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COMPACT OPERATORS AND TENSOR PRODUCTS IN (DF)SPACES

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This is a study of [weakly] compact linear operators with domains in (DF) spaces and ranges in (F) spaces, And we investigate the projective tensor product $E\otimes_\pi F$ and its completion $E\widetilde{\otimes}_\pi F$ when E and F are various weaker forms (ω-barrelled, C-barrelled and sequentially barrelled) of barrelled (DF) spaces and Banach-Mackey (DF) spaces.

The space of continuous linear operators from an locally convex space E into an locally convex space F is denoted by $\mathcal{L}(E,F)$. An operator $A\in\mathcal{L}(E,F)$ is called [weakly] compact, if there exists a zero neighborhood U in E such that $A(U)$ is [weakly] relatively compact in F. We obtain the following main results.

THEOREM 3.1. *Let E be a (DF) space and F an (F) space. Then $A\in\mathcal{L}(E,F)$ is compact if and only if its dual $A'\in\mathcal{L}(F'_b, E'_b)$ is compact.*

THEOREM 4.1. *Let E and F be (DF) spaces, Then*

(a) *if E and F are ω-barrelled, so are $E\otimes_\pi F$ and $E\widetilde{\otimes}_\pi F$*

(b) *if E and F are C-barrelled, so are $E\otimes_\pi F$ and $E\widetilde{\otimes}_\pi F$.*

(c) *if E and F are sequentially barrelled, so are $E\otimes_\pi F$ and $E\widetilde{\otimes}_\pi F$.*

(d) *if E and F are Banach-Mackey, so are $E\otimes_\pi F$ and $E\widetilde{\otimes}_\pi F$.*

THEOREM 4.2. *Let E and F be (DF) spaces. Then the following are equivalent.*

(a) *$E\otimes_\pi F$ (resp. $E\widetilde{\otimes}_\pi F$) is ω-barrelled.*

(b) *$E\otimes_\pi F$ (resp. $E\widetilde{\otimes}_\pi F$) is C-barrelled.*

(c) *$E\otimes_\pi F$ (resp. $E\widetilde{\otimes}_\pi F$) is sequentially barrelled.*

(d) *$E\otimes_\pi F$ (resp. $E\widetilde{\otimes}_\pi F$) is Banach-Mackey.*

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