive program which applies data to GRID4 and to MERIT4 and receives output information via MERIT4.

GRID4 is a subroutine which exercises a grid search in a merit hypersurface of up to eight dimensions by calling subroutine MERIT4, with the assistance of subroutines NORMAL and UNNORM and REGION.

MERIT4 is a subroutine of which the function is to generate the figure of merit ordinate, with the assistance of subroutines MOBIL, CIRGEN, BOUND, SCAN, and response to interrogations of GRID4 in its search for the extreme merit ordinate.

MOBIL is a subroutine which exercises mobility analysis of robot arms.

CIRGEN is a subroutine which generates circular arc using two graphic RAMS.

SCAN is a subroutine which scans the graphic areas to find whether specifiec dots are set or reset.

All the subroutine programs which were written in BASIC are listed in the reference (10).

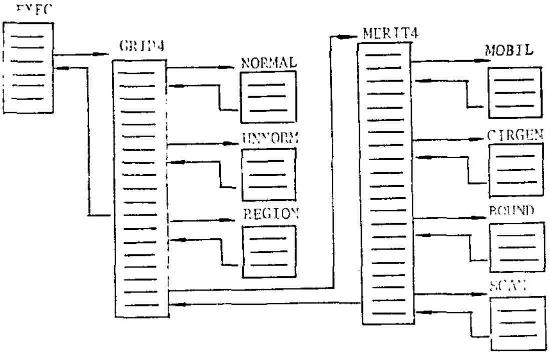


Fig. 5 *Arrangement of programs for synthesis*

4. Results and Discussions

Some examples are presented to show the synthesis of planar robot arms using the proposed method.

Example 1. Designa three-link robot of which accessible region is most close to the following prespecified working positions:

P_1(100.00, 15.00), P_2(110.00, 15.00), P_3(109.39, 10.61), P_4(105.00, 15.00), P_5(105.00, 30.00), P_6 (109.39, 55.61) P_7(130.00, 60.00), P_8(140.00, 60.00)

The following solution is obtained using the proposed synthesis method.

Location of robot base: (26.13, 49.46)

Link length: l_1=59.767, l_2=38.616, l_3=21.736

Relative angular displacement of joints for the prespecified working positions are listed in Table 1.

Table 1 Relative angular displacement of joints

N	DX(cm)	DY(cm)	θ_1(°)	θ_2(°)	θ_3(°)
1	100.00	15.00	67.929	88.512	15,408
2	110.00	15.00	71.602	75.891	15.273
3	109.39	10.61	75.273	73.454	16.764
4	105.00	15.00	69.765	81.287	17.983
5	105.00	30.00	56.914	87.001	19.843
6	109.39	55.61	40.391	83.618	20.744
7	130.00	60.00	55.078	52.347	16.402
8	140.00	60.00	67.929	27.398	17.214

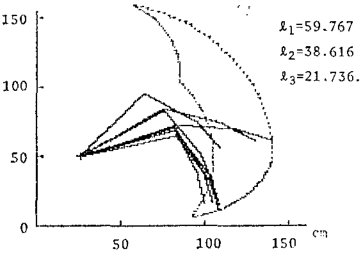


Fig. 6 Accessible region of example 1

Example 2. Find the location of the existing RHINO XR-1 Robot such that the robot can do the jobs at the specified positions with the work region being minimum. Since the link ratios are given, only difference between example 1 and 2 is in the number of design variables to be searched: the design variables to be searched are locations of the robot base and the merit function is the minimum work area which can barely cover the specified positions. The lengths of links l_1, l_2, l_3 are 23cm, 23cm, 10cm and specified work positions are as follows:

P_1(33.69, 74.15), P_2(70.90, 43.21), P_3(56.47, 2.42) P_4(41.35, 61.91), P_5(49.13, 32.55)

The location of robot and relative angular displacement are found as follows.

Location of robot base: (20.12, 20.46)

Table 2 Relative angular displacement of joints

N	DX(cm)	DY(cm)	θ_1(°)	θ_2(°)	θ_3(°)
1	33.69	74.15	10.00	0.50	22.09
2	70.90	43.21	59.00	9.14	8.33
3	56.47	2.42	63.00	82.03	19.47
4	41.35	61.91	10.00	9.47	87.31
5	49.13	32.55	10.00	81.92	76.37

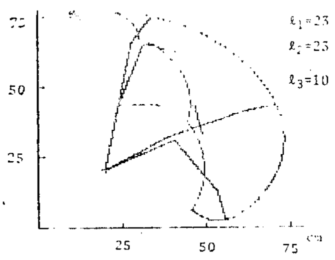


Fig. 7 Accessible region of example 2

This example shows that the present synthesis method can be applied to find the location of the existing robot so that it may perform the jobs at prespecified working positions more effectively.

As shown in previous examples, kinematically optimal design data of robot arm, such as location of base point and link dimension and relative angualr displacement of joint, can be obtained using proposed synthesis procedure. Though the object function of optimization is to make accessible region close to prespecified work region, it may be possible to select appropriate sets of joint angular displacements which can be used to avoid structural problems or limit positions of robot hand in response to a special design problem and pratice of design. However, it should also be considered that it is difficult for robot arm to move fast near to the boundary of accessible region in the view point of dynamics(11). So it is required to extend work region to some degree for pratical design. It is required to extend present investigations further to include the industrial robots which have various joint types and move in three-dimensional space. In addition to kinematic analysis, dynamic analysis is being studied presently by using inertia tensor concept to make this approach applicable to more pratical problems.

5. Conclusion

The present investigation deals with the study of determining accessible region for planar three-link robot arms using graphical method, because an analytical method suggested to two-link robot is not appropriate to three-link robot problems.

And here a synthesis procedure for three-link robot arm is developed utilizinga graphical method in a microcomputer and combining this with grid search technique. This enables to determine the kinematically optimal design data of three-link robot arm. To show the synthesis procedure, two examples are given for practical design of three-link robot arms.

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